

*INTRODUCTION*

## INTRODUCTION

Decision Making is the act of choosing between two or more courses of action. Decision-making can also be regarded as a problem-solving activity terminated by a solution deemed to be satisfactory among several alternative possibilities. It is, therefore, a reasoning or emotional process which can be rational or irrational and can be based on explicit assumptions or implicit assumption. There are processes and techniques to improve decision making and the quality of decisions. In most of real world situations, usually decision makers are confronted with multiple criteria to be considered before any decision can be made. This is the case of Multi Criteria Decision Making (MCDM); a case with the aim to find the overall preferences among the available alternatives.

Multiple criteria decision making (MCDM) is a modeling and methodological tool for dealing with complex engineering problems. MCDM is generally an approach of solving decision problems involving many criteria, factors or objectives. In solving a MCDM problem, one needs to know the importance or weights of the not equally important attributes and also the evaluations of the alternatives with respect to the attributes. There have been different methods on MCDM and the most known is Analytic Hierarchy Process (AHP), which is initially proposed by Saaty (1980). This process makes it possible to incorporate judgments on tangible data as well as intangible data.

In the classic AHP model, the comparison values over different criteria that are provided by the decision maker are represented by crisp numbers within the 1–9 scale. However, in some realistic situations, the decision makers might be reluctant or unable to assign the crisp evaluation values to the comparison judgments due to his/her limited knowledge or the subjectivity of the qualitative evaluation criteria or the variations of individual judgments in group decision making. Hence, the conventional AHP seems to be inadequate to explicitly capture the important assessments for deriving the priorities in these situations. To overcome this issue, the fuzzy set theory, developed by Zadeh (1965), was introduced to AHP, and then, the traditional AHP was extended to the Fuzzy AHP (FAHP), where each pairwise comparison judgment is represented as a fuzzy number that is described by a membership function.

The earliest work in Fuzzy AHP appeared in Laarhoven and Pedrycz (1983), which is a direct extension of Saaty's AHP method. In this method authors compared fuzzy ratios described by triangular membership functions. Logarithmic least-square was used to derive the local fuzzy priorities. The most important problem of the method is related to the possibility of obtaining a normal and convex fuzzy number. The other problem of the method is that it requires cumbersome calculations.

Another extension of Saaty's AHP method was given by Buckley (1985) who incorporates fuzzy comparison ratios to the classical AHP. In this method Buckley used trapezoidal fuzzy numbers as membership functions.

Chang, Da-Yong (1996) introduced a new approach for handling Fuzzy AHP. In this method author used a row mean method called Extent Analysis Method to derive priorities for comparison ratios in the context of triangular fuzzy numbers.

Cheng C.H. (1997) proposed a new Fuzzy AHP algorithm based on grade value of membership function. In this method the author used Fuzzy AHP method and Shannon's entropy concepts to calculate aggregate weights.

Pan, Nang-Fei (2008) developed a new Fuzzy Analytic Hierarchy Process Method which employs triangular and trapezoidal fuzzy numbers and the  $\alpha$ -cut concept to deal with the imprecision inherent to the process of subjective judgment.

Debmallya Chatterjee and Bani Mukherjee (2010) introduced a new Fuzzy AHP method with the use of triangular fuzzy numbers together with Normalized Geometric Mean Method.

A Modified Fuzzy Analytic Hierarchy Process was developed by Serhat Aydin and Cengiz Kahraman (2011). In this method decision makers use not only positive triangular fuzzy numbers but also negative triangular fuzzy numbers in pairwise comparison matrices.

Chou, Chien-Chang and Yu, Ker-Wei (2013) proposed a new Hybrid Fuzzy Analytic Hierarchy Process (AHP) algorithm to deal with the decision making problems in an uncertain and multiple-criteria environment.

Zeshui Xu and Huchang Liao (2013) extended the classic AHP and the FAHP into the Intuitionistic Fuzzy AHP (IFAHP) in which the preferences are represented by intuitionistic fuzzy values.

There are different Fuzzy Analytic Hierarchy Process methods available in the literature. The main aim of this thesis is to study different Fuzzy AHP methods with applications to real life problems.

