# *Introduction*

# INTRODUCTION

Matrices and Graphs play a vital role in the analysis and study of several of the real world problems.

The study of bialgebraic structures led to the invention of new notions like birings, bivector spaces, linear bialgebra, bigroupoids, bisemigroups, etc. But most of these are abstract algebraic concepts expect, the bisemigroup being used in the construction of biautomatons. Therefore it is important to construct non-abstract bistructures which can give itself for more and more lucid applications. Bimatrices are one of the non-abstract bistructures which have enormous applications. The notion of bimatrices is introduced by Florentin Smarandache, Vasantha Kandasamy, W.B and Ilanthenral, K., [10 ]

Uncertainty or indeterminacy happen to be one of the major factors in almost all real world problems. When uncertainty is modeled fuzzy theory is used and when indeterminacy is involved neutrosophic theory is used. Most of the fuzzy models which deal with the analysis and study of unsupervised data make use of bimatrices and bigraphs. The neutrosophic models are fuzzy models that permit the factor of indeterminacy. It also plays a significant role and utilizes the concepts of neutrosophic matrices and neutrosophic graphs.

The new concept of fuzzy interval matrices and neutrosophic interval matrices introduced by Florentin Smarandache and Vasantha Kandasamy, W.B will their applications in engineering, medical, industrial, social and psychological problems. These models are mainly useful when the data is an unsupervised one and when one needs a multi-expert model.

The fuzzy and neutrosophic tools like fuzzy cognitive maps invented by Bart Kosko and neutrosophic cognitive maps introduced by Florentin Smarandache et.al help in the analysis of real world problems and they happen to be mathematical tools which can give the hidden pattern of the problem under investigation.

The main aim of this thesis is to study the applications of bimatrices, fuzzy bimatrices, neutrosophic bimatrices and fuzzy interval bimatrices to real world problems.

The first chapter is devoted to the study of the basic concepts of bimatrices, fuzzy bimatrices, neutrosophic bimatrices and fuzzy interval bimatrices.

"A bimatrix A is defined as the union of two rectangular array of numbers $A_1$ and $A_2$ arranged into rows and columns. It is written as follows $A = A_1 \cup A_2$ where $A_1 \neq A_2$ with

$$A_1 = \begin{bmatrix} a^1_{11} & a^1_{12} & \cdots & a^1_{1n} \\ a^1_{21} & a^1_{22} & \cdots & a^1_{2n} \\ \cdots & & & \cdots \\ a^1_{m1} & a^1_{m2} & \cdots & a^1_{mn} \end{bmatrix}$$

and

$$A_2 = \begin{bmatrix} a^2_{11} & a^2_{12} & \cdots & a^2_{1n} \\ a^2_{21} & a^2_{22} & \cdots & a^2_{2n} \\ \cdots & & & \cdots \\ a^2_{m1} & a^2_{m2} & \cdots & a^2_{mn} \end{bmatrix}$$

'$\cup$' is just the notational convenience (symbol) only ".

The first section of first chapter deals with bimatrices. Bimatrix operations are defined with interesting examples. Some fundamental properties regarding these operations are also studied. The definitions of square bimatrix, mixed bimatrix, symmetric bimartix, skew symmetric bimatrix, subbimatrix are introduced with examples. The bideterminant and biinverse of a square bimatrix are also defined with examples.

Neutrosophic bimatrices are studied in section 1.2. A neutrosophic matrix is a matrix with entries from the neutrosophic field (The definition of neutrosophic field is given in 1.2.4). If $A_1$ and $A_2$ are two neutrosophic matrices then $A=A_1 \cup A_2$ is called the neutrosophic bimatrix. Different types of neutrosophic bimatrices are defined with examples.

Section 1.3 deals with fuzzy bimatrices.

"Let $A=A_1 \cup A_2$ where $A_1$ and $A_2$ are two distinct fuzzy matrices with entries from the interval [0,1]. Then $A=A_1 \cup A_2$ is called the fuzzy bimatrix".

In this section, operations on fuzzy matrices are defined and different types of fuzzy bimatrices are introduced with examples.

