Introduction

Topology is a relatively new branch of mathematics; most of the research in topology has been done since 1900. Topology often does not solve a problem by itself, but contributes important understanding, settings, and tools. Initially developed in the late nineteenth century and early twentieth century to provide basis for abstract mathematical analysis, topology gradually became an influential subject, reaching many achievements in the mid and late twentieth century. Topology often does not stand alone: nowadays there are fields such as algebraic topology, differential topology, geometric topology, combinatorial topology, and quantum topology.

Topology is a mathematical subject that studies shapes. The term "Topology" comes from the Greek words "topos" (place) and "ology" (study). Topology is a part of geometry that does not concern distance. Topological objects are more relaxed than in geometry: beside moving around (allowed in geometry), stretching or bending are allowed in topology (not allowed in geometry). Topology thus literally means „The study of surfaces". It is also called as a "Science of Position". Topology features prominently in differential geometry, global analysis, algebraic geometry, theoretical physics etc.

General topology normally considers local properties of spaces and is closely related to analysis. It generalizes the concept of continuity to define topological spaces, in which limits of sequences can be considered. In Data Mining, the Computational Topology for Geometric Design and Molecular Design, Computer Aided Geometric Design and Engineering Design (briefly CAGD), Digital Topology, Information Systems, Non-Commutative Geometry and its application to Particle Physics one can observe the influence made in the realms of applied research by general topological spaces, properties and structures.

Computer Scientists concerned with problems in Pattern Recognition, Image Analysis and related areas have found that fundamental concepts of topology are useful tools. Although the usual definitions of topology are generally not studied to the analysis of digital pictures, they are easily modified so that notions such as connected component, continuous function, homotopy type, fundamental group, the Hausdorff metric and others can be efficiently and profitably employed.

The mathematical challenge of digital topology lies in the fact that a digital image is a lattice-point approximation of a Euclidean space. Since Euclidean metric imposes a discrete topology on a set of lattice points, it is necessary to use a non-Euclidean foundation as the basis of a theory that allows us to use topological properties in this setting.

Having Open sets as a powerful tool for defining topological spaces, Stone (1937) defined the notion of Regular open sets in his novel paper which related the theory of Boolean Rings to General Topology. Velicko (1968) introduced the concept of δ-open sets as a generalization of the regular open sets. This set was introduced to investigate the characterization of H-closed spaces. Levine (1963) studied the notion of semi-open sets as a weaker form of the open sets. He also initiated the study of generalized closed sets (briefly, g-closed sets) in 1970.

Norman Levine (1970) initiated the idea of continuous functions. Noiri, T (1980) introduced δ-continuous functions. Functions and of course irresolute functions at and among the most important and most researched points in the whole of mathematical science. In 1972, Crossley and Hilderband introduced the notion of irresoluteness.

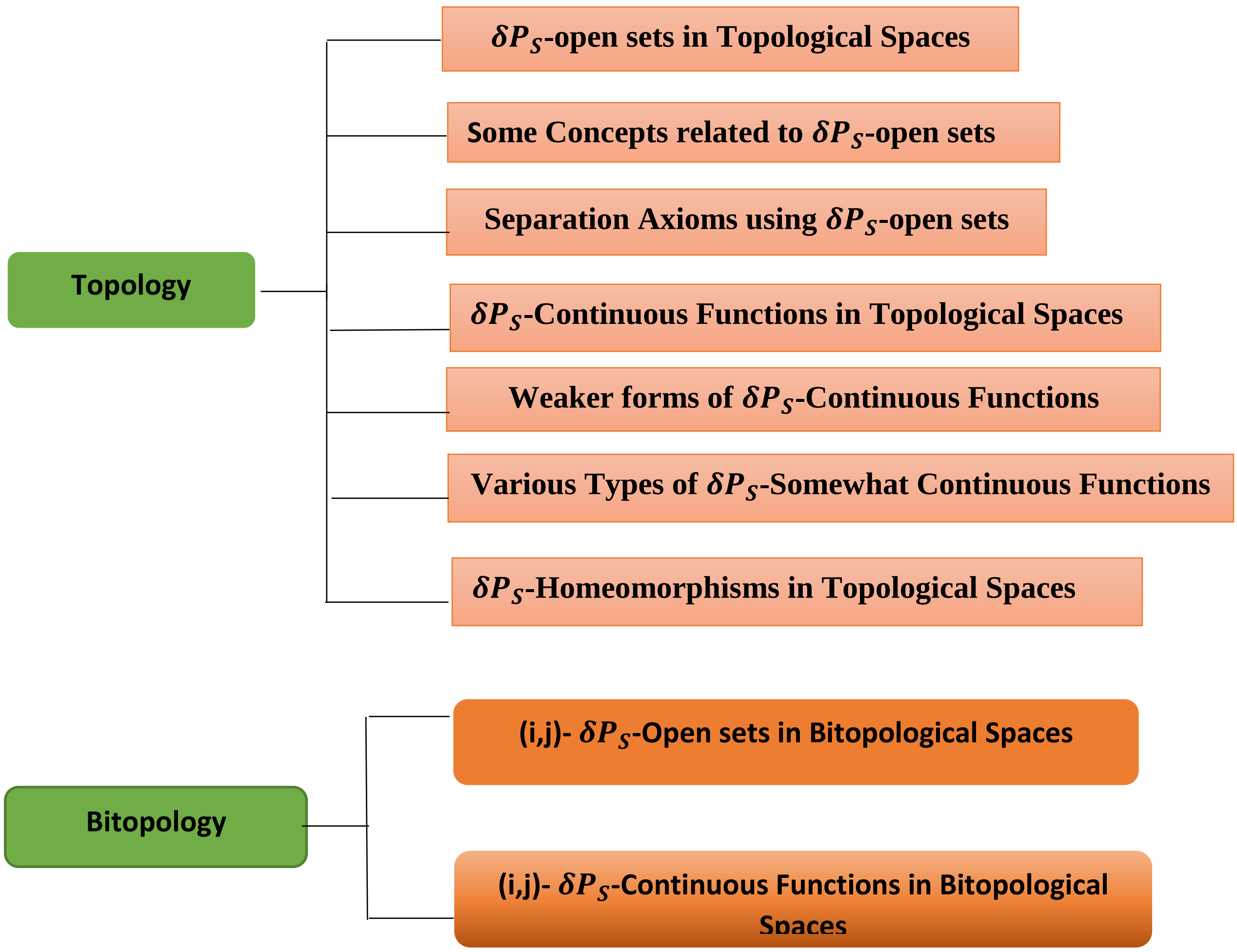
Open maps and closed maps are very useful in topological spaces. The concept of homeomorphisms plays an important role in topological spaces. For researchers on various closed sets, the study is not complete without extending their definitions to open (closed) maps and homeomorphisms.

Gentry (1971) introduced the concepts of somewhat continuous functions and somewhat open functions which are Zdenek Frolik's functions except that they had dropped the requirement that the functions be onto. These ideas are also closely related to the idea of weakly equivalent topologies which was first introduced by Youngblood (1965).

Kelly.J.C. (1963) introduced the idea of bitopological spaces and thereafter the theory has been developed by different mathematicians from different aspects. It is confined in considering the pairwise properties of the two topologies and their interrelations. Mohammed et al. provided the concept of δ-open sets in bitopological spaces.

The endeavour of the present work is to introduce the concept of δP_S-open sets, to analyze their relations with various open sets and to obtain their properties and some characterizations. This new notion is properly placed between P_S-open sets and δ-preopen sets.

The deliberations in this research work include the following topics.



Notations: Throughout the thesis, the following notations are used.

- (X, τ), (Y, σ) and (Z, η) denote non-empty topological spaces on which no separation axioms are mentioned, unless it is stated specifically.
- In all the diagrams, A → B represents A implies B but not conversely and A ←/→ B represents A and B are independent.

Methodology: The analysis has been done by the following methodologies.

- Analytical method of comparing δP_S-open sets with some existing open sets
- Constructing Counter examples
- Analysis of preservation of topological properties by δP_S-open sets
- Obtaining Characterization theorems

Chapter I deals with the study of the preliminary definitions and results which are used to accomplish the research work.

In **Chapter** the author has defined a new kind of open sets in the topological space (X, τ) called δP_S-open sets, combining the concepts of δ-preopen and semi-closed sets. This class of sets lies between the classes of P_S-open and δ-preopen sets. The behaviour of δP_S-open sets in various spaces such as locally indiscrete, hyperconnected, extremally disconnected, semi-T_1, s-regular spaces are discussed and various interesting results are obtained.

Important Definitions and Results:

- A subset A of a space X is called a $\boldsymbol{\delta P_S}$**-open set** if A is a δ- preopen set and for each x ∈ A, there exists a semi-closed set F such that x ∈ F⊆ A.

- A subset A of a space X is $\boldsymbol{\delta P_S}$**-open** if and only if A is a δ-preopen set and A is a union of semi-closed sets.

- Any union of δP_S-open sets is a δP_S-open set.

- Every regular open (resp., P_S-open, clopen, δ-open and θ-open) set is a δP_S-open set.

- If A and B are δP_S-open subsets of a topological space (X, τ) and if the family of all δ-preopen sets in X forms a topology on X, then $A \cap B$ is a δP_S-open set and hence the family of δP_S-open sets forms a topology on X.

