Economic Analysis of Continuous

Sampling Plan

Suganya,S

(12PMA019)

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore-641 043

In Partial Fulfilment of the Requirements for the

Degree of Master of Science in Mathematics

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Signature of the Supervisor

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Introduction

INTRODUCTION

The present scenario of global competition and the increasing awareness demand the assessment of the quality of the product in the every manufacturing industry. Analysis of the behavior of the production processes is essential for estimating the product quality in order to improve the quality as well as to compare their performance. Several sampling plans are available in statistical quality control to study the behavior of the product units derived from various production processes.

Among the available sampling plans continuous sampling plans are widely used to monitor product units submitted from continuous production processes. Continuous sampling plans find their applications in ammunition loading, manufacturing of photographic film, cloth and yarn, glass molding and also in man power scheduling. Continuous sampling plans are used when the production is continuous and the formation of inspection lots for inspection may not be feasible as in conveyorised production line. Dodge (1943) introduced the first continuous sampling plan known as CSP- 1 plan. In its simplest form continuous sampling inspection consists of alternating between sequences of screening and sampling inspection where the severity of inspection depends upon the discovery and spacing of defectivet units. The screening phase ends when a given number of consecutive product units are found to be defect free. Upon termination of the screening phase begin with a sampling phase wit stated frequency. Sampling inspection is terminated when a defective article occurs in inspection then 100% inspection will be reinstated. This procedure continuous indefinitely.

The continuous sampling plan of Dodge has been applied to monitor processes which satisfy the following assumptions:

(i) The probability of each article being created a defective, p is constant,

(ii) Infinite numbers of articles arrive for inspection in the order of production

(iii) Qualities of successive items of production are independent.

The cost minimization becomes a key issue in the area of modern industry. The economic approach in developing and designing the sampling plans is the need of the hour due to modern economic orientation. The introduction of functions evaluating losses and as a result costs are inevitable. Cost of designing a sampling plan plays a vital role in its administration. An economic approach of the continuous sampling plan became essential to study the behavior of plan in terms economy.Therefore the evaluation of the behavior of the CSP-1 plans in terms of economic costs based on the fundamental cost categories the internal and the external costs.

The basic costs involved in the implementation of a continuous sampling plan are the inspection cost, the replacement cost and the acceptance cost. Inspection cost expresses the labour cost of the inspector and the cost of using the inspection equipment and can also be considered constant. The acceptance cost expresses the cost of not inspecting a non-conforming item during the sampling phase and is related to the cost of the loss of the customer’s goodwill.

The cost of inspection includes the labour cost of the inspector and the cost of using the inspection equipment and other facilities and therefore not a constant. This motivated the researcher to study the designing of CSP-1 under linear inspection cost.

The acceptance cost expresses the cost of not inspecting a non-conforming item during the sampling phase and is related to the cost of the loss of the customer’s good will. This warranted designing of CSP-1 under linear acceptance cost.

These aspects motivated the researcher to study the economic analysis of continuous sampling plan.

Synopsis

SYNOPSIS

This dissertation traces the development of economic analysis of CSP-1 continuous sampling plan with the assumption that the number of articles arrive for inspection is infinite and the probability of each article being created defective is constant.

The relevant literature for the preparation of dissertation is given in Review of Literature.

Basic concepts include the terms, definitions, distributions and notations used in preparing this dissertation.

Chapter I presents the derivation of performance measures of continuous sampling plan CSP-1 when the product units are submitted to assure product quality, from a long run production process having the probability of producing a non-conforming item is constant. Performance measures are derived using Markov-chain approach. Its performance measures are numerically evaluated and analysed.

In Chapter II the economic performance of CSP-1, under specified outgoing quality limit (AOQL) and an assumption of linearly variable acceptance cost is considered. A mathematical programming model is developed to determine the unique combination of the parameters i and f of CSP-1 with minimum total cost.

Chapter III discusses the designing of an economically based CSP-1 continuous sampling plan under linear inspection cost. By assuming that the per unit inspection cost is linearly proportional to the average number of inspections per inspection cycle, by solving the modified Cassady et al.'s model, to arrive with the required level of product quality and the minimum total expected cost.

Flow chart and transition probability matrix are presented to explain the operating procedure and derivations of steady state probabilities. Construction of tables to facilitate the user in selection of plans for certain specified specifications is also carried out.

Results of the study are indicated in Summary and Conclusion. A list of Bibliography is added at the end.

Profile of the study

The following research articles are reviewed to prepare this dissertation.

i. Continuous sampling plan under an acceptance cost of linear form by Farmakis and Eleftheriou (2007).

ii. Economic design of continuous sampling plan under linear inspection cost by Chen and Chou (2002).

Review of Literature

REVIEW OF LITERATURE

A brief review of literature on designing continuous sampling plan CSP-1 using economic concept for monitoring the quality of constant production process is presented.

Dodge (1943) first devised a sampling inspection plan for continuous production called continuous sampling plan and designated it as CSP-1.In its simplest form continuous sampling inspection consists of alternating between sequence of screening and sampling inspection.

The CSP-1 plans proposed by Dodge have been applied to monitor processes which satisfy the following assumptions:

i. the probability of each article being created a defective ,p is constant,
ii. infinite numbers of articles arrive for inspection in the order of production and
iii. qualities of successive items of production are independent.

Dodge (1943) indicated how to calculate the long-run average fraction inspected (AFI) and the average outgoing quality limit (AOQL) for a CSP-1 plan.

Liberman (1953) derived an expression for an unrestricted AOQ under rectifying inspection and Enders (1969) derived an expression for the UAOQL under non-rectifying inspection for CSP-1 continuous sampling plan. Resnikoff (1960) have addressed the problem of achieving the minimum AFI for the CSP-1 plan.

Case et.al.(1973) discussed the evaluation of performance measures in the presence of inspection errors in CSP-1 plan. They developed a compensating sampling plan, considering inspection errors, which yield the desired actual average outgoing quality limit (AOQL). Enders (1979) discussed the minimum AFI plan in case of the occurrence of inspection error.

Wang and Chen (1997) further modified Enders's (1979) method and formulated a minimum average fraction inspection model under inspection errors.

Stephens (1979) suggested the suitability of continuous sampling plans and presented the review of various types of continuous sampling plans with their performance measures. Stephens (1981) provides the CSP-1 for consumer production. Govindaraju (1989) provided tables for the selection of CSP-1 plan for a given set of conditions (AOQ, AOQL) and (LQL, AOQL).

Yang (1983) derived performance of continuous sampling plan for short run production processes using renewal process approach and formulated CSP-1 plan as a discrete renewal process.

Kackar (1986) and Barker (1986) defined as a primal objective the determination of the ideal target values and the minimization of the total losses to society, both to the producer and the consumer. Ghosh (1988) explores the continuous sampling plan that minimizes the amount of inspection.

Liu and Aldag (1993) have presented the AOQ and AFI functions for short run CSP-1 plan. Shankar and Mohapatra (1994) suggested procedures and tables to find a unique combination of (i,f) that will achieve AOQL requirement and also minimizes the average amount of inspection when the process level p is known.

Haji and Haji (2004) employed a renewal reward process on CSP-1 to develop optimal policies in terms of the clearance number.

McShane and Turnbull (1991) examined the probability limits on outgoing quality for continuous sampling plans.Chen and Chou(2000) discusses the problem of determining cost optimal CSP-1 plan when quality fluctuates between two levels according to a two-state time homogeneous Markov chain following the McShane-Turnbull (1992) correlated input model.

Chen and Chou (2004) developed a procedure to find parameters (i,f) that will meet the AOQL requirement, while also minimizing the AFI for the CSP-1 plan when the process average p and AOQL are known. Chen (2005) presented how to calculate the outgoing quality limit for short-run CSP-1 plan based on numerical method.

The method of Cassady et. al. (2000) allows formulation of a mathematical programming model and determination of the parameter values that ensure minimization of the total expected cost per item produced. Cassady et.al. (2000) analysed Deming's kp rule using an economic model of the CSP-1.

Chen and Chou (2002) have derived the economic design of continuous sampling plan under linear inspection cost.

Lin and Yu (2009) determined the optimal inspection policy for minimizing the long run average unit cost. Yu et.al. (2009) investigated a mixed policy of precise inspection and CSP-1with inspection errors and return cost.

Farmakis and Eleftheriou (2007a) derived an economic model for continuous sampling plan and their performance. Eleftheriou and Farmakis (2007b) proposed the designing of CSP-1 under the assumption that the acceptance cost is linearly proportional to the average number of items not inspected in the sampling phase.

Farmakis and Eleftheriou (2009) examined the economic performance of CSP-1 under a specified AOQL with linearly varying acceptance cost. Eleftheriou and Farmakis (2009) studied the behavior of continuous sampling plan CSP-1 in terms of quadratically varying acceptance cost.