The study is so designed to present the following:

- i. Fuzzy Analytic Hierarchy Process Method
- ii. Modified Fuzzy Analytic Hierarchy Process Method
- iii. Hybrid Fuzzy Analytic Hierarchy Process Method
- iv. Intuitionistic Fuzzy Analytic Hierarchy Process Method
- v. Extent Analysis Method On Fuzzy Analytic Hierarchy Process
- vi. Entropy Based Fuzzy Analytic Hierarchy Process Method
- vii. Application Of Extent Analysis Method On Fuzzy Analytic Hierarchy Process
- viii. Application Of Modified Fuzzy Analytic Hierarchy Process Method

The first chapter deals with preliminary definitions and notations.

The second chapter is devoted to the study of the following Fuzzy AHP Methods

- i. Fuzzy AHP Method using Triangular Fuzzy Numbers (Laarhoven and Pedrycz's Approach) (1983)
- ii. Fuzzy AHP Method using Triangular Fuzzy Numbers (Debmallya Chatterjee and Bani Mukherjee's Approach)(2010)
- iii. Fuzzy AHP Method using Trapezoidal Fuzzy Numbers (Buckley's Approach) (1985)
- iv. Fuzzy AHP Method using Combination of Triangular Fuzzy Numbers and Trapezoidal Fuzzy Numbers (Pan, Nang-Fei's Approach) (2008)

Section 2.1 deals with Laarhoven and Pedrycz's approach of Fuzzy AHP Method using Triangular Fuzzy Numbers. In this method the criteria weights and alternative ratings are given by linguistic variables, expressed by triangular fuzzy numbers. The Lootsma's logarithmic least-squares method is used to derive fuzzy weights and fuzzy performance scores. The steps outlining the algorithm of a group multi-criteria decision making using Triangular Fuzzy Numbers are given in detail.

Section 2.2 deals with Debmallya Chatterjee and Bani Mukherjee's approach of Fuzzy AHP Method using Triangular Fuzzy Numbers. In this method the triangular fuzzy numbers is used to make the pairwise comparison matrix and Normalization of Geometric Mean (NGM) method is used to compute weights from pairwise comparison matrix. This method is illustrated in detail in section 2.2.

Section 2.3 deals with Buckley's Approach of Fuzzy AHP method using trapezoidal fuzzy numbers. In this method fuzzy comparison ratio is incorporated to the classical AHP and uses trapezoidal fuzzy numbers as membership functions. The geometric mean method is employed to calculate the fuzzy weights for each fuzzy matrix, and these are combined in the usual manner to determine the final fuzzy weights for the alternatives. This method is explained in this section.

Section 2.4 deals with Pan, Nang-Fei's Approach of Fuzzy AHP Method using combination of Triangular Fuzzy Numbers and Trapezoidal Fuzzy Numbers. To derive fuzzy weights from group evaluations, the max–min aggregation and center-of-gravity (COG) defuzzification techniques are utilized because of their simplicity and efficiency. Furthermore, the  $\alpha$  –cut concept is applied to describe specific levels of uncertainty associated with the decision environment. This proposed approach is straightforward and its execution is faster than other Fuzzy AHP methods.

In Chapter III, Modified Fuzzy Analytic Hierarchy Process method proposed by Serhat Aydin and Cengiz Kahraman (2011) is studied. This method includes modifications in Fuzzy Multiple Criteria Decision-Making theory to strengthen the comprehensiveness and reasonableness of the decision making process using Fuzzy AHP. Also this method enables multiple decision makers on evaluation and uses triangular fuzzy scale that

includes both positive and negative fuzzy numbers in order to evaluate hierarchy. The methodology includes simple mathematical calculations, and it yields triangular fuzzy numbers of alternatives' weights. At the last step, obtained alternatives' weights are ranked by Centroid Based Distance Method proposed by Wang et.al (2006). This method is illustrated in this chapter.

Chapter IV is devoted to the study of a new Hybrid Fuzzy AHP method proposed by Chou, Chien-Chang and Yu, Ker-Wei (2013). This method deals with the decision making problems in an uncertain and multiple-criteria environment. Originally, the Fuzzy AHP approach is a systematic method for the alternative choice and justification problems that combines the concept of fuzzy sets theory and the hierarchical structure analysis. But most of the Fuzzy AHP methods cannot evaluate the qualitative and quantitative criteria simultaneously. Therefore, Chien-Chang Chou and Ker-Wei Yu proposed a new hybrid approach which integrates AHP method and fuzzy sets theory to carry out a complete evaluation of qualitative and quantitative criteria simultaneously. The steps outlining the algorithm of this method are given in detail.