The following diagram shows the relation between δP_S-open sets with some existing open sets in topological spaces:

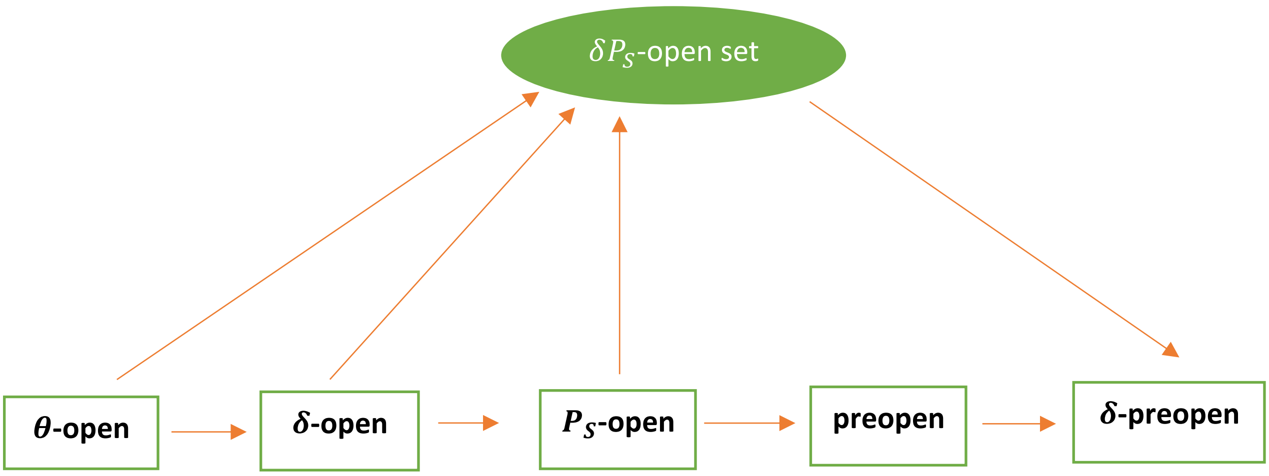


Figure 2.1

The following fascinating characterizations of δP_S-open sets in different topological spaces are also proved in this chapter.

- If a space X is semi-T_1, then $\delta P_S O(X) = \delta PO(X)$.
- In a hyperconnected space, $SC(X) \subseteq \delta P_S O(X)$.
- In a locally indiscrete space, $\delta P_S O(X) = \tau$.
- If A ϵ $\beta O(X) \cap P_S O(X)$, then $A \in \delta P_S O(X)$
- If a topological space (X, τ) is s-regular, then $\tau \subseteq \delta P_S O(X)$.
- For any space $(X, \tau), \delta P_S O(X, \tau) = P_S O(X, \tau_s)$
- For any topological space, if A ϵ $\delta PO(X)$ and either A ϵ $\eta O(X) \cup S\theta O(X)$, then A ϵ $\delta P_S O(X)$.
- Let (X, τ) be any extremally disconnected space. If $A \in \theta SO(X)$, then $A \in \delta P_S O(X)$.

➢ A subset B of a space X is called **δP_S-closed** if X\B is δP_S-open set. The family of all δP_S-closed subsets of a topological space (X,τ) is denoted by $\delta P_S C(X, \tau)$ or $\delta P_S C(X)$.

- The **δP_S -closure of A** (briefly $\delta P_S Cl(A)$) in a topological space (X, τ) is defined to be the intersection of all δP_S-closed sets containing A.
- For a topological space X δP_S-closure satisifies the axioms of Kuratowski's closure axioms.
- For any subsets a and B of X,
  - $\delta P_S Cl(A \cap B) \subseteq \delta P_S Cl(A) \cap \delta P_S Cl(B)$.
  - For $A \subseteq X, \delta P_S Cl(A) \subseteq Cl_\delta(A)$.

➢ A point $x \in X$ is said to be **δP_S- interior point** of A if there exists a δP_S-open set U containing x such that $U \subseteq A$. The set of all δP_S-interior points of A is said to be δP_S-interior of A and is denoted by **δP_S- $Int(A)$.**

Chapter III deals with various notions related to δP_S-open sets namely δP_S-neighborhood, δP_S -limit point, δP_S-derived set, δP_S -frontier, δP_S -boundary, δP_S -exterior and δP_S -saturated set. Properties related to these concepts are studied. Additionally, δP_S -open sets are characterized using the concept of grill in topological spaces. Necessary examples are obtained.

Important Definitions and results:

➢ A subset N of a topological space (X, τ) is called a $\boldsymbol{\delta P_S}$**-neighborhood** of $x \in X$ if there exists a δP_S-open set Q such that $x \in Q \subseteq N$. The set of all δP_S-neighborhood of x is denoted by $\boldsymbol{\delta P_S N(x)}$.

❖ If a subset N of a topological space X is δP_S-open, then N is a δP_S-neighborhood of each of its points.

❖ If F is a δP_S-closed subset of a topological space and $x \in X \setminus F$, then there exists a δP_S-neighborhood N of x such that $N \cap F = \emptyset$.

➢ Let A be a subset of a space X. A point $x \in A$ is said to be a $\boldsymbol{\delta P_S}$**-limit point** of A if for each δP_S-open set U containing x, $U \cap (A - \{x\}) \neq \emptyset$. The set of all δP_S-limit points of A is called the δP_S-derived set of A and is denoted by $D_{\delta P_S}(A)$.

❖ Let $A \subseteq X$. If A is δP_S-closed then $D_{\delta P_S}(A) \subseteq$ A.

❖ In a topological space X, for any subset A, $Cl_{\delta P_S}(A) = A \cup D_{\delta P_S}(A)$.

➢ For a subset A of a topological space (X, τ), $\boldsymbol{\delta P_S}$**-frontier** of A is denoted by $\delta P_S F(A)$ and defined as $\delta P_S F(A) = \delta P_S Cl(A) \setminus \delta P_S Int(A)$.

❖ For a subset A of a topological space (X, τ), the following results are true.

- $\delta P_S Cl(A) = \delta P_S Int(A) \cup \delta P_S F(A)$.
- $\delta P_S Int(A) \cap \delta P_S F(A) = \emptyset$.
- $\delta P_S F(A) = \delta P_S Cl(A) \cap \delta P_S Cl(X \setminus A)$.
- δP_SF(A) is δ-closed.
- δP_SF(A) = δP_SF(X \ A).
- $Fr_\delta(\delta P_S F(A)) \subseteq \delta P_S$F(A).

❖ Let A ⊆ B and δP_SInt(B) = ∅ then δP_S F(A) ⊆ δP_S F(B).

➢ For a subset A of a topological space (X, τ), $\boldsymbol{\delta P_S}$**-boundary** of A is denoted by $\boldsymbol{\delta P_S B(A)}$ and defined as δP_S B(A) = A \ δP_S Int(A).

- For a subset A of a topological space (X, τ), the following results are true.
  - $\delta P_S B(\emptyset) = \emptyset$.
  - δP_SB(X) = ∅.
  - A = δP_SInt(A) ∪ δP_SB(A).
  - If A is δP_S-open if and only if $\delta P_S B(A) = \emptyset$.
  - δP_SInt(A) ∩ δP_SB(A) = ∅.
  - δP_SB(δP_SInt(A)) = ∅.

- For a subset A of a topological space (X,τ), $\boldsymbol{\delta P_S}$**-exterior** of A is denoted by $\boldsymbol{\delta P_S}$**E(A)** and defined as δP_SE(A) = X \ δP_SCl(A).

- For a subset A of a topological space (X, τ), the following results are true.
  - Ext_δ(A) ⊆ δP_SE(A).
  - δP_SE(X) = ∅.
  - δP_SE(∅) = X.
  - δP_SE(A) = δP_SInt(X \ A).
  - If A ⊆ B then δP_SE(B)⊆δP_SE(A).
  - δP_SE(A ∪ B) ⊆ δP_SE(A) ∪ δP_SE(B).
  - δP_SE(A ∩ B) ⊇ δP_SE(A) ∩ δP_SE(B).
  - δP_SE(δP_SE(A)) = δP_SInt(δP_Scl(A)).
  - δP_SE(A) = δP_SE(X \ δP_SE(A)).
  - X = δP_SInt(A) ∪ δP_SE(A) ∪ δP_SF(A).
  - δP_SInt(A) ⊆ δP_SE(δP_SE(A)).

- A subset A of a topological space (X, τ) is said to be $\boldsymbol{\delta P_S}$**-saturated** if $\delta P_S Cl(\{x\}) \subseteq A$ for every x ∈ A. The set of all δP_S-saturated sets in (X, τ) is denoted by $\boldsymbol{\delta P_S}$**Sat(X).**

- Every δP_S-closed set is a δP_S-saturated set.

- Let $(X, \delta P_S O(\tau), \mathcal{G})$ be a grill δP_S-space. We define a function $\psi_{\delta P_S}\mathcal{G}: P(X) \to P(X)$, called the **$\delta P_S$ -operator** associated with the grill G and $\delta P_S O(\tau)$, and is defined by $\psi_{\delta P_S}\mathcal{G}(A) = \{x \in X | U \cap A \in \mathcal{G}$, for all δP_S-open set U containing $x\}$.