Basic concepts

BASIC CONCEPTS

Terms, definitions and notations employed in developing this dissertation are presented under the head Basic Concepts.

Acceptance Sampling

A methodology that deals with procedures by which decisions to accept or not accept are based on the result of the inspection of sample is acceptance sampling. According to Professor Dodge (1969), the major areas of acceptance sampling are:

i. Lot-by lot sampling by the method of attributes, in which each unit in a sample is inspected on a go-not-go basis for one or more characteristics
ii. Lot –by-lot sampling by the method of variable, in which each unit in a sample is measured for a single characteristic, such as weight or strength
iii. Continuous sampling of a flow of units in which each unit inspected by the method of attributes.

Moving Product

Moving product refers to product which is flowing fast at the inspection station. In the typical case the product moves on a conveyor belt or assembly line which are operated manually or by mobile materials handling equipments.

Conforming Unit

Conforming unit is one which meets the acceptance criteria established for the characteristic being considered.

Non-Conforming Unit

Non-conforming unit is one which does not meet the acceptance criteria established for the characteristic being considered.

Process Average

The percent defective of product submitted by the supplier for inspection. The phrases process average and percent defective of submitted product are used inter changeably.

Inspection

Inspection is the process of measuring, examining, testing or otherwise comparing the unit of product with the requirements.

100% Inspection

100% inspection means the inspection of every unit of product for the defects concerned listed for an inspection station .The two terms screening and 100% inspection are used inter - changeably.

Sampling Inspection

Sampling inspection means the inspection for the defects concerned only for randomly selected units.

Continuous Sampling Inspection

Continuous sampling inspection is the examination or testing of units of product as they move fast in an inspection station. Only those units of product found by the inspector or screening screw to be non-conforming are corrected or replaced with conforming units. The rest of the units are allowed to continue down the production line as conforming material.

Continuous Sampling Plan (CSP)

Continuous sampling plan is a plan intended for continuous flow of individual units of product that involves acceptance or non-acceptance on a unit-by-unit basis and uses alternate periods of 100 percent inspection and sampling inspection depending on the quality of the observed product.

Single- Level - Continuous Sampling

Single-level continuous sampling is a sampling inspection of consecutively produced units in which fixed sampling rate alternates with 100% inspection depending on the quality of the observed product.

Clearance Number

The number of consecutive conforming units required to adjust inspection action in continuous sampling. It is denoted by i.

Sampling Frequency

The desired ratio between the number of units of production and product units randomly selected and inspected at an inspection station during the sampling inspection. The procedure used in selecting the sampling units given each unit of product presented during periods of sampling inspection an equal chance of being selected and inspected. It is denoted by f.

Process Quality

A statistical measure to portray the quality of product from a given process is known as process quality .The most commonly used measure of process quality is the percentage or proportion of non-conforming units in the process. It is also known as process average.

Acceptable Quality Level

The maximum percentage or proportion of variant units in a lot or batch that , for the purpose of acceptance sampling can be considered satisfactory as a process average. It is denoted as AQL.

Limiting Quality Level

The percentage or proportion of variant units in a batch or lot which , for the purpose of acceptance sampling the consumer wishes the probability of acceptance to be restricted to a specific small value. It is denoted as LQL.

Producers Risk

For a given sample plan, the probability of not accepting a lot quality of which has a designated numerical value representing a level which it is generally desired to accept. It is represented by α.

Consumer Risk

For a given sample plan, the probability of acceptance of a lot the quality of which has a designated numerical value representing a level which it is seldom desired to accept. It is represented by β.

Operating Characteristic (OC) Curve

Associated with each sampling plan there is an OC curve which portrays the performance of the sampling plan against good and poor quality. OC curve shows graphically the interrelationship of risks, probability and qualities of a given sampling plan.

Average Outgoing Quality

The expected quality of outgoing product, following the use of an acceptance sampling plan for given value of incoming product quality. It is represented by AOQ. Wortham and Mogg (1970) have given expressions for AOQ under nine different policies adopted for single sampling plans. AOQ is approximated by $p.P_a$ (p).

Average Outgoing Quality Limit

For a given sampling plan, Average Outgoing Quality Limit is defined as the maximum of AOQ over all possible levels of incoming quality. It is represented by AOQL.

Average Fraction of Inspection

Fraction of a product that will be inspected over the long run if the process average is stable. It is denoted as AFI.

These definitions are from American National Standard Institute/ American Society for Quality Control (ANSI/ASQC) standard A_2.

Markov – Chain

A process $\{X_n\}$, n=0, 1,2,………. is called a Markov-chain if for all n,

$$P\,[X_n = a_n | X_{n-1}=a_{n-1}, X_{n-2}=a_{n-2},\ldots\ldots\ldots,X_0=a_0] = p[X_n=a_n|X_{n-1}=a_{n-1}]$$

Transition Matrix

A more complete characterization of the one-step transitions of a Markov-chain with their corresponding probabilities provided in a matrix form is called the transition matrix.

Stable Process

Manufacturing process which produces defective units with constant probability is known as stable process.

Glossary of symbols used

The following is the list of symbols which are used in this CSP-1plan.

p	-lot quality or process quality
q	-1-p
i	-clearance number of screening inspection
f	-sampling frequency
c	-acceptance number
$P(S_m)$	-steady-state probability for the state S_m
p_{ij}	- the probability that the process transits from state s_i to state s_j in one step

P_a (p)	- probability of acceptance during the sampling phase when the submitted product is of quality p
α	- producers risk
β	- consumers risk
AOQ (p)	- average outgoing quality when the submitted product is of quality p
AOQL	- average outgoing quality limit
p_L	-the value of p for which AOQL is attained
U	-the expected number of units inspected during the period of screening inspection
V	-the expected number of units inspected during the periods of sampling inspection
F	-the average fraction of units inspected in the long run
C_s	-inspection cost per item
C_r	-replacement cost per item
C_a	-acceptance cost per item

Chapter - I

CHAPTER I

Continuous sampling plan - CSP-1

This chapter presents the operating procedure and conditions of application of CSP-1.Performance measures of CSP-1 plan such as average fraction inspected, average outgoing quality, the operating characteristic function, average number of units inspected during screening and sampling inspection are derived using Markov-chain approach.

Conditions of Applications

i. There is a continuous flow of units from the production process and units are offered for inspection one by one in the order of production.
ii. Process quality level is stable for a process producing materials where the probability of article being created defective is constant.

Operating Procedure of CSP-1

i. At the outset, inspect 100% of the units consecutively as produced and continue such inspection until i units in succession are found free of defects.
ii. When i units in succession are found free of defects, discontinue 100% inspection and inspect only a fraction f of the units selecting individual sample units one at a time from the flow of product in such a manner as to assure an unbiased sample.
iii. If a sample unit is found defective, revert immediately to a 100% inspection of succeeding units and continue until again i units in succession are found free of defects.
iv. Correct or replace all defective units found with good units.

CSP-1 is characterized by two parameters i and f where i is an integer and f is a fraction, the parameter i is related to screening inspection and the parameter f is related to sampling inspection. The operating procedure of CSP- 1 is given in Fig. 1.1

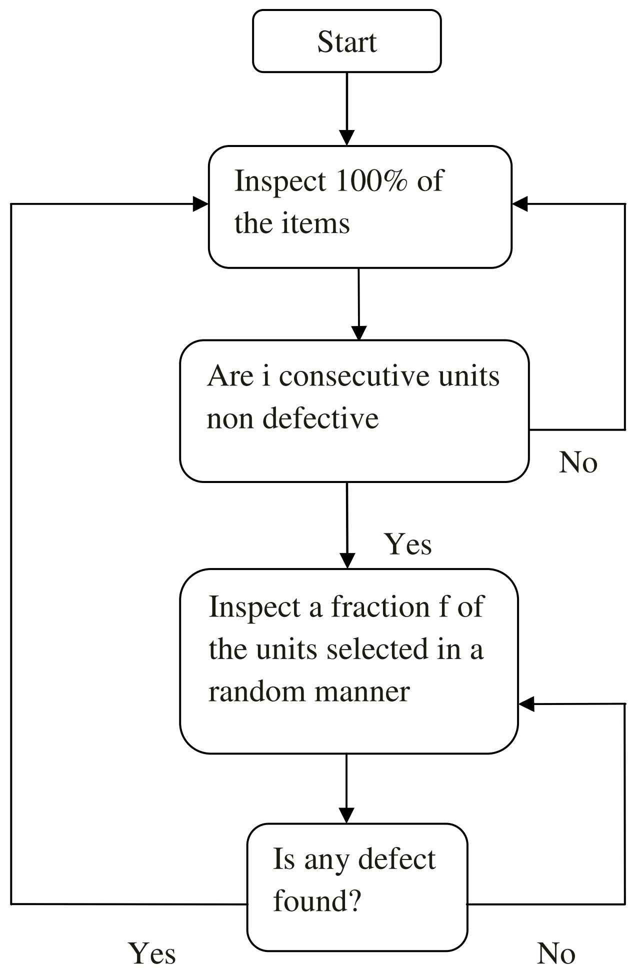

Fig.1.1 Operating procedure of CSP-1

Procedure of CSP-1 plan as a Markov-chain

Let $[X_n]$ (n=1,2,……) denote a discrete parameter Markov- chain with finite state space (S_K) (k=1,2,……..,i+4).The states of the process are defined as

$S_k = H(k-1)$ (k=1,2,………,i+1)

=100% inspection is being conducted and including the latest article inspected the last (k-1) consecutive articles were non-defectives.

