Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category 'A' by MHRD (now MoE)
Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test I –February 2025
Semester -II

Class: I PG **Time:2 Hrs**
Major: Mathematics **Max.Marks:60**

23MMAC08 - Real Analysis - II

Course Outcomes:

CO1: Distinguish between the Lebesgue and Riemann integrals.
C02:Spply the concept of Lebesgue integral to broader class of functions.
CO3:Test the convergence using Riemann's localization theorem.
CO4:Solve problems in a closed form using Fourier integrals.
CO5:Evauate the multiple integrals using iterated integration.

PART-A

Choose the correct answer **6x1=6**

1. If $f(x) \leq g(x)$ a.e. on I ,then CO1K2

a.$\int_I \boldsymbol{f} \leq \int_I \boldsymbol{g}$ b.$\int_I \boldsymbol{f} \geq \int_I \boldsymbol{g}$ c.$\int_I \boldsymbol{f} = \int_I \boldsymbol{g}$d.$\int_I (\boldsymbol{f}+\boldsymbol{g}) \leq \int_I \boldsymbol{f} + \int_I \boldsymbol{g}$

2. If $\boldsymbol{f}^+$ and $\boldsymbol{f}^-$ are non negative functions, then CO1K2

a.$|f| = f^+ + f^-$ b.$|f| = f^+ - f^-$c.$f = f^+ + f^-$ d. $f = f^+ - f^-$

3. If f is measurable and bounded on a bounded interval, then CO2K2

a.$f \in L(I)$ b. $f \in M(I)$c.$f \in U(I)$d. $|f| \in L(I)$

4. If $\boldsymbol{\mu(S) = +\infty}$,then CO2K2

a.$\chi_S$is measurable but not Lebesgue-integrable
b. χ_S is measurable also Lebesgue-integrable
c. χ_S is not measurable but Lebesgue-integrable
d. χ_S is not measurable and not Lebesgue-integrable

5. The value of $\boldsymbol{\varphi_{2n-1}(x)}$is CO3K2

a.$cosnx/\sqrt{\pi}$ b. $sinnx/\sqrt{\pi}$ c. $1/\sqrt{\pi}$ d.$1/\sqrt{2\pi}$

6.An orthonormal system of complex –valued functions on every interval of length 2π

when n=0,1,2,.. CO3K1

a. $e^{inx}/\sqrt{2\pi}$ b. $e^{inx}/\sqrt{\pi}$c. $e^{inx}/2$ d. e^{inx}/π

PART-B

Answer ALL questions **3x6=18**

7.a. If $f \in U(I)$ and $\{\boldsymbol{s_n}\},\{\boldsymbol{t_m}\}$ are sequences generating f. Prove

that $\lim_{n\to\infty} \int_I \boldsymbol{s_n} = \lim_{m\to\infty} \int_I \boldsymbol{t_m}$. CO1K3

(or)

7.b. State and Prove Levi theorem for upper functions. CO1K3

8.a. If f is Riemann-integrable on [a,b] $\forall b \geq a$ and ∃ a positive constant M such that

$\int_a^b |f(x)|dx \leq M$, Prove that both f anf $|f|$ are improper Riemann-integrable on

$[a, +\infty)$. CO2K4

(or)

8.b. If A and B are disjoint measurable sets, show that $\mu(A \cup B) = \mu(A) + \mu(B)$. CO2K4

9.a. Write Parseval's formula and establish it. CO3K3

(or)

9.b. State and Prove Bessel's inequality. CO3K3

PART-C

Answer ALL questions **3x12=36**

10. a.If $\{s_n\}$ is a decreasing sequence of nonnegative step functions such that $s_n \searrow 0$ a.e. on an intervalI, Show that $\lim_{n\to\infty} \int_I S_n = 0$. CO1K4

(or)

10.b. State and Prove additive property of the integral with respect to the interval of integration. CO1K4

11.a. State and prove Lebesgue dominated convergence theorem. CO2K5

(or)

11.b. (i) If S and T are measurable, so is $S - T$.

(ii) If S_1, S_2, ... are measurable, so are $\cup_{i=1}^{\infty} S_i$and $\cap_{i=1}^{\infty} S_i$. CO2K4

12.a. If $\{\emptyset_0, \emptyset_1, \emptyset_2, \ldots\}$ is an orthonormal on I and $f \in L^2(I)$and the sequence of functions $\{s_n\}$ and $\{t_n\}$ on I as follows.$s_n(x) = \sum_{k=0}^{n} c_k \emptyset_k(x)$ and $t_n(x) = \sum_{k=0}^{n} b_k \emptyset_k(x)$ where $c_k = (f, \emptyset_k)$ for k = 0,1,2,… and $b_0, b_1, b_2, \ldots$, are arbitrary complex numbers. Show that for each n, $\|f - s_n\| \leq \|f - t_n\|$ ----(*). Moreover equality holds in (*) if and only if $b_k = c_k$fork=0,1,2…n. CO3K5

(or)

12.b. State and Prove Riesz-Fischer theorem. CO3K5

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