- Let $(X, \delta P_S O(\tau))$ be a δP_S -space and $\mathcal{G}, \mathcal{H}$ be two grills on X. Then for a subset A of X, the following conditions are valid:
    - $\mathcal{G} \subseteq \mathcal{H} \Rightarrow \psi_{\delta P_S}\mathcal{G}(A) \subseteq \psi_{\delta P_S}\mathcal{H}(A)$
    - $\psi_{\delta P_S}(\mathcal{G} \cup \mathcal{H})(A) \supseteq \psi_{\delta P_S}\mathcal{G}(A) \cup \psi_{\delta P_S}\mathcal{H}(A)$
- Let $(X, \delta P_S O(\tau), \mathcal{G})$ be a grill δP_S -space. Then for any two subsets A and B of X the following conditions hold:
    - $\psi_{\delta P_S}\mathcal{G}(A) \subseteq \psi_{\delta P_S}\mathcal{G}(B), if\ A \subseteq B.$
    - $\psi_{\delta P_S}\mathcal{G}(A \cup B) = \psi_{\delta P_S}\mathcal{G}(A) \cup \psi_{\delta P_S}\mathcal{G}(B)$
    - $\psi_{\delta P_S}\mathcal{G}(A) = \emptyset, if\ A \notin \mathcal{G}$
    - $\psi_{\delta P_S}\mathcal{G}(A) \subseteq \psi_{\delta}\mathcal{G}(A)$
    - $\psi_{\delta P_S}\mathcal{G}(A) \subseteq\ \delta P_S Cl(A).$
- Let $(X, \delta P_S O(\tau), \mathcal{G})$ be a grill δP_S -space. Then for any two subsets A and B of X, the following conditions hold:
    - $\psi_{\delta P_S}\mathcal{G}(A) \setminus \psi_{\delta P_S}\mathcal{G}(B) \subseteq \psi_{\delta P_S}\mathcal{G}(A \setminus B)$
    - If $B \notin \mathcal{G}, \psi_{\delta P_S}\mathcal{G}(A \cup B) = \psi_{\delta P_S}\mathcal{G}(A) = \psi_{\delta P_S}\mathcal{G}(A \setminus B)$
    - If $(A \setminus B) \cup (B \setminus A) \notin \mathcal{G}$, then $\psi_{\delta P_S}\mathcal{G}(A) = \psi_{\delta P_S}\mathcal{G}(B)$
- Let $(X, \delta P_S O(\tau), \mathcal{G})$ be a grill δP_S-space and $A \subseteq X$. If V is δP_S-open set containing x then $\psi_{\delta P_S}\mathcal{G}(A) = \psi_{\delta P_S}\mathcal{G}(V \cap A)$.

Chapter IV deals with separation axioms using δP_S-open sets which involves P_S-open sets. Spaces such as δP_S-T_i, $(i = 0,1,2)$ are defined. The properties of the associated spaces are studied. The author has also investigated the properties among all the spaces. The non-impliance of each is substantiated by counter examples. This chapter is committed to other type of separation axioms by δP_S-open sets namely, ${}_{\delta P_S}T_{\delta}$, ${}_{\delta P_S}T_{\theta}$, ${}_{\delta P_S}T_{P_S}$ and ${}_{\delta P}T_{\delta P_S}$. In each space the corresponding sets coincide

[For example in, $_{\delta P_S}T_\delta, \delta P_S O(X) = \delta O(X)$]. Also, δP_S-convergence, δP_S-accumulation, δP_S-open cover are defined and using these results δP_S-Compact space is defined. Some of the existing results were analysed using the newly defined spaces.

Important Definitions and Results:

- A topological space is **$\delta P_S T_0$-space** if to each pair of distinct points x,y of X, there exists a δP_S-open set containing one, but not the other.
- A topological space X is $\delta P_S T_0$ space if and only if $\delta P_S Cl\{x\} \neq \delta P_S Cl\{y\}$, for every pair of distinct points x, y of X.
- Every semi-regular subspace of a $\delta P_S T_0$ space is $\delta P_S T_0$ space.
- A function $f:(X,\tau) \to (Y,\sigma)$ is said to be a **point δP_S-closure 1-1** if $x, y \in X$ such that $\delta P_S Cl\{x\} \neq \delta P_S Cl\{y\}$, then $\delta P_S Cl\{f(x)\} \neq \delta P_S Cl\{f(y)\}$.
- If $f:(X,\tau) \to (Y,\sigma)$ is a point δP_S-closure 1-1 function and X is $\delta P_S T_0$ space, then f is 1-1.
- Let $f:(X,\tau) \to (Y,\sigma)$ be a mapping from $\delta P_S T_0$ space X into $\delta P_S T_0$ space Y. Then f is point δP_S closure 1-1 if and only if f is 1-1.
- Let $f:(X,\tau) \to (Y,\sigma)$ be an injective, continuous and open function. If Y is $\delta P_S T_0$, then X is $\delta P_S T_0$.
- A topological space X is **$\delta P_S T_1$-space** if to each pair of distinct points x, y of X, there exists a pair of δP_S-open sets containing x but not y, and the other containing y but not x.
- Every open (or semi-regular) subspace of a $\delta P_S T_1$ space is $\delta P_S T_1$-space.
- Let $f:(X,\tau) \to (Y,\sigma)$ be an injective, continuous and open function. If Y is $\delta P_S T_1$, then X is $\delta P_S T_1$-space.
- A topological space X is **$\delta P_S T_2$-space** if to each pair of distinct points x, y of X, there exists pair of disjoint δP_S-open sets, one containing x and the other containing y.
- For a topological space X, the following statements are equivalent:
    - X is $\delta P_S T_2$-space.
    - If $x \in X$, then there exists a δP_S-neighborhood N(x) of x such that $y \notin \delta P_S\, Cl(N(x))$.
    - For each $x \in X$, $\cap \{(\delta P_S\, Cl(N): N$ is a δP_Sneighborhood of $x\} = \{x\}$.
- Every open (or semi-regular) subspace of a $\delta P_S T_2$ space is $\delta P_S T_2$-space.
- Let $f:(X,\tau) \to (Y,\sigma)$ be an injective, continuous and open function. If Y is $\delta P_S T_2$, then X is $\delta P_S T_2$-space.

✓ The following diagram represents the implication between the separation axioms that we have defined in this paper and examples show that no other implications hold between them.

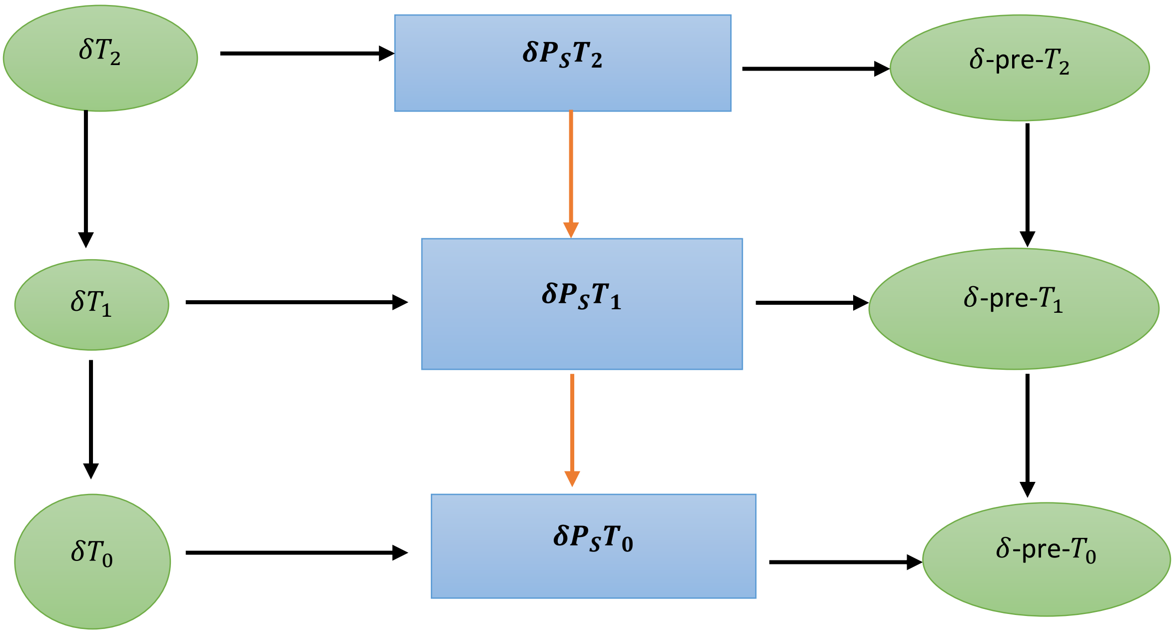


Figure 4.1

A topological space (X, τ) is said to be a

- $_{\delta P_s}T_{\delta}$**-space** if every δP_S-open subset of (X, τ) is δ-open in (X, τ).
- $_{\delta P_s}T_{\theta}$**-space** if every δP_S-open subset of (X, τ) is θ-open in (X, τ).
- $_{\delta P_s}T_{P_s}$**-space** if every δP_S-open subset of (X, τ) is P_S-open in (X, τ).
- $_{\delta P}T_{\delta P_s}$**-space** if every δ-pre-open subset of (X, τ) is δP_S-open in (X, τ).

❖ If X is semi-T_1, then X is $_{\delta P}T_{\delta P_s}$-space.

❖ If X is discrete, then X is $_{\delta P}T_{\delta P_s}$-space.

❖ Every $_{\delta P_s}T_{\delta}$-space is a hyperconnected space.

❖ In a space if $\theta O(X) = \delta PO(X)$, then (X, τ) is $_{\delta P_s}T_{\theta}$, $_{\delta P_s}T_{\delta}$, $_{\delta P_s}T_{P_s}$, and $_{\delta P}T_{\delta P_s}$-spaces.