$S_{i+2} =$ SD

= sampling inspection is in effect and the last article submitted was inspected and found to be defective.

S_{i+3} = SND

= sampling inspection is in effect and the last article submitted was inspected and found to be non- defective.

S_{i+4} = SN

= sampling inspection is in effect and the last article submitted for inspection was not inspected.

The set of (i+4) states defined above completely describes the mutually exclusive phases of inspection for the CSP-1 plan. The CSP-1 procedure applied to a process which is in statistical control can be viewed as discrete parameter, finite, recurrent, irreducible, aperiodic Markov chain. States and transitions of the CSP-1 procedure are shown in Fig .1.2.

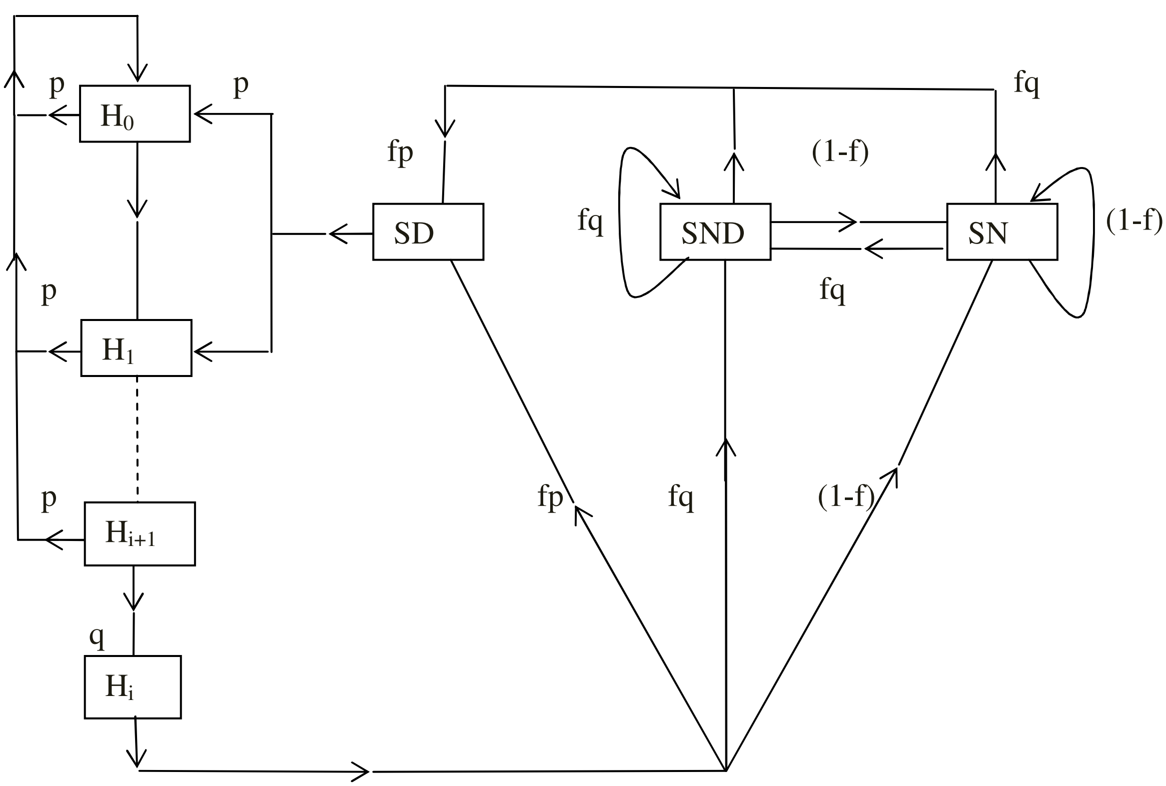

Fig. 1.2 States and transitions of the CSP-1 plan

The transitional matrix defining the transition probabilities is presented for the CSP-1 plan in Table 1.1.

Table 1.1 Transition probability matrix of CSP -1 plan.

i^{th} state

(i-1)th state

	H_0	H_1	H_2	.	.	H_{i-1}	H_i	SD	SND	SN
H_0	p	q	0	.	.	0	0	0	0	0
H_1	p	0	q	.	.	0	0	0	0	0
H_2	p	0	0	.	.	0	0	0	0	0
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
H_{i-1}	p	0	0	.	.	0	q	0	0	0
H_i	0	0	0	.	.	0	0	fp	fq	1-f
SD	p	q	0	.	.	0	0	0	0	0
SND	0	0	0	.	.	0	0	fp	fq	1-f
SN	0	0	0	.	.	0	0	fp	fq	1-f

The vectors of limiting probabilities are

$$\pi_{H0} = p(\pi_{H0} + \pi_{H1} + \pi_{H2} + ... + \pi_{Hi-1} + \pi_{HD})$$

$$\pi_{H1} = q(\pi_{H0} + \pi_{D})$$

$$\left.\begin{array}{l} \pi_{H2} = q\pi_{H1} \\ \pi_{H3} = q\pi_{H2}... \\ \pi_{Hi-1} = q\pi_{Hi-2} \\ \pi_{Hi} = q\pi_{Hi-1} \end{array}\right\} \quad (1.1)$$

$$\pi_{D} = fp\pi_{Hi} + fp\pi_{ND} + fp\pi_{S} \quad (1.2)$$

$$\pi_{ND} = fp\pi_{Hi} + fp\pi_{ND} + fp\pi_{S} \quad (1.3)$$

$$\pi_D = (1-f)\pi_{Hi} + (1-f)\pi_{ND} + (1-f)\pi_S \tag{1.4}$$

From (1.1)

$$\pi_{Hi} = q^{i-1}\pi_{H1} \tag{1.5}$$

From (1.2), (1.3) and (1.4)

$$\pi_D = \pi_{Hi} \tag{1.6}$$

$$\pi_{ND} = q\pi_{Hi} /(1-q) \tag{1.7}$$

$$\pi_S = (1-f)/ f(1-q)\pi_{Hi} \tag{1.8}$$

The sum of the steady state probabilities is unity. Therefore the steady state probabilities are,

$$\pi_{H0} + \pi_{H1} + \pi_{H2} + ... + \pi_{Hi-1} + \pi_{Hi} + \pi_D + \pi_S = 1$$

$$\left.\begin{aligned} \pi_{Hk} &= fpq^k /(f + q^k(1-f)), k = 1,2,....i \\ \pi_{H0} &= fp(1-q^i)/(f + q^i(1-f)) \\ \pi_D &= fpq^i /(f + q^i(1-f)) \\ \pi_{ND} &= fq^{i+1} /(f + q^i(1-f)) \\ \pi_S &= q^i(1-f)/(f + q^i(1-f)) \end{aligned}\right\} \tag{1.9}$$

Performance Measures

Performance measures of CSP-1 are

i. The average number of units inspected under the screening inspection is

$$U = \frac{(1-q^i)}{(pq^i)}$$

ii. The average number of units passed under the sampling inspection is

$$V = \frac{1}{fp}$$

iii. The average fraction of units inspected in the long run is

$$F = \frac{U + fV}{U+V} = \frac{f}{(f+q^i(1-f))}$$

iv. The average fraction of units accepted on a sampling basis is

$$P_a(p) = \frac{V}{(U+V)} = \frac{q^i}{(f+q^i(1-f))}$$

v. The average outgoing quality is

$$AOQ\ (p) = p\ (1\text{-}F) = \frac{pq^i(1-f)}{f+q^i(1-f)}$$

Numerical Results

The efficiency of the CSP-1 plan is evaluated by means of the performance measures on the basis of numerical values with respect to change in parameters of i, f, and p.

Effect of change of i on OC, AOQ, U, V are determined and presented in Table 1.2 for fixed f and p.