Basic definitions and properties of interval bimatrices, fuzzy interval bimatrices and neutrosophic interval bimatrices are studied with interesting examples in sections 1.4, 1.5 and 1.6 respectively.

Two new notions of graphs namely bigraphs and neutrosophic bigraphs which plays one role of representing the fuzzy models and neutrosophic models.

The first section of this chapter deals with bigraphs and matrix representation of bigraphs are introduced in the second chapter.

"$G=G_1 \cup G_2$ is said to be a bigraph if $G_1$ and $G_2$ are two graphs such that $G_1$ is not a subgraph of $G_2$ or $G_2$ is not a subgraph of $G_1$ , i.e., they have either distinct vertices or edges".

Different types of bigraphs namely vertex glued bigraph, edge glued bigraph, strong subgraph glued bigraph, weighted bigraph and directed bigraph are defined with examples.

Section 2.2 deals with neutrosophic bigraphs (Definition 2.2.12) and its properties. Different types of neutrosophic bigraphs are defined and its properties are studied.

To every neutrosophic graph G there is a neutrosophic matrix associated with it and with every neutrosophic bigraph, there is a neutrosophic bimatrix associated with it. Interesting examples are illustrated for neutrosophic matrix representation of a neutrosophic graph and neutrosophic bimatrix representation of a neutrosophic bigraph.

Fuzzy congnitive maps have a major role to play mainly when the data concerned is an unsupervised one. Further they help to analyze the data by directed graphs and connection matrices. Suppose some unsupervised data having two sets of disjoint attributes is to be analyzed using fuzzy cognitive maps, they would have two directed graphs or a bigraph related with it and so a bimatrix is a dynamical bisystem which will give the bihidden pattern. To work for such types of models and construct such models, fuzzy cognitive bimaps are introduced.

Suppose there are situations in which no relation can be determined between some of the nodes of an unsupervised data, then indeterminacy is involved in that model. To introduced indeterminacy in fuzzy cognitive maps, a generalized structure called neutrosophic cognitive maps are introduced.

Chapter III deals with fuzzy cognitive bimaps and neutrosophic cognitive bimaps.

In section 3.1 fuzzy cognitive bimaps are studied. Definition of fuzzy cognitive maps and several of its interrelated definitions with interesting examples are given. The definition of fuzzy cognitive bimaps is introduced.

In section 3.2 neutrosophic cognitive bimaps are studied.

Chapter IV devoted to the study of applications of bimatrices to some fuzzy models.

This chapter illustrates how bimatrices are utilized in fuzzy models. Here, a model which is analyzing the problem faced by an industry is considered. All problems faced while running an industry or a factory cannot be put as a statistical data. Several of them are feelings involving a great deal of uncertainty and impreciseness. In order to run the industry smoothly and with atleast some profit, one should try to analyze the problem. To get some sort of frictionless feelings among workers, among the financers and above all the relation between the workers and boss i.e., what we mean the relation between the employee and the employers. Thus to have good profit, the sales should be good which indirectly means the impact of their products in the public has a good standing and rapport. So the problem involved is multidimensional. Illustration of this model is given in this chapter.

Chapter V deals with the applications of neutrosophic matrices and neutrosophic bimatrices to neutrosophic models. As an application of neutrosophic matrices, the child labor problem prevalent in India is modeled and this model is demonstrated in the first part of this chapter.

As an application of neutrosophic bimatrices, a model which describes the important factors influencing good business is illustrated in the second part of this chapter.

Chapter VI deals with the applications of fuzzy interval matrices and fuzzy interval bimatrices to some fuzzy models. In the first part of this chapter, a new model called fuzzy cognitive interval maps model is introduced as an application of fuzzy interval matrices. A rough sketch of this model is given and a symptom disease model in children taking five attributes using three experts is illustrated.

Suppose there are several experts analyzing a problem and if each one of them accept to work with two sets of concepts with different numbers say m and n, $n \neq m$ and some opt to give opinion on m concepts and others opt to give opinion on n concepts, a new model called the fuzzy interval bimatrix model, which has the capacity to work on two sets of experts opinions taking the two sets of concepts is introduced. In the second part of this chapter, this fuzzy interval bimatrix model is constructed and demonstrated.