❖ From the above definitions we have the following diagram:

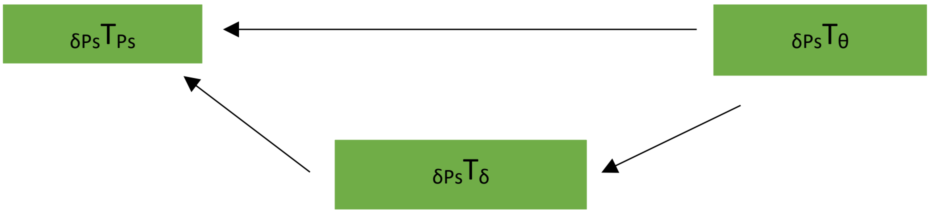


Figure 4.2

- A filter base $\mathfrak{J}$ in a topological space (X,τ) $\boldsymbol{\delta P_S}$**-converges** (resp., $\boldsymbol{\delta P_S}$**-θ-converges**) to a point $x \in X$ if for every δP_S-open set V containing x, there exists an F ∈ $\mathfrak{J}$ such that $F \subseteq V$ (resp., $F \subseteq \delta P_S Cl(V)$).

- A filter base $\mathfrak{J}$ in a topological space (X,τ) $\boldsymbol{\delta P_S}$**-accumulates** (resp., $\boldsymbol{\delta P_S}$**-θ-accumulates**) to a point $x \in X$ if $F \cap V \neq \emptyset$ (resp., $F \cap \delta P_S Cl(V) \neq \emptyset$), for every δP_S-open set V containing and every $F \in \mathfrak{J}$.

- If $\mathfrak{J}$ is a maximal filter base in a topological space (X,τ), then $\mathfrak{J}$ δP_S-converges (resp., δP_S-θ-converges) to a point $x \in X$ if and only if $\mathfrak{J}$ δP_S-accumulates (resp., δP_S-θ- accumulates) to a point x.

- Let $\mathfrak{J}$ be a filter base in a topological space (X,τ). If $\mathfrak{J}$ δP_S-converges to a point $x \in X$, then $\mathfrak{J}$ δ-converges to a point x.

- Let $\mathfrak{J}$ be a filter base in a topological space (X,τ) and E is any semi-closed set containing x. If there exists an $F \in \mathfrak{J}$ such that $F \subseteq E$ (resp., $F \subseteq \delta P_S Cl(E)$), then $\mathfrak{J}$ δP_S- converges (resp., δP_S-θ-converges) to a point $x \in X$.

- Let $\mathfrak{J}$ be a filter base in a topological space (X,τ) and E is any semi-closed set containing x. If there exists an $F \in \mathfrak{J}$ such that $F \cap E \neq \emptyset$ (resp., $F \cap \delta P_S Cl(E) \neq \emptyset$), then $\mathfrak{J}$ is δP_S-accumulation (resp., δP_S-θ-accumulation) to a point $x \in X$.

- If a function $f: (X,\tau) \rightarrow (Y,\sigma)$ is δP_S-continuous (resp., almost δP_S-continuous), then for each point $x \in X$ and each filter base $\mathfrak{J}$ in X δP_S-converging to x, the filter base $f(\mathfrak{J})$ is convergent (resp., δ-convergent) to $f(x)$.

- Let $\{Q_\alpha : \alpha \epsilon J\}$ be a collection of δP_S-Open subsets of X whose union is X. Then $\{Q_\alpha : \alpha \in J\}$ is called a δP_S-open cover, if for every $\boldsymbol{\delta P_S}$**-open cover** $\{Q_\alpha : \alpha \in J\}$ of X, there exists a finite subset J_0 of J such that $X = \cup \{Q_\alpha : \alpha \epsilon J_0\}$

- A topological space (X, τ) is called $\boldsymbol{\delta P_S}$**- Compact** if for every δP_S-open cover of X has a finite subcover.

- If every semi-closed cover of a space X has a finite subcover, then X is δP_S-compact.

- For a topological space δP_S-Compact space is a P_S-Compact Space
- If a topological space (X,τ) is strongly compact, then it is δP_S-compact.
- If a topological space (X, τ) is δP_S-compact then it is nearly compact.
- If (X, τ) is locally indiscrete, then X is compact if and only if X is δP_S-compact.

- If (X, τ) be s-regular and X is δP_S-compact, then it is compact.
- For any topological space (X, τ). The following statements are equivalent:
  - (X, τ) is δP_S-compact,
  - Every maximal filter base $\mathfrak{I}$ in X δP_S-converges to some point $x \in X$.
  - Every filter base $\mathfrak{I}$ in X δP_S-accumulates to some point $x \in X$.
  - For every family $\{F_\alpha : \alpha \in \Delta\}$ of δP_S-closed subsets of X such that $\cap \{F_\alpha : \alpha \in \Delta\} = \emptyset$, there exists a finite subset Δ_0 of Δ such that $\cap \{F_\alpha : \alpha \in \Delta_0\} = \emptyset$.

- A subset A of a topological space (X, τ) is said to be $\boldsymbol{\delta P_S}$**-set** (resp., $\boldsymbol{\delta P_S}$**-compact subspace**) if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of A by δP_S-open subsets of (X, τ) (resp., by δP_S-open subsets of A), there exists a finite subset Δ_0 of Δ such that $A \subseteq \cup \{V_\alpha : \alpha \in \Delta_0\}$ (resp., $A = \cup \{V_\alpha : \alpha \in \Delta_0\}$).

- A subset A of a space X is δP_S-set (resp., δP_S-compact subspace) if and only if for every cover of A by δP_S-open sets of X (resp., by δP_S-open sets of A) has a finite subcover.

- Let A be a subset of a topological space (X, τ). If every cover of A by semi-closed subsets of X (resp., by semiclosed subsets of A) has a finite subcover, then A is δP_S-set (resp., δP_S-compact subspace).

- For any topological space (X,τ). The following statements are equivalent:
  - A is δP_S-set (resp., δP_S-compact subspace),
  - Every maximal filter base $\mathfrak{J}$ on X which meets A δP_S-converges to some point of A,
  - Every filter base $\mathfrak{J}$ on X which meets A δP_S-accumulates to some point x ∈ X.
  - For every family $\{F_\alpha : \alpha \in \Delta\}$ of δP_S-closed subsets of (X, τ) such that

    $[\cap \{F_\alpha : \alpha \in \Delta\}] \cap A = \emptyset$, there exists a finite subset Δ_0 of Δ such that

    $[\cap \{F_\alpha : \alpha \in \Delta_0\}] \cap A = \emptyset$.
- A space X is δP_S-compact if and only if every proper δP_S-closed set of X is δP_S-set.
- If there exists either a proper regular semi-open or a δ-open subset A of a topological space (X, τ) such that A and $X \backslash A$ are δP_S-compact subspace, then X is also δP_S-compact.
- If either G ∈ RSO(X) or G ∈ τ or G is a δ-open set G of a space X is δP_S-compact subspace, then G is δP_S-set.
- If a regular open set G of a space X is δP_S-set, then G is δP_S-compact subspace.
- Let A and B be subsets of a space X. If A is δP_S- closed set and B is δP_S-set, then $A \cap B$ is δP_S-set.
- Let Y be any regular open subspace of a space X and A be any subset of Y. Then A is δP_S-set of X if and only if A is δP_S-set of Y.

Chapter V is on various types of continuities using δP_S-open sets. Here, δP_S-continuity, quasi δP_S-continuity, perfectly δP_S-continuity, totally δP_S-continuity, strongly δP_S-continuity and contra δP_S-continuity are defined with their properties being discussed. The association between these continuities is studied with the support of counter examples

Important Definitions and Results:

- A function $f : (X, \tau) \to (Y, \sigma)$ is called $\boldsymbol{\delta P_S}$**-continuous** at a point $x \in$ X if each open set V of Y containing f (x), there exists a δP_S-open set U of X containing x such that

$f(U) \subseteq V$. If f is δP_S-continuous at every point of X, then it is called δP_S-continuous.

- The following diagram indicates the relation between δP_S-continuous functions and the existing continuous functions:

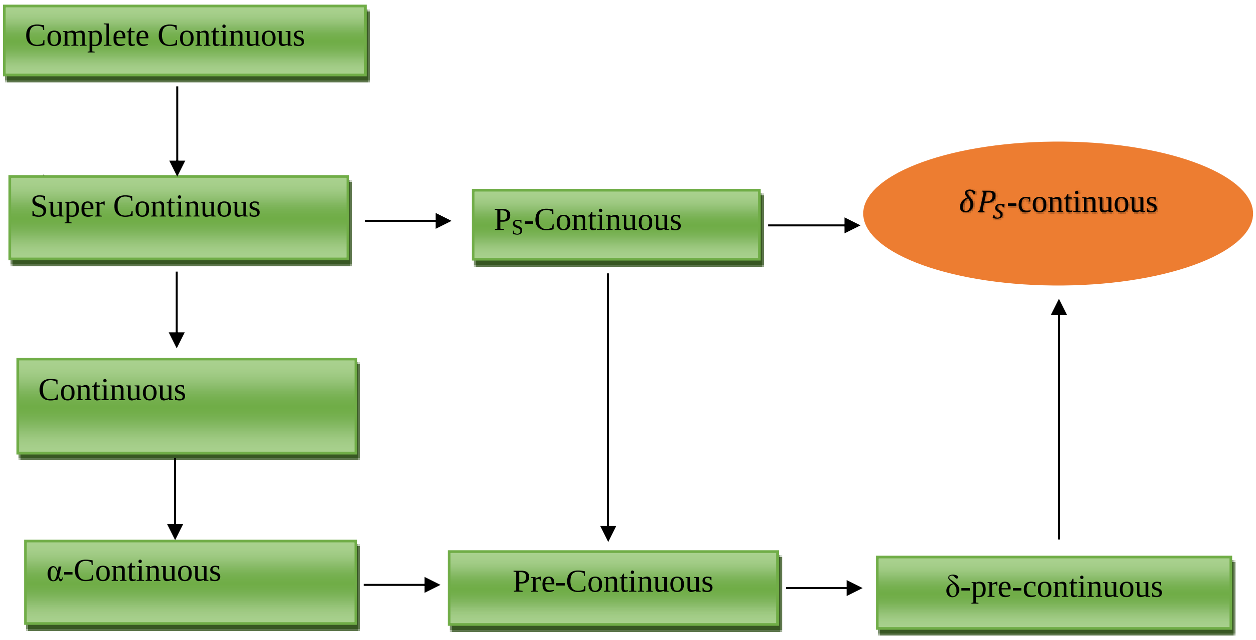


Figure 5.1

- For a function $f: (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:
  - f is δP_S-continuous.
  - $f^{-1}(V)$ is a δP_S-open set in X, for each open set V in Y.
  - $f^{-1}(F)$ is a δP_S-closed set in X, for each closed set F in Y.
  - $f(\delta P_S Cl(A)) \subseteq Cl(f(A))$, for each $A \subseteq X$.
  - $\delta P_S Cl(f^{-1}(B)) \subseteq f^{-1}(Cl(B))$, for each $B \subseteq Y$.
  - $f^{-1}(B) \subseteq \delta P_S Int(f^{-1}(B))$ for each $B \subseteq Y$.
  - $Int(f(A)) \subseteq f(\delta P_S Int(A))$, for each $A \subseteq$ X.

- A function $f: (X, \tau) \to (Y, \sigma)$ is δP_S-continuous if and only if f is δ-precontinuous and for each $x \in$ X and each open set V of Y containing $f(x)$, there exists a semi-closed set F in X containing x such that $f(F) \subseteq$ V.

- If $f: (X, \tau) \to (Y, \sigma)$ is a continuous and an open function and V is a δP_S-open set of Y, then $f^{-1}(V)$ is a δP_S-open set of X.
- Let X be an extremally disconnected space. If the function $f: (X, \tau) \to (Y, \sigma)$ is almost θs-continuous, then $f: (X, \tau) \to (Y, \sigma_S)$ is δP_S-continuous.
- Let $f: (X, \tau) \to (Y, \sigma)$ be a function. Let B be any basis for τ in Y. Then f is δP_S-continuous if and only if for each $B \in \mathcal{B}$, $f^{-1}(B)$ is a δP_S-open subset of X.
- The pasting lemma for δP_S-continuous functions is proved.
- A map $f: (X, \tau) \to (Y, \sigma)$ is called **perfectly $\boldsymbol{\delta P_S}$-continuous** if the inverse image of every δP_S-open set in (Y, σ) is a clopen set in (X, τ).
- Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be any two functions. Then their composition $g \circ f: (X, \tau) \to (Z, \eta)$ is perfectly δP_S-continuous if g is perfectly δP_S-continuous and f is continuous.
- Every strongly continuous function is a perfectly δP_S-continuous function.
- A function $f: (X, \tau) \to (Y, \sigma)$ is called **strongly $\boldsymbol{\delta P_S}$-continuous** if the inverse image of every subset of (Y, σ) is δP_S-clopen in (X, τ).
- If f is a strongly δP_S-continuous and g is any function then $g \circ f$ is a strongly δP_S-continuous.
- A function $f: (X, \tau) \to (Y, \sigma)$ is called **quasi $\boldsymbol{\delta P_S}$-continuous** if the inverse image of every δP_S-open set in (Y, σ) is open in (X, τ).
- A function $f: (X, \tau) \to (Y, \sigma)$is quasi δP_S-continuous if and only if the inverse image of every δP_S-closed set in (Y, σ) is closed in (X, τ).
- Let (X, τ) be a partition space, (Y, σ) be a topological space and $f: (X, \tau) \to (Y, \sigma)$ be any function. Then f is quasi δP_S-continuous.
- Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be any two functions. Then their composition $g \circ f: (X, \tau) \to (Z, \eta)$ is continuous if g is δP_S-continuous and f is quasi δP_S-continuous.
- A function $f: (X, \tau) \to (Y, \sigma)$ is called **totally $\boldsymbol{\delta P_S}$-continuous** if the inverse image of every open subset of (Y, σ) is δP_S- clopen in (X, τ)
- Every totally δP_S-continuous function is δP_S-continuous but not conversely.

- ❖ For topological spaces (X, τ), (Y, σ), (Z, η), $f:(X,\tau)\rightarrow(Y,\sigma)$, $g:(Y,\sigma)\rightarrow(Z,\eta)$ and $g\circ f:(X,\tau)\rightarrow(Z,\eta)$ then if f is a totally δP_S-continuous and g is a continuous (resp. super continuous) then $g\circ f$ is a totally δP_S-continuous.
  - ❖ Every strongly δP_S-continuous function is a totally δP_S-continuous function but not conversely.
- ➢ A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be **contra δP_S-continuous**, for every open subset V of Y, $f^{-1}(V)$ is δP_S-closed.

✓ The following figure explains the relation between contra δP_S-continuous functions and already existing continuous functions:

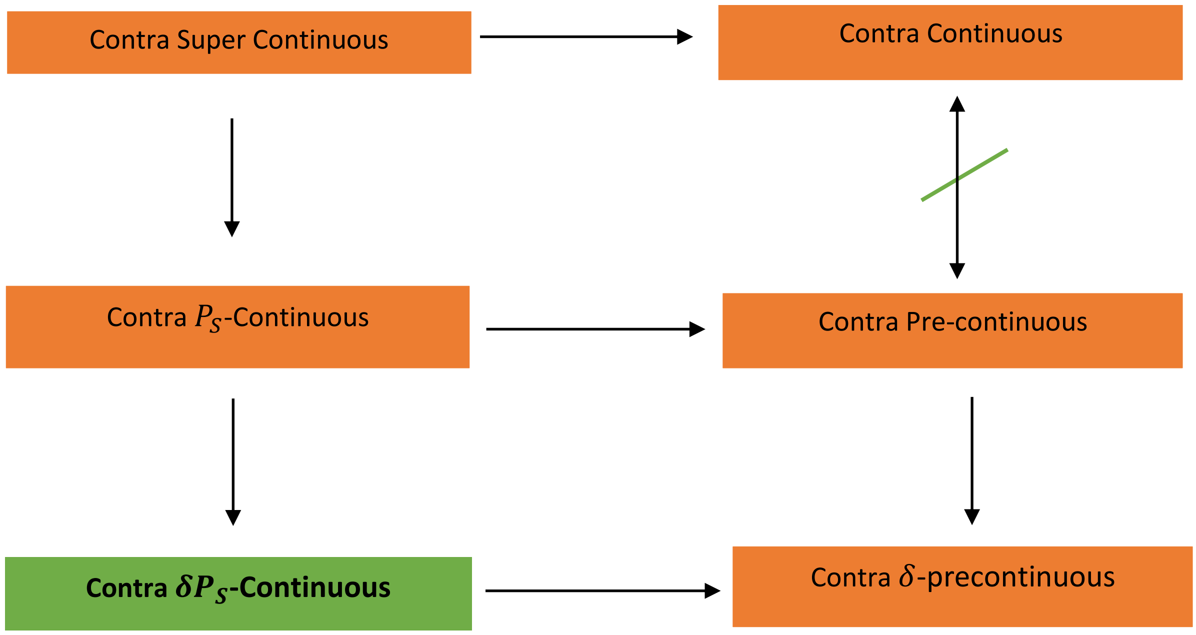


Figure 5.2

- ❖ For a function $f:(X,\tau)\rightarrow(Y,\sigma)$, the following statements are equivalent:
  - f is contra δP_S-continuous.
  - For every closed subset F of Y, $f^{-1}(F)$ is δP_S-open.
  - $f(\delta P_S Cl(A))\subseteq \ker(f(A))$ for every subset A of X.
  - $\delta P_S Cl(f^{-1}(B))\subseteq f^{-1}(\ker(B))$ for every subset B of Y.
- ❖ Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function and X is semi- T_1 (resp., locally indiscrete) space. Then f is contra δP_S-continuous if and only if f is contra δ-precontinuous (contra continuous).

- If the function $f:(X,\tau) \to (Y,\sigma)$ is almost contra δ-precontinuous and either S-continuous or θ-irresolute, then $f:(X,\tau) \to (Y,\sigma_S)$ is contra δP_S-continuous.
- Let X be a semi- T_1 (resp., locally indiscrete) space. A function $f:(X,\tau) \to (Y,\sigma)$ is almost contra δ-precontinuous (resp., (θ,s)-continuous) if and only if $f:(X,\tau) \to (Y,\sigma_S)$ is contra- δP_S-continuous.
- The pasting lemma for contra δP_S-continuous functions is proved.
- A function $f:(X,\tau) \to (Y,\sigma)$ is said to have **δP_S-closed graph** if for every $(x,y) \in X \times Y - G(f)$ there exists a δP_S-open set U of X such that $x \in U \subseteq X$ and an open set V such that $y \in V \subseteq Y$ for which $(x,y) \in U \times V \subseteq X \times Y - G(f)$.
- If $f:(X,\tau) \to (Y,\sigma)$ is contra δP_S-continuous and Y is T_2, then $G(f)$ is δP_S-closed.
    - A function $f:(X,\tau) \to (Y,\sigma)$ is said to have a **contra δP_S-closed graph** if, for every $(x,y) \in X \times Y - G(f)$, there exists a δP_S-open set U of X such that $x \in U \subseteq X$ and a closed set F such that $y \in F \subseteq Y$ for which $(x,y) \in U \times F \subseteq X \times Y - G(f)$.
- A function $f:(X,\tau) \to (Y,\sigma)$ is contra δP_S-continuous and Y is Urysohn, then $G(f)$ is contra δP_S-closed.
- Let $f_\alpha:(X,\tau) \to (Y_\alpha,\sigma)$ be a function for every $\alpha \in \mathcal{A}$ and let $f: X \to \prod_{\alpha\in\mathcal{A}} Y_\alpha$ be the product function given by $f(x) = (f_\alpha(x))_{\alpha\in\mathcal{A}}$ for every $x \in X$. If f is contra δP_S-continuous, then f_α is contra δP_S-continuous for every $\alpha \in A$.