Table 1.2 indicates that,

i. increase in p, increases the value of U, AFI, AOQ, and decreases the value of V and $P_a(p)$ for fixed i and f .

ii. increase in i, increases the value of U, AFI and decreases the value of $P_a(p)$, AOQ for fixed p and f.

iii. increase in i, does not alter the value of V for fixed p and f and

iv. the AOQ value of increases to a certain limit and it tends to decreases.

Effect of change of f for fixed i and p on OC, AOQ, AFI values of CSP-1 are obtained and presented in Table 1.3.

Numerical values Table 1.3 reveal that

i. increases in p, increases the value of U, AFI, AOQ and decreases the value of V, $P_a(p)$ for fixed i and f

ii. for varying f, the value of AFI increases and the value of V, $P_a(p)$ and AOQ decreases for fixed i and p and

iii. the value of AOQ increases to a certain point and then it tends to decreases.

Table 1.2 Effect of i on performance measures of CSP-1

	i =20 , f=1/3					i =100,f=1/3				
P	U	V	AFI	$P_a(p)$	AOQ	U	V	AFI	$P_a(p)$	AOQ
0.001	20.2244	3000	0.33779	0.99331	0.0006617	105.2369	3000	0.35592	0.96611	0.0006439
0.005	21.0898	600	0.35597	0.96605	0.0032197	130.1603	600	0.45217	0.82173	0.0027390
0.010	22.2634	300	0.37938	0.93091	0.0062060	173.1999	300	0.57734	0.63398	0.0042265
0.015	23.5286	200	0.40350	0.89473	0.0089473	235.5058	200	0.69385	0.45921	0.0045919
0.020	24.8896	150	0.42822	0.85766	0.0114354	327.0184	150	0.79036	0.31445	0.0041925
0.025	26.3695	120	0.45343	0.81984	0.0136639	463.0358	120	0.86278	0.20581	0.0034300
0.030	27.9644	100	0.47902	0.78146	0.0156292	667.6462	100	0.91315	0.13026	0.0026052
0.035	29.6906	85	0.50484	0.74272	0.0173302	978.8269	85	0.99057	0.08428	0.0019663
0.040	31.5609	75	0.53078	0.83273	0.0187685	1456.8942	75	0.96736	0.04895	0.0013053
0.045	33.5888	66	0.55668	0.66496	0.0199488	2198.2902	66	0.98037	0.02943	0.0008829
0.050	35.7903	60	0.58242	0.62636	0.0208789	3358.0930	60	0.98829	0.01755	0.0005849
0.055	37.0702	54	0.60784	0.58823	0.0215685	5187.3318	54	0.99306	0.01040	0.0003813
0.060	40.7835	50	0.63282	0.55076	0.0220304	8094.4228	50	0.99590	0.00613	0.0002452
0.065	43.6157	46	0.65724	0.51413	0.0222791	12747.6943	46	0.99759	0.00360	0.0001562
0.070	46.7021	42	0.68095	0.47852	0.0223308	20246.2649	42	0.99859	0.00211	0.0000985
0.075	49.6439	40	0.70215	0.44298	0.0221489	32404.2045	40	0.99917	0.00123	0.0000614
0.080	53.7450	37	0.72601	0.41098	0.0219189	52245.0250	37	0.99941	0.00071	0.0000380
0.085	57.7632	35	0.75476	0.38313	0.0217109	84809.4745	35	0.99972	0.00041	0.0000233
0.090	62.1594	33	0.76728	0.34906	0.0209437	138704.3834	33	0.99983	0.00024	0.0000143
0.095	66.9742	31	0.78638	0.32042	0.0202934	227831.8068	31	0.99990	0.00072	0.0000086
0.100	72.2526	30	0.80440	0.29339	0.0195593	377348.4906	30	0.99994	0.00007	0.0000050

Table 1.3 Effect of f on performance measures of CSP-1

	i=50,f=1/3					i=50, f =1/2				i=50, f =2/3			
p	U	V	AFI	$P_a(p)$	AOQ	V	AFI	$P_a(p)$	AOQ	V	AFI	$P_a(p)$	AOQ
0.001	51.2974	3000	0.34454	0.98318	0.0006554	2000	0.51250	0.97499	0.0004879	1500	0.67769	0.96693	0.0003223
0.005	56.2974	600	0.39114	0.98318	0.0030442	400	0.56233	0.87533	0.0021883	300	0.71986	0.84041	0.0014006
0.010	65.2876	300	0.45248	0.82127	0.0054750	200	0.62305	0.75389	0.0037694	150	0.76775	0.69674	0.0023224
0.015	75.2708	200	0.51562	0.72655	0.0046969	133	0.68041	0.63918	0.0047937	100	0.80981	0.57054	0.0028527
0.020	87.2986	150	0.57859	0.63211	0.0084280	100	0.73304	0.53390	0.0053390	75	0.84596	0.46204	0.0030807
0.025	101.8499	120	0.63939	0.63939	0.0090150	80	0.73304	0.43992	0.0054990	60	0.87642	0.37071	0.0030892
0.030	119.5260	100	0.69631	0.45552	0.0091105	66	0.82097	0.35805	0.0053707	50	0.90168	0.29494	0.0029493
0.035	141.0834	85	0.74804	0.37793	0.0088184	57	0.85586	0.28827	0.0050447	42	0.92233	0.23299	0.0027182
0.040	141.0834	75	0.79379	0.30930	0.0082482	50	0.88504	0.22990	0.0045981	37	0.93901	0.18294	0.0024392
0.045	199.9138	66	0.83327	0.25008	0.0075042	44	0.90905	0.18188	0.0040923	33	0.95236	0.14291	0.0021436
0.050	239.9262	60	0.86663	0.20004	0.0066683	40	0.92855	0.14289	0.0035723	30	0.96295	0.11114	0.0018523

Chapter - II

CHAPTER II

ECONOMIC DESIGN OF CSP-1 PLAN WITH LINEAR ACCEPTANCE COST

In this chapter the economic design of CSP-1 under linear acceptance cost is presented, under specified outgoing quality limit (AOQL) and a realistic assumption of linearly varying acceptance cost is considered. A mathematical programming model is developed to determine the unique combination of the plan's parameters (i, f) having minimum total cost.

A continuous sampling plan by Dodge (1943) involves the alternating periods of screening inspection and sampling inspection with the assumption that the manufacturing process is statistically controlled. The modern approach to continuous sampling plan is characterized by economic objectives.

The basic costs involved in the implementation of a continuous sampling plan are the inspection cost, the replacement cost and the acceptance cost. Inspection cost expresses the labour cost of the inspector and the cost of using the inspection equipment and can be considered constant (per unit) during a quality control process. The replacement cost reflects the cost of replacing an item found non-conforming and can also be considered constant. The acceptance cost expresses the cost of not inspecting a non-conforming item during the sampling phase and is related to the cost of the loss of the customer's goodwill.

The total expected cost of implementing CSP-1 per unit produced during one inspection cycle is dependent on the following three costs:

The fixed cost of inspecting an item is c_s . Let c_s be the random variable expressing the inspection cost per item produced. It is easily derived that

$$E(c_s) = c_s \, AFI(p),$$

where AFI (p) is the long run average proportion of items inspected .

The fixed cost of replacing an item that has been found to be non-conforming is c_r. This cost includes the cost of producing the replacement item, the inventory and obsolescence costs associated with storing the replacement items, and the costs of reworking or disposing of the found non-conforming item.

Let c_a be the random variable expressing the replacement cost per item produced .Obviously,

$E(c_r) = c_r\, AFI(p)p.$

The cost of accepting (without inspection) non-conforming item during the sampling phase includes warranty costs, service costs, liability costs and the costs of the loss of the customer's goodwill.

Let c_a be the random variable expressing the acceptance cost per item produced .Then

$$E(c_a) = c_a[1\text{-}AFI(p)]p$$

The relationship among the three costs is

$$c_s \leq c_r \leq c_a \qquad (2.1)$$

The values of c_r and c_s are usually fixed and basically dependent on the kind of products, the product features or the consumer's demands and the structure of the production line. The value of c_a however cannot be considered constant because it is composed by additional fluctuating costs reflecting the variable consumer's loss of goodwill (dependent on the quantity of defective items encountered) the cost of withdrawing defective items from the market and the cost of preserving the market's tolerance .Therefore the economic design of a continuous sampling plan is more realistic if it involved a variable acceptance cost

This economic design is expressed through the following assumptions which are also considered as fundamental for formulating a mathematical programming model

i. The cost of inspection of an item and replacement of an item are considered constant.

ii. The cost of accepting a non-conforming item found during the sampling phase is linearly proportional to the average number of items not inspected in the sampling phase.

iii. The production process is under statistical control the probability of an item to be non-conforming is constant and is equal to p.

iv. The inspection is perfect, and

v. Non-conforming items are repaired or replaced,

The total expected cost is then given by,

$$E(c) = E(c_s)+E(c_a)+E(c_r) \tag{2.2}$$

If U is the expected number of items inspected during the 100% inspection phase and V is the expected number of items passing during the sampling phase of an inspection cycle of CSP-1 plan then,

$$U = \frac{1-q^i}{pq^i} \tag{2.3}$$

$$V = \frac{1}{fp} \tag{2.4}$$

where $q = 1-p$

Average fraction inspected is

$$AFI(p) = \frac{U + fV}{U + V}$$

$$= \frac{f}{f + q^i(1-f)} \tag{2.5}$$

The total expected cost per item is

$$E(C) = c_s AFI + c_a p(1-AFI) + c_r pAFI$$

$$= \frac{c_s f + c_a (1-f)pq^i + c_r fp}{f + (1-f)q^i} \tag{2.6}$$

However according to the second assumption, the acceptance cost is variable and is given by

$$C_a = \lambda + \mu(1-f)Vp \tag{2.7}$$

where λ is constant, the fixed portion of acceptance cost and μ is constant, a variable portion of acceptance cost .

