



# Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)  
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC  
Coimbatore - 641 043, Tamil Nadu, India

## Continuous Internal Assessment Test II – October 2025 Semester V

Class : III UG  
Branch : Mathematics

Time : 2 Hours  
Max. Marks : 60

### 23BMAC09 – Set Theory and Metric Spaces

#### Course Outcomes:

CO1: learn basic facts about the cardinality of a set.

CO2: understand several standard concepts of metric spaces and their properties like openness and closedness.

CO3: identify the continuity of a function defined on metric spaces and homeomorphisms.

CO4: know the concepts of compactness, Bolzano-Weierstrass property and Totally Bounded sets.

CO5: recognize the difference between connected and disconnected sets and their properties.

#### PART A

6 x 1 = 6

#### Choose the Correct Answer

- The sequence  $\{\frac{1}{n}\}$  converges to the limit \_\_\_\_\_ CO1K1  
a) 0            b)  $\infty$             c) n            d) 1
- Which of the following is a complete metric space? CO3K2  
a)  $R_n$             b)  $C_n$             c)  $C^n$             d) all the three
- Which of the following is not true? CO3K2  
a) A compact metric space is separable  
b) A closed subset of a compact metric space is complete  
c) A totally bounded metric space is separable  
d) Not every bounded subset of  $R^2$  is totally bounded
- A function  $f: X \rightarrow Y$  is said to be a homeomorphism if CO3K2  
a) f is bijective            b) f is continuous            c)  $f^{-1}$  is continuous            d) all the three
- In a metric space  $(X, d)$ , the two sets A and B are said to be separated if CO4K2  
a)  $A \cap \bar{B} = \phi$             b)  $\bar{A} \cap B = \phi$             c) both (a) & (b)            d)  $A \cap B = \phi$
- The nowhere dense in R under the usual metric is CO5K1  
a) Any singleton set            b) any infinite set  
c) either (a) or (b)            d) neither (a) nor (b)

#### Part B

3 x 6 = 18

#### Answer ALL questions

- a. Prove that in a metric space every convergent sequence has a unique limit. CO3K3  
(or)
- b. Let  $(X, d)$  be a metric space and  $(Y, d_Y)$  is a subspace, then prove that  $Y$  is complete  $\Rightarrow Y$  is closed. CO3K3

8. a. Let  $(X, d)$  and  $(Y, \rho)$  be metric spaces and  $f: X \rightarrow Y$  be a function. Then prove that  $f$  is continuous iff  $f^{-1}(G)$  is open in  $X$  whenever  $G$  is open in  $Y$ . CO4K3
- (or)
- 8.b. Let  $(X, d)$  and  $(Y, \rho)$  be metric spaces and  $f: X \rightarrow Y$  be a uniformly continuous function. If  $\{x_n\}$  is a Cauchy sequence in  $X$ , then prove that  $\{f(x_n)\}$  is Cauchy sequence in  $Y$ . CO4K4
- 9.a. Prove that a closed subset of a compact metric space is compact. CO5K4
- (or)
- 9.b. Prove that a totally bounded metric space is bounded. CO5K4

**Part C**

**3 x 12 = 36**

**Answer ALL questions**

10. a. State and prove Cantor's intersection theorem. CO3K4
- (or)
10. b. Let  $(X, d)$  be a metric space. If  $\{G_n\}$  is a sequence of dense open sets in  $X$ , then prove that  $G = \bigcup_{n=1}^{\infty} G_n$  is dense in  $X$ . CO3K3
11. a. State and prove Baire's Category theorem. CO4K3
- (or)
- 11.b. State and prove Heine-Borel theorem in a metric space  $(X, d)$ . CO4K4
- 12.a. Prove that continuous image of a compact set is compact. CO5K4
- (or)
- 12.b. Prove that continuous image of a connected set is connected. CO5K4

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