Chapter VI gives a new weaker form of δP_S-continuity including almost δP_S-continuity and weakly δP_S-continuity. In this chapter, almost δP_S-continuity and weakly δP_S-continuity are introduced and studied. Characterizations and some relationships are investigated and obtained.

Important Definitions and Results:

- A function $f:(X,\tau) \to (Y,\sigma)$ is called **almost δP_S-continuous function** at a point $x \in$ X if for each open set V of Y containing $f(x)$, there exists a δP_S-open set U of X containing x such that f(U) $\subseteq$ IntCl(V). If f is almost δP_S-continuous at every point of X, then it is called almost δP_S-continuous.

- If a function $f: (X, \tau) \to (Y, \sigma)$ is δ-continuous, then f is almost δP_S-continuous.

- The association of almost δP_S-continuous functions with other continuous functions are depicted in the following diagram:

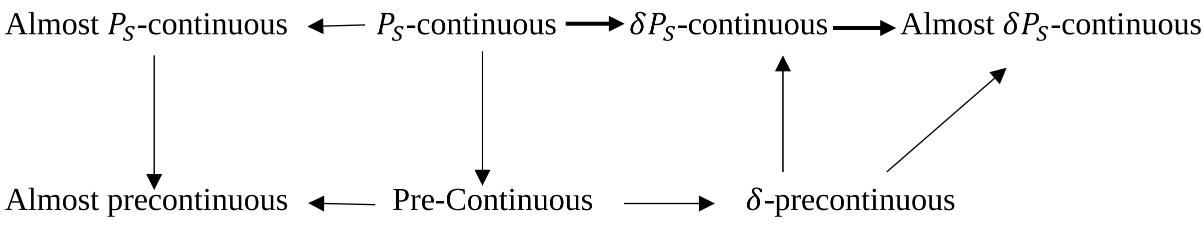


Figure 6.1

- Let $f: (X, \tau) \to (Y, \sigma)$ be an almost δP_S-continuous function and let V be any open subset of Y. If x ∈ δP_SCl $f^{-1}(V) \setminus f^{-1}(V)$, then f (x) ∈ δP_SCl V.

- If a function $f: (X, \tau) \to (Y, \sigma)$ is almost precontinuous. Then the following statements are equivalent:

  - f is almost δP_S-continuous.
  - For each x ∈ X and each open set V of Y containing f (x), there exists a semi-closed set F in X containing x such that f (F) ⊆ Int δCl V.
  - For each x ∈ X and each open set V of Y containing f (x), there exists a semi-closed set F in X containing x such that f (F) ⊆ sCl V.
  - For each x ∈ X and each regular open set V of Y containing f (x), there exists a semi-closed set F in X containing x such that f (F) ⊆ V.
  - For each x ∈ X and each δ-open set V of Y containing f (x), there exists a semi-closed set F in X containing x such that f (F) ⊆ V.

- A function $f: (X, \tau) \to (Y, \sigma)$ is almost δP_S-continuous if and only if $f: (X, \tau) \to (Y, \sigma_S)$ is δP_S-continuous.

- Let X be a locally indiscrete space. Then the function $f: (X, \tau) \to (Y, \sigma)$ is almost δP_S -continuous if and only if $f: (X, \tau) \to (Y, \sigma_S)$ is continuous.

- A function $f: (X, \tau) \to (Y, \sigma)$ is almost δP_S-continuous, if for each x ∈ X, there exists a regular open set A of X containing x such that f |A: A → Y is almost δP_S-continuous.

- The pasting lemma for almost δP_S-continuious functions is proved.
- Let $f:(X,\tau)\to(Y,\sigma)$ be almost δP_S-continuous surjection and A be either δ-open or regular semi-open subset of X. If f is an open function, then the function g : A → f (A), defined by g (x) = f (x) for each x ∈ A, is almost δP_S-continuous.
- Let $f:(X,\tau)\to(Y,\sigma)$ Y be almost δP_S-continuous. If Y is a preopen subset of Z, then $f:(X,\tau)\to(Z,\eta)$ is almost δP_S-continuous.
- The composition of almost δP_S-continuous functions need not be almost δP_S-continuous. So, some modifications are derived.
- If Y is a hyperconnected space, then every function $f:(X,\tau)\to(Y,\sigma)$ is almost δP_S-continuous.
- If a function $f:(X,\tau)\to(Y,\sigma)$ is almost δP_S-continuous and semi-open, then f (δP_SCl V) ⊆ δP_SCl f (V) for each open set V of X.
- If a function $f:(X,\tau)\to(Y,\sigma)$ is irresolute and preopen. Then f is almost δP_S-continuous if and only if δP_SCl f^{-1}(V) = f^{-1}(δP_SCl V) for each semi-open set V of Y.

➢ A function $f:(X,\tau)\to(Y,\sigma)$ is said to be **δP_S-preopen** if for every δP_S-open subset U of X, $f(U)$ is preopen.

- If $f:(X,\tau)\to(Y,\sigma)$ is δP_S-preopen and contra δP_S-continuous, then f is almost δP_S-continuous.

➢ A function $f:(X,\tau)\to(Y,\sigma)$ is called **weakly δP_S-continuous** if for each $x\in X$ and each open set V of Y containing f(x), there exists a δP_S-open set U of X containing x such that $f(U)\subseteq \delta Cl(V)$.

- The following figure explains the newly defined continuous function and the already existing functions:

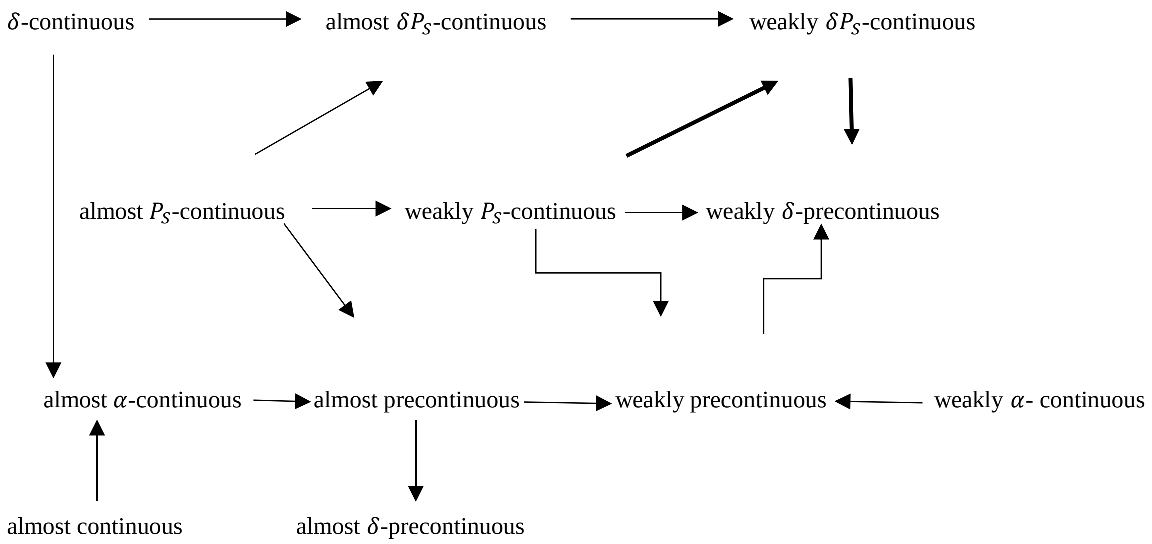


Figure 6.2

- If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is weakly δP_S-continuous, then for each x ∈ X and each θ-open set V of Y containing f (x), there exists a δP_S- U in X containing x such that $f(U)\subseteq V$.
- Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function. If for each $x\in X$ and each regular closed set R of Y containing $f(x)$, there exists a δP_S-open set U in X containing x such that $f(U)\subseteq R$, then f is weakly δP_S-continuous.
- If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is weakly δP_S-continuous, then the inverse image of each θ- open set of Y is a δP_S-open set in X.
- If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is weakly δP_S-continuous, then for each $x\in X$ and each open set V of Y containing f (x), there exists a semi-closed set F in X containing x such that $f(F)\subseteq Cl(V)$.
- For a function $f:(X,\tau)\rightarrow(Y,\sigma)$, thefollowing statements are equivalent:
  - f is weakly δP_S-continuous.
  - $\delta P_S Clf^{-1}(IntCl(B))\subseteq f^{-1}(Cl(B))$ for each $B\subseteq Y$
  - $f^{-1}(Int(B))\subseteq\ \delta P_S Intf^{-1}(Cl\big(Int(B)\big))$ for each $B\subseteq Y$
  - $f^{-1}(Int(Cl\,V))\subseteq\ \delta P_S Intf^{-1}(ClV)$ for each open setV of Y
  - $f^{-1}(V)\subseteq\delta P_S Int(f^{-1}(Cl(V))$ for each regular open set V of Y.
  - $\delta P_S(Cl\left(f^{-1}\big(Int(F)\big)\right)\subseteq f^{-1}(F)$, for each regular closed set F of Y.