The analytical form of the total expected cost per item produced, given that the acceptance cost is not constant, is

$$E(C) = \frac{c_s f^2 + c_r f^2 p + (1-f)pq^i((1-f)\mu + \lambda f}{f(q^i(1-f) + f)} \tag{2.8}$$

where, c_s – the inspection cost per unit produced,

c_a – the cost of accepting a non-conforming unit,

c_r – the cost of replacing a non-conforming unit found during inspection,

AFI – the average fraction inspection,

i – the clearance number of 100% inspection stage,

f – the sampling frequency of the sampling inspection stage,

p – the incoming proportion defective,

U – the expected number of items inspected during 100% inspection phase,

V – the expected number of items passing during the sampling phase,

λ - the fixed portion of acceptance cost,

μ - the variable portion of acceptance cost,

q – 1-p.

Minimization of the total expected cost reduces to the non-linear programming model as,

Minimize

$$E(C) = \frac{c_s f^2 + c_r f^2 p + (1-f^2) pq^i \mu + f(1-f) pq^i \lambda}{f(q^i(1-f)+f)} \qquad (2.9)$$

subject to

$$\max_{0 \le p \le 1} (AOQ) = AOQL$$

$i \ge 0$, an integer

$0 \le f \le 1$, a fraction

where AOQ is the average outgoing quality and AOQL is specified by the consumer which is maximum value of AOQ. Dodge (1943) for CSP-1 plan derived that

$$p_l = \frac{iAOQL + 1}{i+1} \qquad (2.10)$$

$$f = \frac{(1-p_l)^{i+1}}{iAOQL + (1-p_l)^{i+1}} \qquad (2.11)$$

where p_l is the value of p where AOQL is achieved.

Therefore, equation (2.11) gives the value of f (for a given value of i) for which AOQL is achieved. Thus the mathematical programming model (2.9) of CSP-1 reduces to

Minimize

$$E(C) = \frac{c_s f^2 + c_r f^2 p + (1-f)^2 pq^i \mu + f(1-f)pq^i - 1}{f(q^i(1-f)+f)} \qquad (2.12)$$

subject to

$$f = \frac{(1-\frac{iAOQL+1}{i+1})^{i+1}}{iAOQL + (1-\frac{iAOQL+1}{i+1})^{i+1}}$$

i≥0 , an integer

0≤f≤1, a fraction

For given values of i , and using the constraint of the model (2.12), a unique combination of (i, f) can be obtained that ensures minimum E(c) .

Numerical Example

Numerical illustrations are provided to explain the effect of the cost parameters on the minimum cost. The probable values of the parameters λ and μ are chosen to ensure that

i. the value of c_a is greater than the inspection and replacing costs ,
ii. the value of c_a is at most five times the replacing cost,

For example, for c_s=1 ,c_r=20, AOQL= 0.1%, and p=0.25% , a combination of λ and μ that ensures relation (2.1) and at the same time total minimum cost is $\lambda = 1, \mu = 10, i = 551, f = 0.277$ and the acceptance cost per unit produced is 27.043.

The effect of the incoming fraction defective (p)

Table 2.1 is obtained for minimum expected cost for various values of incoming quality p, and assumed parameters $\lambda = 1, \mu = 8$, c_s=1, c_r=20 with AOQL=0.1%.

Table 2.1 gives the AFI, E(c),c_a values for various incoming fraction defective.

Table values indicate that

i. i decreases as p increases,

ii. f increases as p increases,

iii. AFI increases as p increases,

iv. E(C) increases with increase in p and

v. acceptance cost c_a decreases as p increases.

These results indicate that as p increases it is more likely that 100% inspection will be re-instated.

The effect of the variable portion of the acceptance cost (μ)

Table 2.2 is constructed to study the effect of variable portion of acceptance cost on AFI, E(C), and c_a.

Table values show that

i. i decreases for increase in μ,

ii. f increases for increase in μ,

iii. AFI increases for increase in μ,

iv. E(c) increases for increase in μ and

iv. C_a increases for increase in μ

The effect of screening cost (c_s)

Table 2.3 is obtained to study the effect of screening cost on i, f, AFI,E(c) and c_a.

Table values indicates that

Table 2.1 Results for CSP-1 under different values of incoming quality, p ($\lambda = 1, \mu = 8$,c_s=1,c_r=20,AOQL=0.1%)

p	i	f	AFI	E(c)	c_a
0.0020	752	0.1871	0.5091	0.5646	35.7599
0.0021	709	0.2031	0.5308	0.5850	32.3935
0.0022	670	0.2190	0.5508	0.6042	29.5326
0.0023	633	0.2354	0.5695	0.6224	26.9783
0.0024	600	0.2514	0.5867	0.6395	24.8253
0.0025	569	0.2675	0.6028	0.6556	22.9053
0.0026	541	0.2832	0.6176	0.6709	21.2529
0.0027	515	0.2987	0.6315	0.6853	19.7854
0.0028	491	0.3139	0.6445	0.6990	18.4859
0.0029	469	0.3287	0.6565	0.7119	17.3396
0.0030	449	0.3429	0.6678	0.7242	16.3336
0.0031	430	0.3570	0.6785	0.7359	15.4089
0.0032	413	0.3703	0.6884	0.7470	14.6065
0.0033	397	0.3833	0.6978	0.7576	13.8723
0.0034	382	0.3960	0.7066	0.7678	13.2021
0.0035	368	0.4083	0.7149	0.7775	12.5920
0.0036	355	0.4202	0.7228	0.7868	12.0385
0.0037	343	0.4315	0.7302	0.7958	11.5386
0.0038	332	0.4422	0.7373	0.8044	11.0895
0.0039	321	0.4533	0.7440	0.8127	10.6489
0.0040	311	0.4636	0.7504	0.8207	10.2557

Table 2.2 Results for CSP-1 under different values of $\mu, (\lambda = 1, c_s=1, c_r=20, AOQL=0.1\%, p=0.25\%)$

μ	i	f	AFI	E(c)	c_a
1	650	0.2277	0.6000	0.6345	4.3916
2	636	0.2341	0.6002	0.6378	7.5450
3	623	0.2401	0.6005	0.6410	10.4927
4	611	0.2459	0.6008	0.6441	13.2651
5	600	0.2514	0.6012	0.6471	15.8908
6	589	0.2569	0.6017	0.6500	18.3497
7	579	0.2622	0.6022	0.6529	20.6998
8	569	0.2675	0.6028	0.6556	22.9053
9	560	0.2724	0.6033	0.6583	25.0366
10	551	0.2774	0.6040	0.6609	27.0429
11	543	0.2820	0.6046	0.6635	29.0066
12	535	0.2866	0.6053	0.6660	30.8630
13	528	0.2908	0.6059	0.6684	32.7059
14	520	0.2956	0.6067	0.6708	34.3598
15	513	0.2999	0.6074	0.6731	36.0159
16	507	0.3036	0.6080	0.6754	37.6929
17	500	0.3081	0.6088	0.6776	39.1801
18	494	0.3119	0.6096	0.6798	40.7024
19	488	0.3159	0.6103	0.6819	42.1518
20	482	0.3198	0.6111	0.6840	43.5296

i. in increasing c_s i increases,

ii. in increasing c_s f decreases,

iii. in increasing c_s AFI decreases,

iv. in increasing c_s E(c) increases and

v. in increasing c_s C_a increases

Numerical values provide the intuition that as the screening cost c_s increases, the i increases and f decreases gradually and E(c) increases rapidly.

