Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University, Estd. u/s 3 of UGC Act 1956 Category 'A' by MHRD)
Re-accredited with 'A++' Grade by NAAC. Recognised by UGC Under Section 12 B
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – November 2024
III Semester

Class : II PG
Major : Mathematics

Time: 3 Hours
Max. Marks: 100

23MMAC14 Complex Analysis

Course Outcomes:

CO1 : Use Poisson formula and Mean-value property in Harmonic functions.
CO2: Expand Taylor's series and Laurent's series for a given function.
CO3: Convert various functions in to canonical product form.
CO4: Identify elliptic functions.
CO5: Apply We ierstrass functions in Brownianmotion.

Part A 10 x 1 = 10

Choose the Correct Answer

1. A constant multiple of a harmonic function is _________. CO1K1
   a. analytic b. constant c. harmonic d. conjugate harmonic

2. Two harmonic functions u and v is said to be conjugate harmonic if ___________. CO1K1
   a. $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}, \frac{\partial u}{\partial y}=\frac{-\partial v}{\partial x}$
   b. $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}, \frac{\partial u}{\partial y}=\frac{-\partial v}{\partial y}$
   c. $\frac{\partial u}{\partial x}=\frac{-\partial v}{\partial y}, \frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$
   d. $\frac{\partial u}{\partial x}=\frac{-\partial v}{\partial x}, \frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$

3. Taylor series is _________ in the largest disk about z_0 contained in Ω. CO2K2
   a. divergent
   b. convergent
   c. neither convergent nor divergent
   d. absolutely Convergent

4. The radius of convergence of the Taylor series is _________ the shortest distance from a point z_0 to the boundary of Ω. CO2K1
   a. $\geq$ b. $\leq$ c. $\neq$ d. $=$

5. Every function which is __________ in the whole plane is the quotient of two entire function. CO3K1
   a. analytic b. meromorphic c. harmonic d. conjugate harmonic

6. When n is a +ve integer, then $\gamma(n) =$ _______. CO3K2
   a. $n!$ b. $(n-1)!$ c. $(n+1)!$ d. n

7. The isolated singularities of $F(\zeta)$ are either removable singularities or poles, then F is a __________ function. CO4K1
   a. entire b. meromorphic c. rational d. analytic

8. A module with isolated points is said to be ________. CO4K1
   a. unimodular b. discrete c. canonical basis d. period module

9. The equation $\eta_1\omega_2 - \eta_2\omega_1 = 2\pi i$ is known as _________. CO5K1
   a. Differential equation
   b. Legendre's equation
   c. Weierstrass ρ-function
   d. Single-valued function

10. The value of $\begin{vmatrix} \rho(z) & \rho'(z) & 1 \\ \rho(u) & \rho'(u) & 1 \\ \rho(u+z) & -\rho'(u+z) & 1 \end{vmatrix} =$ _________. CO5K2
    a. 1 b. $\rho(z)$ c. $\rho(u)$ d. 0

Part B $5 \times 6 = 30$

Answer ALL questions

Each answer should not exceed 400 words or two pages

11. a. State and prove the maximum principle of harmonic function. CO1K3

(or)

11.b. Prove that $\int_\gamma u_1^* du_2 - u_2^* du_1 = 0$, where u_1 and u_2 are harmonic in a region Ω. CO1K4

12.a. State and prove Hurwitz's theorem. CO2K3

(or)

12.b Prove that Laurent development is unique. CO2K4

13. a Derive the necessary and sufficient condition for the absolute convergence of the infinite product $\prod_{n=1}^{\infty}(1+a_n)$ is, the convergence of the series. $\sum_{n=1}^{\infty}|a_n|$. CO3K4

(or)

13. b. Prove that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$. CO3K4

14. a. Prove that any two basis of the same module are connected by a unimodular Transformation. CO4K4

(or)

14.b. Prove that an elliptic function without poles is a constant. CO4K4

15. a. Show that any even elliptic function with periods w_1, w_2 can be expressed in the form $\prod_{k=1}^{\infty}\frac{\rho(z)-\rho(a_k)}{\rho(z)-\rho(b_k)}$ provided that ρ is neither a zero nor a pole. CO5K5

(or)

15.b. Prove that $\frac{\rho'(z)}{\rho(z)-\rho(u)} = \zeta(z-u)+\zeta(z+u)-2\zeta(z)$. CO5K4

Part C $5 \times 12 = 60$

Answer ALL questions

Each answer should not exceed 800 words or four pages

16. a. State and prove the mean value property. CO1K4

(or)

16.b. Derive the two forms of Poisson's formula. CO1K4

17.a. Derive the Laurent's series. CO2K4

(or)

17.b. Derive the Taylor's series. CO2K4

18.a. State and prove Mittag-Leffler theorem. CO3K4

(or)

18.b. Prove that $\frac{\pi^2}{sin\pi z} = \sum_{n=-\infty}^{\infty}\frac{1}{(z-n)^2}$. CO3K5

19.a. Prove that a discrete module consists of zero alone, of the integral multiples $n\omega$ of a single complex number $\omega \neq 0$, or of all linear combinations $\eta_1\omega_1 + \eta_2\omega_2$ with integral coefficients of two numbers ω_1, ω_2 with non real ratio $\omega_2|\omega_1$. CO4K5

(or)

19.b. Prove that the sum of the residues of an elliptic function is zero. CO4K4

20.a. Prove that $\rho(z) = \frac{1}{z^2} + \sum_{\omega\neq 0}\left(\frac{1}{(z-\omega)^2}-\frac{1}{\omega^2}\right)$. CO5K4

(or)

20.b. State and prove the addition theorem for the ρ-function. CO5K4

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