- $\delta P_S(Cl(f^{-1}(Int(F))) \subseteq f^{-1}(Cl(Int(F)))$, for each closed set F of Y.
- $\delta P_S(Cl(f^{-1}(V))) \subseteq f^{-1}(Cl(V))$, for each open set V of Y.
- $f^{-1}(Int(F)) \subseteq \delta P_S(Int(f^{-1}(F))$, for each closed set F of Y.

❖ For a function $f:(X,\tau) \to (Y,\sigma)$, thefollowing statements are equivalent:

- f is weakly δP_S-continuous.
- $f(\delta P_S Cl(A)) \subseteq Clf(A)$, for each subset A of X.
- $Int_\theta f(A) \subseteq f(\delta P_S IntA)$, for each subset A of X.
- $f^{-1}(Int_\theta B) \subseteq \delta P_S Intf^{-1}(B)$, for each subset B ofY.
- $\delta P_S Clf^{-1}(B) \subseteq f^{-1}(Cl_\theta B)$, for each subset B of Y.

❖ A function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous if and only if $\delta P_S Clf^{-1}(IntCl_\theta(B)) \subseteq f^{-1}(Cl_\theta(B))$ for each subset B of Y.

❖ If a function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous, then $f:(X,\tau) \to (Y,\sigma_\theta)$ is δP_S-continuous.

❖ Let X be a locally indiscrete space. Then the function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous if and only if $f:(X,\tau) \to (Y,\sigma_\theta)$ is continuous.

❖ The pasting lemma for weakly δP_S-continuous functions is derived.

❖ Let $f:(X,\tau) \to (Y,\sigma)$ be a weakly δP_S-continuous function and for each x ∈ X. If Y is any subset of Z containing f(x), then $f:(X,\tau) \to (Z,\eta)$ is weakly δP_S-continuous.

❖ The composition of weakly δP_S-continuous functions need not be weakly δP_S-continuous. So, some modifications are derived.

❖ If $f:(X,\tau) \to (Y,\sigma)$ is a weakly δP_S-continuous function and Y is almost regular (resp., Extremally disconnected space), then f is almost δP_S-continuous.

❖ Let $f:(X,\tau) \to (Y,\sigma)$ be a function and X is a semi-T_1 space. Then f is weakly δP_S-continuous if and only if f is weakly δ-precontinuous.

❖ If $f:(X,\tau) \to (Y,\sigma)$ is a semi-continuous function. Then f is weakly continuous if and only if f is weakly δP_S-continuous.

❖ If $f:(X,\tau) \to (Y,\sigma)$ is weakly θs-continuous and weakly δ-precontinuous, then f is weakly δP_S-continuous.

- If a function $f:(X,\tau)\to(Y,\sigma)$is semi-continuous and almost open, then f is weakly δP_S- continuous if and only if δP_SClf^{-1}(V) = f^{-1}(δP_SClV)for each open set V of Y.

In **Chapter VII** author deals with the concepts of somewhat continuity using δP_S-open sets. Various types of somewhat functions related to δP_S-open sets namely, somewhat δP_S-continuous, somewhat almost δP_S-continuous, somewhat δP_S-irresolute, somewhat δP_S-open and somewhat almost δP_S -open functions are defined. Many properties and characterizations of newly defined somewhat functions are presented.

Important Definitions and Results:

- A function f is said to be **somewhat $\boldsymbol{\delta P_S}$-continuous** if for $U\in\sigma$ and $f^{-1}(U)\neq\emptyset$, there exists a non-empty δP_S-open set V in (X,τ) such that $V\subseteq f^{-1}(U)$.
- Every δP_S-continuous function is somewhat δP_S-continuous function.
- If f is somewhat δP_S-continuous and g is continuous, then $g\circ f$ is somewhat δP_S-continuous.
- A subset A of (X,τ) is δP_S-dense in (X,τ) if there is no proper δP_S-closed set C in (X,τ) such that $A\subseteq C\subseteq X$.
- For a surjective function f, the following statements are equivalent:
  - f is somewhat δP_S-continuous.
  - If C is a closed subset of (Y,σ) such that $f^{-1}(C)\neq X$, then there is a proper δP_S-closed subset D of (X,τ) such that $f^{-1}(\mathcal{C})\subseteq D$.(Equivalently, if $\mathcal{C}$ is an open subset of (Y,σ) such that $f^{-1}(C)\neq X$, then there exists a proper δP_S-open subset D of (X,τ) such that $f^{-1}(C)\subseteq D$).
  - If A is a δP_S-dense subset of (X,τ), then $f(A)$ is a dense subset of (Y,σ).
- If X is a set and τ and σ are topologies on X, then τ is said to be δP_S**-equivalent** to σ provided if $U\in\sigma$ and U ≠ ∅, then there is a non-empty δP_S-open set V in (X,τ) and $V\subseteq U$.
- The results using δP_S-equivalence are obtained here.
- A function f is said to be **somewhat almost $\boldsymbol{\delta P_S}$-continuous** if for every $U\in\delta PO(Y,\sigma)$ and $f^{-1}(U)\neq\emptyset$ there exists a non-empty $V\in\delta P_SO(X,\tau)$ such that $V\neq\emptyset$ and $V\subseteq f^{-1}(U)$.

- Every somewhat almost δP_S-continuous function is a somewhat δP_S-continuous.
- The composition of two somewhat almost δP_S-continuous functions is somewhat almost δP_S-continuous.
- A function $f:(X,\tau) \to (Y,\sigma)$ is said to be **somewhat δP_S-irresolute** if for U ∈ δP_SO (Y, σ) and $f^{-1}(U) \neq \emptyset$, there exists a non-empty δP_S-open set V in (X, τ) such that V $\subseteq f^{-1}(U)$.
- If f and g are somewhat δP_S-irresolute functions from $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$ then $g \circ f$ is δP_S-irresolute.
- Let f be a function and X = A∪B, where A, B ∈ RO(X, τ). Then, if the restriction functions $f|_A$: (A; $\tau|_A$) → (Y, σ) and $f|_B$: (B; $\tau|_B$) → (Y, σ) are somewhat δP_S-irresolute, then f is a somewhat δP_S-irresolute function.
- The results using δP_S-equivalence are obtained here.

- A function f is said to be **somewhat δP_S-open** provided that if U∈ τ and U ≠ ∅, then there exists a non-empty δP_S-open set V in (Y, σ) such that V ⊆ f(U).

- Let f be an open function and g be somewhat δP_S-open. Then $g \circ f$ is somewhat δP_S-open.
- For a bijective function f, the following are equivalent:
  - f is somewhat δP_S-open.
  - If C is a closed subset of (X, τ), such that f(C) ≠ Y, then there is a δP_S-closed. subset D of (Y, σ) such that D ≠ Y and f(C) ⊆ D.
- f is somewhat δP_S-open if and only if for a non-empty open subset A⊆ (X, τ), δP_SInt(f(A)) ≠ ∅.
- The following statements are equivalent:
  - f is somewhat δP_S-open
  - If A is a δP_S-dense subset of (Y, σ), then f^{-1}(A) is a dense subset of (X, τ).
- Let f be somewhat δP_S-open and A be any open subset of (X, τ). Then $f|_A$: (A, $\tau|_A$) → (Y, σ) is somewhat δP_S-open.

- Let f be a function and X = A∪B, where $A, B \in \tau$. Then, if the restriction functions $f|_A$ and $f|_B$ are somewhat δP_S-open, then f is somewhat δP_S-open.

- Two topologies τ and τ^* for X are δP_S-equivalent if and only if the identity functions $i_\tau : (X, \tau) \to (X, \tau^*)$ and $i_{\tau^*} : (X, \tau^*) \to (X, \tau)$ are both somewhat δP_S-open.

- A function $f: (X, \tau) \to (Y, \sigma)$ is said to be **somewhat almost δP_S-open** provided that if U ∈ δPO(X, τ) and U ≠ ∅, then there exists a non-empty δP_S-open set V in (Y, σ) such that V ⊆ f(U).

- Every somewhat almost δP_S-open function is a somewhat δP_S-open function.

- The composition of two somewhat almost δP_S-open functions is a somewhat almost δP_S-open function.

- If $f: (X, \tau) \to (Y, \sigma)$ is somewhat almost δP_S-open then for a subset A ∈ $\delta PO(X, \tau)$, $\delta P_S Int(f(A)) \neq \emptyset$.

- Let f be a function and X = A∪B, where $A, B \in \delta PO(X, \tau)$. Then if $f|_A$ and $f|_B$ are somewhat almost δP_S-open, then f is somewhat almost δP_S-open.