Table 2.3 Results for CSP-1 under different values of screening cost, c_s

($\lambda = 1, \mu = 8$, c_r=20, AOQL=0.1%, p=0.25%)

c_s	i	F	AFI	E(c)	c_a
1	569	0.2675	0.6028	0.6556	22.9053
2	610	0.2464	0.6004	1.2572	25.4657
3	626	0.2387	0.6004	1.8578	26.5120
4	635	0.2345	0.6003	2.4581	27.1126
5	641	0.2318	0.6002	3.0583	27.5178
6	645	0.2300	0.6001	3.6585	27.7902
7	647	0.2290	0.6000	4.2586	27.7902
8	649	0.2282	0.6000	4.8586	28.0643
9	651	0.2273	0.6000	5.4587	28.2020
10	652	0.2268	0.6000	6.0587	28.2710
11	654	0.2259	0.6000	6.6588	28.4094
12	654	0.2259	0.6000	7.2588	28.4094
13	655	0.2255	0.6000	7.8588	28.4787
14	656	0.2250	0.6000	8.4589	28.5482
15	657	0.2246	0.6000	9.0589	28.6178

The effect of the replacement cost (c_r)

Table 2.4 is constructed to study the behavior of the performance measures of CSP-1 for various replacement cost c_r.

Table values show that

i. i increases with increasing c_r
ii. f decreases with increasing c_r
iii. AFI decreases with increasing c_r
iv. E(c) increases with increasing c_r and
v. C_a increases with increasing c_r

Table 2.4 Results for CSP-1 under different values of replacement cost, c_r

($\lambda = 1, \mu = 8$, c_s=1, AOQL=0.1%,p=0.25%)

c_r	i	F	AFI	E(c)	c_a
15	568	0.2681	0.6028	0.6481	22.8450
16	569	0.2675	0.6028	0.6496	22.9053
17	569	0.2675	0.6028	0.6511	22.9053
18	569	0.2675	0.6028	0.6526	22.9053
19	569	0.2675	0.6028	0.6541	22.9053
20	569	0.2675	0.6028	0.6556	22.9053
21	569	0.2675	0.6028	0.6572	22.9053
22	570	0.2670	0.6027	0.6587	22.9658
23	570	0.2670	0.6027	0.6602	22.9658
24	570	0.2670	0.6027	0.6617	22.9658
25	570	0.2670	0.6027	0.6632	22.9658

The effect of the incoming quality p

The economic performance of CSP-1 is examined with respect to the incoming quality p for certain assumed values. Table 2.5 gives the numerical results with $\lambda = 1, \mu = 8$, c_s=1,c_r=20,AOQL=0.1%. Table 2.5 shows that increases in p values are accompanied with increase in f and decrease in i.

In this chapter an attempt is made to minimize the total expected cost under a specified average outgoing quality limit (AOQL) and under the assumption that the acceptance cost c_a varies linearly with the not inspected, non-conforming items during sampling phase. The effect of change in the set of cost parameters on the economic performance of CSP-1 is analyzed.

Table 2.5 Results for CSP-1 under different values of defective p ($\lambda = 1, \mu = 8$, c_s=1,c_r=20,AOQL=0.1%)

p	i	f	AFI	E(c)	c_a
0.0020	760	0.0966	0.5087	0.1462	35.8157
	758	0.1867	0.5116	0.5638	35.8494
	756	0.1878	0.5180	0.5644	35.5985
	752	0.1871	0.5191	0.5646	35.7599
0.0021	720	0.2088	0.5453	0.5757	31.3141
	718	0.2100	0.5461	0.5764	31.0871
	716	0.2117	0.5475	0.5977	30.7893
	709	0.2031	0.5308	0.5850	30.3935
0.0022	680	0.2225	0.5614	0.6118	28.9550
	678	0.2239	0.5622	0.6127	28.7302
	676	0.2239	0.5622	0.6127	28.7302
	670	0.2190	0.5508	0.6042	29.5326
0.0023	643	0.2367	0.5768	0.6271	26.7980
	641	0.2381	0.5772	0.6272	26.5993
	639	0.2396	0.5786	0.6290	26.3889
	633	0.2354	0.5695	0.6224	26.9783
0.0024	620	0.2425	0.5864	0.6347	25.9896
	618	0.2440	0.5878	0.6400	25.7665
	615	0.2464	0.5891	0.6417	25.4675
	600	0.2514	0.5867	0.6395	24.8253
0.0025	579	0.2646	0.6053	0.6583	23.2343
	576	0.2672	0.6065	0.6584	22.9401
	572	0.2707	0.6085	0.6594	22.5530
	569	0.2675	0.6028	0.6556	22.9053
0.0026	551	0.2785	0.6184	0.6706	21.7253
	548	0.2813	0.6198	0.6724	21.4393
	545	0.2841	0.6213	0.6733	21.1590
	541	0.2832	0.6176	0.6709	21.2529

Continuations of Table 2.5

p	i	f	AFI	E(c)	c_a
0.0027	535	0.2936	0.6385	0.6923	20.2479
	530	0.2985	0.6408	0.6942	19.8006
	545	0.3035	0.6431	0.6955	19.3591
	541	0.2987	0.6315	0.6853	19.7854
0.0028	510	0.3077	0.6501	0.7032	18.9993
	505	0.3129	0.6525	0.7040	18.4292
	490	0.3182	0.6484	0.7014	18.1414
	491	0.3139	0.6445	0.6990	18.4859
0.0029	487	0.3215	0.6611	0.7152	17.8833
	484	0.3247	0.6623	0.7171	17.6381
	480	0.3293	0.6644	0.7181	17.2939
	469	0.3287	0.6565	0.7119	17.3396
0.0030	463	0.3382	0.6726	0.7282	16.6546
	641	0.3406	0.6736	0.7293	16.7143
	459	0.3430	0.6746	0.7303	16.3236
	449	0.3429	0.6678	0.7242	16.3336
0.0031	445	0.3497	0.6818	0.7396	15.8767
	443	0.3522	0.6828	0.7395	15.7143
	440	0.3559	0.6842	0.7406	15.4782
	430	0.3570	0.6785	0.7359	15.4089
0.0032	425	0.3652	0.6920	0.7498	14.9058
	423	0.3679	0.6932	0.7510	14.7450
	420	0.3718	0.6946	0.7526	14.5164
	413	0.3703	0.6884	0.7470	14.6065
0.0033	408	0.3783	0.7010	0.7609	14.1472
	393	0.3899	0.7010	0.7601	13.5180
	390	0.3942	0.7026	0.7611	13.2942
	397	0.3833	0.6978	0.7576	13.8723

Continuation of Table 2.5

p	i	f	AFI	E(c)	c_a
0.0034	389	0.3956	0.7112	0.7718	13.2224
	386	0.4000	0.7128	0.7736	13.0000
	384	0.4029	0.7139	0.7747	12.8560
	382	0.3960	0.7066	0.7678	13.2021
0.0035	379	0.4011	0.7167	0.7789	12.9451
	374	0.4087	0.7196	0.7815	12.5742
	371	0.4133	0.7212	0.7901	12.3563
	368	0.4083	0.7149	0.7775	12.5920
0.0036	365	0.4136	0.7245	0.7885	12.3423
	362	0.4186	0.7262	0.7897	12.1296
	359	0.4230	0.7280	0.7916	12.0385
	355	0.4202	0.7228	0.7868	11.9125
0.0037	353	0.4239	0.7383	0.8035	11.8723
	350	0.4288	0.7332	0.7985	11.6567
	347	0.4337	0.7349	0.7999	11.4459
	343	0.4315	0.7302	0.7958	11.5386
0.0038	340	0.4368	0.7389	0.8055	11.3150
	338	0.4402	0.7402	0.8070	11.1735
	336	0.4436	0.7414	0.8078	11.3042
	332	0.4422	0.7373	0.8044	11.0895
0.0039	330	0.4456	0.7449	0.8131	10.9533
	327	0.4508	0.7466	0.8151	10.7462
	324	0.4561	0.7530	0.8215	10.6189
	321	0.4533	0.7440	0.8127	10.6489
0.0040	319	0.4570	0.7515	0.8215	10.5054
	317	0.4606	0.7527	0.8225	10.3686
	314	0.4660	0.7545	0.8653	10.1673
	311	0.4636	0.7504	0.8207	10.2557

Chapter - III

CHAPTER III

ECONOMIC DESIGN OF CSP-1 PLAN WITH LINEAR INSPECTION COST

This chapter discusses the designing of an economically based continuous sampling plan CSP-1 under linear inspection cost .The minimum total expected cost per unit produced is obtained for CSP-1 plan by assuming that the per unit inspection cost is linearly proportional to the average number of inspections per inspection cycle.

The procedure of the CSP-1 plan starts with inspecting every unit until i successive units are found free of non-conformities, and then inspect a fraction of f of the units. Procedure of CSP-1 plan has two inspection stages, 100% inspection and sampling inspection, and there is a simple rule to determine when to change between these inspection stages .The average quality limit is widely used as the primary index of the CSP-1 plan to study its performance.