Chapter V deals with Intuitionistic Fuzzy Analytic Hierarchy Process (IFAHP) proposed by Zeshui Xu and Huchang Liao. Intuitionistic Fuzzy Set, introduced by Atanassov (1986) is characterized by three functions expressing the degree of belongingness, the degree of non-belongingness and the degree of hesitation. The concept of Intuitionistic Fuzzy Sets is a generalization of fuzzy sets. But an intuitionistic fuzzy set has many advantages in handling vagueness and uncertainty over a fuzzy set. In this IFAHP method the preferences are represented by intuitionistic fuzzy values. The IFAHP can be used to handle more complex problems, where the decision maker has some uncertainty in assigning preference values to the objects considered. This method proposes a new way to check the consistency of an intuitionistic preference relation and then introduces an automatic procedure to repair the inconsistent one. The proposed method improves the inconsistent intuitionistic preference relation without the participation of the decision maker, and thus, it can save much time and show some advantages over FAHP. This model also develops a novel normalizing rank summation method to derive the priority vector of an intuitionistic preference relation, on which the priorities of the

hierarchy in the IFAHP are derived. The detailed procedure of IFAHP is given in this chapter.

In Chapter VI, an Extent Analysis Method on Fuzzy Analytic Hierarchy Process proposed by Chang, Da-Yong (1996) is studied. This is a new approach for handling Fuzzy AHP. This method uses triangular fuzzy numbers for pairwise comparison scale of Fuzzy AHP, and the extent analysis method for the synthetic extent value of the pairwise comparison. This method is explained in this chapter.

Chapter VII deals with Entropy Based Fuzzy Analytic Hierarchy Process Method proposed by Cheng C.H (1997). The proposed Fuzzy AHP model build membership functions of judgment criteria for all sub-items called fuzzy standard and also the grade of membership function is calculated by using practical data to represent performance scores. In this method Shannon's entropy concepts are used to calculate aggregate weights.

Chapter VIII is devoted to present a decision making model, for "Selection of right school for the children" by collecting data from 100 parents of school children reside in Palakkad district, using Extent Analysis Method on Fuzzy Analytic Hierarchy Process.

In Chapter IX, the author of this thesis presented a mathematical model for the same problem "Selection of right school for the children" by applying Modified Fuzzy Analytic Hierarchy Process using the same data.

Decision-making plays a vital role in every walk of life. Real world decision making problems are very often uncertain or vague in a number of ways. Selecting a bad as good is more dangerous than rejecting a good as bad. Nowadays, decision making is a problem of every common man to take right decision on many routine affairs like education for children, food, transportation, purchase of durables, healthcare, shelter and so on. So what we need is a systematic and organized way to evaluate our choices and figure out which one offers the best solution to our problem.

In countries like India, efforts of the parents contribute much for the progress of every individual. Better the academic opportunities provided by the parents, better will be the progress of their children in the future. The academics effort of the parents starts from

identifying and admitting their children in the best school. It is the basic responsibility of every parent to identify and admit their children in the best school available in their area. So the author of this thesis intended to develop two mathematical models for decision making regarding the selection right school for the children using Extent Analysis Method on Fuzzy Analytic Hierarchy Process and Modified Fuzzy Analytic Hierarchy Process Method. These two models are illustrated in Chapters VIII and IX respectively.

It is worth mentioning that the author of the thesis published two articles related to these applications as detailed below:

- 1) "Selection of the Best School for the Children- A Decision Making Model Using Extent Analysis Method on Fuzzy Analytic Hierarchy Process", *International Journal of Innovative Research in Science, Engineering and Technology*, Vol.3, No. 5, 2014, pp. 12334- 12344 [52].
- 2) "Selection of the Best School for the Children- A Decision Making Model Using Modified Fuzzy Analytic Hierarchy Process", *International Journal of Scientific and Innovative Mathematical Research*, Vol. 2, No. 5, 2014, pp. 426-434 [53].