  - ✓ The following table depicts the composition of defined continuous functions:

Functions	Composition
Somewhat δP_S-continuous	✗
Somewhat almost δP_S-continuous	✓
Somewhat δP_S-irresolute	✓
Somewhat δP_S-open	✗
Somewhat almost δP_S-open	✓

In **Chapter VIII**, notions of δP_S-open and δP_S-closed functions are taken for study and their behaviours are characterized in locally indiscrete space. δP_S-irresoluteness and contra δP_S-irresoluteness are introduced and their properties relating to composition are analyzed. Characteristics of these functions by inducing surjection, bijection on various types of continuity are presented. Two types of homeomorphisms namely δP_S-homeomorphism and $\delta P_S C$ -homeomorphism are developed and their properties are obtained. It is observed that the set of all $\delta P_S C$ -homeomorphisms form a group under composition of functions.

Important Definitions and Results:

- A function $f:(X,\tau)\to(Y,\sigma)$ is
  - $\boldsymbol{\delta P_S}$ **-open** if f(A) is δP_S-open for every open set A in X.
  - $\boldsymbol{\delta P_S}$**-closed** if f(A) is δP_S-closed for every closed set A in X.

- Let $f:(X,\tau)\to(Y,\sigma)$ be a map with $f\big(Int(A)\big)\subseteq \delta P_S Int\big(f(A)\big)$, for every $A\subseteq X$. Then f is δP_S-open.

- Let $f:(X,\tau)\to(Y,\sigma)$ be δP_S-closed then $\delta P_S Cl(f(A))\subseteq f(Cl(A))$, for each $A\subseteq X$.
- A bijective function $f:(X,\tau)\to(Y,\sigma)$ is δP_S -closed iff for each subset B in Y and each open set U in X containing $f^{-1}(B)$, there exists a δP_S -open set V in Y such that
  - $B\subseteq V$ and
  - $f^{-1}(V)\subseteq U$.

- Let $f:(X,\tau)\to(Y,\sigma)$ be any function with (Y,σ) being a locally indiscrete space. Then every δP_S-open function is an open function.

- The composition of δP_S-open functions need not be δP_S-open. So, some modifications are derived.

- If a function $f:X\to Y$ is δP_S-open then for each $x\in X$ and for each neighborhood U of $x\in X$, there exists a δP_S -neighborhood W of $f(x)$ such that $W\subseteq f(U)$.

- A function $f:(X,\tau)\to(Y,\sigma)$ is called
  - $\boldsymbol{\delta P_S}$ **-irresolute** if $f^{-1}(B)$ is δP_S-open in (X,τ) for every δP_S-open set B in (Y,σ).
  - **contra** $\boldsymbol{\delta P_S}$**-irresolute** if $f^{-1}(B)$ is δP_S-open in (X,τ) for every δP_S-open set B in (Y,σ).

- For a function $f:(X,\tau)\to(Y,\sigma)$ with (Y,σ). being a locally indiscrete space, every δP_S-continuous function is δP_S-irresolute.

- Composition of two δP_S-irresolute functions is a δP_S-irresolute function.

- For topological spaces $(X,\tau),(Y,\sigma),(Z,\eta), f:(X,\tau)\to(Y,\sigma), g:(Y,\sigma)\to(Z,\eta)$ and

$g \circ f: (X, \tau) \rightarrow (Z, \eta)$ the following results are true.

- If f is a δP_S-irresolute function and g is a δP_S -continuous (resp. totally δP_S-continuous, strongly δP_S-continuous, contra δP_S-continuous, contra δP_S-irresolute) function then $g \circ f$ is a δP_S -continuous (resp. totally δP_S-continuous, strongly δP_S-continuous, contra δP_S-continuous, contra δP_S -irresolute) function.

- If f is a contra δP_S-irresolute (resp. strongly continuous, quasi δP_S-continuous, perfectly δP_S-continuous) function and g is a δP_S-irresolute function then $g \circ f$ is a contra δP_S- irresolute (resp. strongly continuous, quasi δP_S-continuous, perfectly δP_S-continuous) function.

❖ If $f: (X, \tau) \rightarrow (Y, \sigma)$ is δP_S -irresolute then

- $A \subseteq X, f(\delta P_S Cl(A) \subseteq Cl_\delta(f(A))$.

- $B \subseteq Y, \delta P_S Cl(f^{-1}(B) \subseteq f^{-1}(Cl(A))$

❖ If $f: (X, \tau) \rightarrow (Y, \sigma)$ is surjective, δP_S-irresolute and open with (X, τ) being a locally indiscrete space then every δP_S-open set is open in (Y, σ).

➢ A bijective function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called **δP_S -homeomorphism** if f is both δP_S-continuous and δP_S-open.

❖ If a bijective function $f: (X, \tau) \rightarrow (Y, \sigma)$ is δP_S -continuous then the following statements are equivalent.

- f is δP_S-open;
- f is δP_S -homeomorphism;
- f is δP_S-closed.

❖ Every δP_S-homeomorphism from a locally indiscrete space into another locally indiscrete space is a homeomorphism.

❖ Every δP_S-homeomorphism from a ${}_{\delta P_S}T_\delta$-space into another ${}_{\delta P_S}T_\delta$-space is a homeomorphism.

- A bijective function $f: (X, \tau) \to (Y, \sigma)$ is called **$\delta P_S \mathcal{C}$ -homeomorphism** if both f and f^{-1} are δP_S -irresolute.

- Composition of two $\delta P_S \mathcal{C}$ -homeomorphisms is a $\delta P_S \mathcal{C}$ -homeomorphism.

- The set of all $\delta P_S \mathcal{C}$ -homeomorphisms of (X, τ) onto itself denoted by $\delta P_S \mathcal{C}h(X, \tau)$ forms a group under composition of functions.

- Let $f: (X, \tau) \to (Y, \sigma)$ be a $\delta P_S \mathcal{C}$ -homeomorphism. Then f induces an isomorphism from the group $\delta P_S \mathcal{C}\ h(X, \tau)$ onto the $\delta P_S \mathcal{C}\ h(Y, \sigma)$.

In **Chapter IX**, A parallel study of the work done on δP_S-open sets is developed in bitopological spaces. The various concepts analyzed in previous chapters are extended to bitopological spaces. A new class of open sets called (i, j) δP_S-open sets are established and their association between various existing generalized notions of (i, j) δP_S-open sets are studied in bitopological spaces. Also, (i, j) δP_S-closed sets and various properties like boundary, frontier, interior were defined and their properties are also derived using (i, j) δP_S-open sets.

Further (i, j) δP_S-continuous functions in bitopological spaces, are defined and their properties are discussed in this chapter. Different forms of continuities namely, contra (i, j) δP_S, quasi (i, j) δP_S, almost (i, j) δP_S and weakly (i, j) δP_S-continuous functions are defined and their properties are analyzed.

Important definitions in this chapter:

- A subset A of a bitopological space (X, τ_1, τ_2) is said to be **(i, j)-δP_S-open**, if A is a j-δ-preopen set and for all x in A, there exists an i-semiclosed set F such that $x \in F \subseteq A$.

- A subset B of a space X is called **(i, j) δP_S-closed** if $X \backslash B$ is (i, j) δP_S-open. The family of all (i, j) δP_S-closed subsets of bitopological space (X, τ_1, τ_2) is denoted by (i, j) $\delta P_S C(X, \tau_1, \tau_2)$ or (i, j) $\delta P_S C(X)$.

- If A is a subset of a bitopological space (X, τ_1, τ_2), then **the (i, j) δP_S-interior ((i, j) $\delta P_S Int(A)$)**, the **(i, j) δP_S-closure ((i, j) $\delta P_S Cl(A)$)** and the **(i, j) δP_S-boundary ((i, j) $\delta P_S Bd(A)$** of A are defined as follows:

- $(i,j)\ \delta P_S Cl(A) = \cap \{F: A \subseteq F, X - F \in (i,j)\ \delta P_S O(X)\}$.
- $(i,j)\ \delta P_S Int(A) = \cup \{U: U \subseteq A, U \in (i,j)\ \delta P_S O(X)\}$.
- $(i,j)\ \delta P_S Bd(A) = (i,j)\ \delta P_S Cl(A) - (i,j)\ \delta P_S Int(A)$.

- A subset N of a bitopological space (X, τ_1, τ_2) is called **(i,j) δP_S-neighbourhood** of a subset A of X if there exists an $(i,j)\ \delta P_S$-open set U such that $A \subseteq U \subseteq N$. When $A = \{x\}$, we say that N is $(i,j)\ \delta P_S$- neighbourhood of x.
- Let A be a subset of a bitopological space X, A point x ∈ X is said to be **(i,j) δP_S-limit point** of A if for each $(i,j)\ \delta P_S$-open set U containing $x, U \cap A \neq \emptyset$. The set of all $(i,j)\ \delta P_S$-limit point of A is called $(i,j)\ \delta P_S$-derived set of A and is denoted by $(i,j)\ \delta P_S D(A)$.
- A function $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called **(i,j) δP_S-continuous** at a point $x \in X$, if for each i-open set V of Y containing $f(x)$, there exists $(i,j)\ \delta P_S$-open U of X containing x such that $f(U) \subseteq V$. If f is $(i,j)\ \delta P_S$-continuous at every point x of X, then it is called $(i,j)\ \delta P_S$-continuous.
- A function $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called **almost (i,j) δP_S-continuous** at a point $x \in X$, if for each i-open set V of Y containing $f(x)$, there exists $(i,j)\ \delta P_S$-open U of X containing x such that $f(U) \subseteq i\ Int(i\ Cl(V))$. If f is almost $(i,j)\ \delta P_S$-continuous at every point x of X, then it is called almost $(i,j)\ \delta P_S$-continuous.
- A function $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called **contra (i,j) δP_S-continuous** if $f^{-1}(V)$ is $(i,j)\ \delta P_S$-closed in X for every j-open V of Y.