In this chapter the economic design of the CSP-1 plan under linear inspection cost is presented. The cost of inspection includes the labor cost of the inspector and the cost of using the inspection equipment. The following assumptions are assumed for formulating a model of CSP-1.

i. the per unit inspection cost is linearly proportional to the average number of inspections per inspection cycle

ii. the cost of accepting a non-conforming unit and the cost of replacing a non-conforming unit found during inspection are constant

iii. production process is under statistical control

iv. the inspection is perfect and

v. rectifying inspection is adopted.

The total expected cost of implementing the CSP-1 plan during one inspection cycle, E(c), is

$$E(c) = E(c_1) + E(c_2) + E(c_3)$$

$$= c_s \, AFI + c_a \, p(1 - AFI) + c_r p \, AFI \qquad (3.1)$$

$$= \frac{c_s f + c_a (1-f) p q^i + c_r f p}{f + (1-f) q^i}$$

where, $E(c_1)$ - the expected inspection cost per unit produced,

$E(c_2)$ - the expected acceptance cost per unit produced,

$E(c_3)$ - the expected replacement cost per unit produced,

C_s - the inspection cost per unit inspected,

C_a - the cost of accepting a non-conforming unit,

C_r - the cost of replacing a non-conforming unit found during inspection,

AFI - the average fraction inspected is $\frac{f}{f + (1-f) q^i}$

i - the clearance number of the 100% inspection stage,

f - the sampling frequency of the sampling inspection stage,

p - the incoming proportion defective, and

q - 1-p.

According to linearly proportional inspected cost

$$C_s = a + b\,(U + f\,V) \qquad (3.2)$$

where, a is the constant of the unit inspection cost composed of a fixed portion,

b is the constant of the unit inspection cost composed of a variable portion, U is the average number of units produced during 100% inspection stage for the CSP-1 plan during one inspection cycle is $\frac{1-q^i}{pq^i}$

V is the average number of units passed during sampling inspection stage for theCSP-1 plan during one inspection cycle is $\frac{1}{fp}$

Using (3.2) and (3.1)

$$E(C) = \frac{[a+b(U+fV)]f + c_a(1-f)pq^i + c_r fp}{f+(1-f)q^i}$$

$$= \frac{afpq^i + bf + c_a(1-f)p^2q^{2i} + c_r fpq^2}{pq^i[f+(1-f)q^i]} \qquad (3.3)$$

The problem of minimizing the cost of inspection of CSP-1 plan becomes

Minimize

$$E(C) = \frac{afpq^i + bf + c_a(1-f)p^2q^{2i} + c_r fpq^2}{pq^i[f+(1-f)q^i]} \qquad (3.4)$$

subject to

$\max_{0 \le p \le 1}$ AOQ= p_L

i≥0, an integer

0≤f≤1, a fraction

where AOQ is the average outgoing quality, and p_L is the specified AOQL value. Using Dodge's (1946) results relating to AOQL

$$p_l = \frac{ip_L + 1}{i+1} \qquad (3.5)$$

$$f = \frac{(1-p_L)^{i+1}}{ip_L + (1-p_l)^{i+1}} \tag{3.6}$$

where p_l is the quality level for which AOQL is reached. Using equations (3.5) and (3.6) in (3.4) the problem reduces to,

Minimize

$$E(C) = \frac{afpq^i + bf + c_a(1-f)p^2q^{2i} + c_r fpq^2}{pq^i[f + (1-f)q^i]} \tag{3.7}$$

subject to

$$f = \frac{(1-\frac{ip_L+1}{i+1})^{i+1}}{ip_L + (1-\frac{ip_L+1}{i+1})^{i+1}}$$

i-an integer ≥ 0 and

f- a fraction lying between 0 and 1.

The unique combinations (i, f) that has the minimum E(c) are obtained using search procedure with equations (3.5), (3.6) and (3,7).

Numerical Analysis

Optimal values of (i, f) corresponding to the required level of outgoing quality and the minimum total expected cost are computed for certain numerical values. The specified average outgoing quality limit be p_L=0.1%, and the incoming proportion defective be p=0.15%, and fix the parameters a=4,b=0.6, c_r=8, c_a=16. By solving equation (3.7), the optimal combination of parameters are i^*=198 and f^*=0.6029717 are obtained with AFI=0.671524 and E(c)=364.2816.

The Effect of the Incoming Proportion Defective p

Table 3.1 is constructed for i, f, AFI and E(c) values corresponding to the assumed parameters a=4, b=0.6, c_r=8, c_a=16 and AOQL=0.1% for various values of the incoming quality, p.

From numerical values in Table 3.1 it is observed that the minimum E(c) for a CSP-1 plan has the following properties

i. i decreases with increase in p
ii. f increases with increase in p
iii. AFI increases with increase in p and
iv. E(c) decreases with increase in p.

The effect of constant a

Table 3.2 is constructed for the optimal i and f values along with AFI and E(c) for the various values of a with certain assumed values for parameters b=0.6, c_r=8, c_a=16, p=0.15%and p_L=0.1%.

Table values indicate that

i. increase in a increases i
ii. increase in a decreases f
iii. increase in a increases E(c) and
iv. increase in a increases AFI

Table 3.1 Results for CSP-1 plans under different incoming proportion defective p(a=4, b=0.6, c_r=8, c_a=16, p_L=0.1%)

p	i	f	AFI	E(c)
0.0010	633	0.2354372	0.3671373	416.4707
0.0011	484	0.3185127	0.4432665	413.6680
0.0012	379	0.3986005	0.5109449	404.7619
0.0013	300	0.4753081	0.5723396	392.5569
0.0014	244	0.5407593	0.6236895	378.7373
0.0015	198	0.6029717	0.6715240	364.2816
0.0016	162	0.6579098	0.7136963	349.7721
0.0017	133	0.7067552	0.7513616	335.5534
0.0018	109	0.7505928	0.7855243	321.8173
0.0019	89	0.7897596	0.8164893	308.6765
0.0020	73	0.8229464	0.8432462	296.1743
0.0021	58	0.8556614	0.8700717	284.3286
0.0022	46	0.8830153	0.8930816	273.1292
0.0023	35	0.9090621	0.9155087	262.5553
0.0024	26	0.9310966	0.9349984	252.5796
0.0025	17	0.9538091	0.9556482	243.1698
0.0026	10	0.9719619	0.9726628	234.2930
0.0027	4	0.9878804	0.9880092	225.9188
0.0028	1	0.9960080	0.9960191	218.0385

Table 3.2 Results for CSP-1 plans under different values of a

($b=0.6$, $c_r=8$, $c_a=16$, $p=0.15\%$, $p_L=0.1\%$)

a	i	f	AFI	E(c)
1	196	0.6058703	0.6735327	362.2634
2	196	0.6058703	0.6735327	362.9370
3	198	0.6029717	0.6715240	363.6101
4	198	0.6029717	0.6715240	364.2816
5	199	0.6015275	0.6705245	364.9524
6	199	0.6015275	0.6705245	365.6229
7	201	0.5986542	0.6685396	366.2932
8	201	0.5986542	0.6685396	366.9617

The Effect of constant b(>0)

Table3.3 is constructed for the optimal i and f values along with AFI and E(c) for the specified constant b (>0) with certain assumed values for parameters $a=4$, $c_r=8$, $c_a=16$, $p=0.15\%$and $p_L=0.1\%$.

For a given set of parameters as the positive b increases the i decreases slowly and E(c) increases rapidly as shown in Table 3.3.

Numerical values in Table 3.3 reveal that increase in b(>0)

i. increases in b results in decrease of i

ii. increases in b results in increase of f

iii. increases in b results in increase of AFI and

iv. increases in b results in increase of E(c)

Table 3.3 Results for CSP-1 plans under different constant b>0

($a=4$, $c_r=8$, $c_a=16$, $p=0.15\%$,$p_L=0.1\%$)

b	i	f	AFI	E(c)
0.1	21	0.5831458	0.6578976	62.9420
0.2	204	0.5943747	0.6655909	123.2208
0.3	201	0.5986542	0.6685396	183.4888
0.4	199	0.6015275	0.6705245	243.7539
0.5	199	0.6029717	0.6705245	304.0179
0.6	198	0.6029717	0.6715240	364.2816
0.7	198	0.6029717	0.6715240	424.5449
0.8	198	0.6029717	0.6715240	484.8081
0.9	196	0.6058703	0.6735320	545.0710
1.0	196	0.6058703	0.6735327	605.3334

The Effect of constant b(<0)

Table 3.4 is constructed for the optimal i and f values along with AFI and E(c) for the different b(<0) values with certain assumed values for parameters $a=4$, $c_r=8$, $c_a=16$, $p=0.15\%$and $p_L=0.1\%$. As the negative b increases slowly the i decreases rapidly and E(c) varies slowly as shown in Table 3.4.

Increase in b (<0) is accompanied with

i. increase in i
ii. decrease in f
iii. decrease in AFI and
iv. increase in E(c)

Table 3.4 Results for CSP-1 plan under different constant b<0,

($a=4$, $c_r=8$, $c_a=16$,$p=0.15\%$,$p_L=0.1\%$)

b	i	f	AFI	E(c)
-0.0001	2727	0.00872387	0.3453906	0.0209220
-0.0002	2265	0.01654540	0.3351660	0.0215132
-0.0003	1995	0.02441488	0.3333444	0.0213123
-0.0004	1804	0.03241358	0.3344347	0.0200107
-0.0005	1655	0.04066336	0.3370317	0.0206863
-0.0006	1533	0.04915463	0.3404806	0.0217627
-0.0007	1431	0.05778519	0.3444809	0.0203104
-0.0008	1342	0.06671523	0.3489255	0.0203590
-0.0009	1236	0.07595813	0.3537371	0.0214479
-0.0010	1193	0.08537346	0.3587861	0.0210077
-0.0020	731	0.19470660	0.4200936	0.0211229
-0.0030	461	0.33426050	0.5007605	0.0202488
-0.0040	270	0.50908440	0.6086511	0.0170881
-0.0050	121	0.72825360	0.7626777	0.0169375

The Effect of Replacement Cost c_r

Table 3.5 is constructed for the optimal i and f values along with AFI and E(c) for the replacement cost c_r with certain assumed values for parameters $a=4$, $b=0.6$, $c_a=16$, $p=0.15\%$ and $p_L=0.1\%$ with different values of c_r.

For a given set of parameters as c_r increases the value of E(c) increases gradually but no change occurs in i, f and AFI.

Table 3.5 Results for CSP-1 plan under different replacement cost c_r,

($a=4$, $b=0.6$, $c_a=16$, $p=0.15\%$, $p_L=0.1\%$)

c_r	i	f	AFI	E(c)
4	198	0.6029717	0.671524	364.2775
5	198	0.6029717	0.671524	364.2786
6	198	0.6029717	0.671524	364.2796
7	198	0.6029717	0.671524	364.2806
8	198	0.6029717	0.671524	364.2816
9	198	0.6029717	0.671524	364.2826
10	198	0.6029717	0.671524	364.2836
11	198	0.6029717	0.671524	364.2846
12	198	0.6029717	0.671524	364.2856
13	198	0.6029717	0.671524	364.2866
14	198	0.6029717	0.671524	364.2876
15	198	0.6029717	0.671524	364.2886
16	198	0.6029717	0.671524	364.2897

The Effect of Acceptance Cost c_a

Table 3.6 is constructed for the optimal i and f values along with AFI and E(c) for the acceptance cost c_a with certain assumed values for parameters $a=4$, $b=0.6$, $c_r=8$, $p=0.15\%$ and $p_L=0.1\%$ with different values of c_a.

Numerical values in Table 3.6 reveal that increase in c_a increases E(c) alone.

Table 3.6 Results for CSP-1 plans under different acceptance cost c_a

($a=4$, $b=0.6$, $c_r=8$, $p=0.15\%$, $p_L=0.1\%$)

c_a	i	f	AFI	E(c)
8	198	0.6029717	0.671524	364.2776
9	198	0.6029717	0.671524	364.2781
10	198	0.6029717	0.671524	364.2786
11	198	0.6029717	0.671524	364.2791
12	198	0.6029717	0.671524	364.2796
13	198	0.6029717	0.671524	364.2801
14	198	0.6029717	0.671524	364.2806
15	198	0.6029717	0.671524	364.2811
16	198	0.6029717	0.671524	364.2816
17	198	0.6029717	0.671524	364.2821
18	198	0.6029717	0.671524	364.2826
19	198	0.6029717	0.671524	364.2831
20	198	0.6029717	0.671524	364.2836

The effect of incoming proportion defective of p

Table 3.7 shows that for the given optimal i and f values along with AFI and E(c)for the incoming proportion defective p with certain assumed values for parameters $a=4, b=0.6, c_r=8, c_a=16, p_L=0.1\%$.Numerical results of Table 3.7 reveal that increases in p

i. decreases i
ii. increases f
iii. increases AFI and
iv. decreases E(c)

This chapter gives the designing of CSP-1 plan with minimization of the total expected cost per unit produced under specified level of the product quality with linear inspection cost.

Table 3.7 Results for CSP-1 under different values of p,(a=4, b=0.6,c_r=8,c_a=16,p_L=0.1%)

P	i	f	AFI	E(c)
0.0010	643	0.2308599	0.3635149	416.5579
	639	0.2326822	0.3649583	416.4824
	633	0.2354372	0.3671373	416.4707
0.0011	492	0.3132536	0.4394353	413.7663
	487	0.3165301	0.4418224	413.7503
	484	0.3185127	0.4432665	413.6680
0.0012	386	0.3925673	0.5067358	404.8113
	383	0.3951410	0.5085300	404.7733
	379	0.3986005	0.5109449	404.7619
0.0013	310	0.4646644	0.5650502	392.6391
	305	0.4699498	0.5686664	392.3952
	300	0.4753081	0.5723396	392.5569
0.0014	254	0.5283045	0.6151917	378.8505
	250	0.5332487	0.6185590	378.8116
	244	0.5407593	0.6236895	378.7373
0.0015	208	0.5887289	0.6617153	364.3690
	203	0.5958011	0.6665736	364.3213
	198	0.6029717	0.6715240	364.2816
0.0016	172	0.6420526	0.7025989	349.8506
	167	0.6499214	0.7080863	349.8125
	162	0.6579098	0.7136963	349.7721
0.0017	143	0.6894073	0.7389767	335.6445
	138	0.6980068	0.7450956	335.5922
	133	0.7067368	0.7513616	335.5534
0.0018	119	0.7319191	0.7718477	321.9080
	114	0.7411842	0.7785985	321.8521
	109	0.7505928	0.7855243	321.8173

Continuation of Table 3.7

P	i	f	AFI	E(c)
0.0019	98	0.7718266	0.7718477	308.7627
	93	0.7411842	0.7785985	308.7011
	89	0.7505928	0.7855243	308.6765
0.0020	82	0.8040694	0.8286499	296.2903
	77	0.8144898	0.8366631	296.2067
	73	0.8229464	0.8432462	296.1743
0.0021	67	0.8646596	0.8779837	284.4386
	62	0.8737769	0.8854308	284.3578
	58	0.8830153	0.8930816	284.3286
0.0022	54	0.8646596	0.8779837	273.2319
	50	0.8737769	0.8854308	273.1621
	46	0.8830153	0.8930816	273.1292
0.0023	43	0.8900249	0.8993488	262.6593
	39	0.8994801	0.9073134	262.5790
	35	0.9090621	0.9155087	262.5553
0.0024	33	0.9139015	0.9199396	252.6746
	29	0.9236776	0.9284474	252.6030
	26	0.9310966	0.9349984	252.5796
0.0025	23	0.9385906	0.9418264	243.2246
	20	0.9461612	0.9486552	243.1883
	17	0.9538091	0.9556482	243.1698
0.0026	15	0.9589512	0.9604611	234.3435
	12	0.9667307	0.9677209	234.3038
	10	0.9719619	0.9726628	234.2930
0.0027	8	0.9772296	0.9777060	225.9545
	6	0.9825350	0.9828112	225.9278
	4	0.9878804	0.9880092	225.9188
0.0028	3	0.9905702	0.9906484	218.0619
	2	0.9932752	0.9933126	218.0482
	1	0.9960080	0.9960191	218.0385

Summary and Conclusion

SUMMARY AND CONCLUSION

The dissertation investigates the economic designing of CSP-1 continuous sampling plan for various production processes corresponding to the number of articles arrive for inspection is infinite and the probability of each article being created defective is constant.

The relevant literature for the preparation of dissertation is given in review of literature.

Chapter I explained the derivation of performance measures of continuous sampling plan CSP-1 for production process having the probability of producing a non-conforming item is constant.

In Chapter II, an attempt is made to minimize the total expected cost under, a specified average outgoing quality limit (AOQL) and the more realistic assumption that the acceptance cost c_a varies linearly with the not inspected, non-conforming items during sampling phase.

In Chapter III, the economic design of the CSP-1 plan under linear inspection cost with the required level of product quality with the minimum total expected cost per unit produced is presented.

Recommendation for further study

i. Cost model for systematically varying process for CSP-1 plan may be considered.

ii. Cost model of CSP-1 in the presence of inspection error may be carried out.

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