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Applications of Intuitionistic Fuzzy Generalized γ Closed Sets

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Abstract- ***In this paper, we have investigated the theoretical applications of intuitionistic fuzzy generalized γ closed sets and obtained some important theorems.***

Keywords- ***Intuitionistic fuzzy topology, Intuitionistic fuzzy generalized γ closed sets, intuitionistic fuzzy γ $T_{1/2}$ spaces.***

I. INTRODUCTION

The notion of intuitionistic fuzzy sets was introduced by Atanassov[6] as a generalization of fuzzy sets. In 1997, Coker[2] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we have investigated the theoretical applications of intuitionistic fuzzy generalized γ closed sets and obtained some important theorems.

II. PRELIMINARIES

Definition 2.1[6]: An *intuitionistic fuzzy set*(IFS in short) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2[6]: Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

a) $A \subseteq B$ in and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
b) $A = B$ in and only if $A \subseteq B$ and $A \supseteq B$;
c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$;
d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$;
e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0\sim = \langle x, 0, 1\rangle$ and $1\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3[2]: An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFS, in X satisfying the following axioms:

(i) $0\sim, 1\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4[5]: An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) *intuitionistic fuzzy γ closed set* (IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$
(ii) *intuitionistic fuzzy γ open set* (IFγOS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$

Definition 2.5[5]: Let A be an IFS in an IFTS (X, τ). Then the γ-interior and γ-closure of A are defined as

$\gamma int(A) = \cup \{G / G$ is an IFγOS in X and $G \subseteq A\}$

$\gamma cl(A) = \cap \{K / K$ is an IFγCS in X and $A \subseteq K\}$

Note that for any IFS A in (X, τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A)^c = (\gamma cl(A))^c$.

Result 2.6[8]: Let A be an IFS in (X, τ), then

(i) $\gamma cl(A) \supseteq A \cup (cl(int(A)) \cap int(cl(A)))$
(ii) $\gamma int(A) \subseteq A \cap (cl(int(A)) \cap int(cl(A)))$

Definition 2.7[8]: An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy generalized γ closed set* (IFGγCS for short) if $\gamma cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ). The family of all IFGγCSs of an IFTS (X, τ) is denoted by IFGγC(X).

Definition 2.8[9]: The complement A^c of an IFGγCS A in an IFTS (X, τ) is called an *intuitionistic fuzzy generalized γ open set* (IFGγOS in short) in X.
The family of all IFGγOSs of an IFTS (X, τ) is denoted by IFGγO(X).
Definition 2.9[10]: An IFTS (X, τ) is said to be $IFT_{1/2}$ space if every IFGCS in(X, τ) is an IFCS in (X, τ).
Definition 2.10[1]: An *intuitionistic fuzzy point* (IFP in short), written as $p_{(\alpha,\beta)}$, is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \ \beta) & \text{if } x = p \\ (0, \ 1) & \text{otherwise} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

III. APPICATIONS OF INTUITIONISTIC FUZZY GENERALIZED γ CLOSED SETS

In this section we have discussed some theoretical applications of intuitionistic fuzzy generalized γ closed sets.
Definition 3.1: An IFTS (X, τ) is called an intuitionistic fuzzy γ $T_{1/2}$ ($IF\gamma T_{1/2}$ in short) space if every IFGγCS is an IFγCS in X.
Example 3.2: Let X = {a, b} and $G_1 = \langle x, (0.5_a, 0.5_b), (0.4_a, 0.5_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.5_b)\rangle$. Then τ = {0~, G_1, G_2, 1~} is an IFT on X.
Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a + \nu_b \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.
Thus (X, τ) is an $IF\gamma T_{1/2}$ space, as every IFGγCS is an IFγCS in (X, τ).
Definition 3.3: An IFTS (X, τ) is called an intuitionistic fuzzy γ_g $T_{1/2}$ ($IF\gamma_g T_{1/2}$ in short) space if every IFGγCS is an IFCS in X.
Definition 3.4: An IFTS (X, τ) is called an intuitionistic fuzzy pre γ $T_{1/2}$ ($IFP\gamma T_{1/2}$ in short) space if every IFGγCS is an IFPCS in X.
Definition 3.5: An IFTS (X, τ) is called an intuitionistic fuzzy semi γ $T_{1/2}$ ($IFS\gamma T_{1/2}$ in short) space if every IFGγCS is an IFSCS in X.
Definition 3.6: An IFTS (X, τ) is called an intuitionistic fuzzy α γ $T_{1/2}$ ($IF\alpha\gamma T_{1/2}$ in short) space if every IFGγCS is an IFαCS in X.
Remark 3.7: Not every $IF\gamma T_{1/2}$ space is an $IFT_{1/2}$ space. This can be seen easily by the following example.
Example 3.8: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x,(0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then (X, τ) is an IFTS.
Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / whenever $\mu_a \geq 0.5$, $\mu_b \geq 0.3$, whenever $\mu_a \leq 0.4$, $\mu_b < 0.2$ and $0 \leq \mu_b + \nu_b \leq 1$}.
Since every IFGγCS is an IFγCS in (X, τ), it is an $IF\gamma T_{1/2}$ space. Let $A = \langle x, (0.5_a, 0.8_b), (0.5_a, 0.2_b)\rangle$, and A ⊆ 1~, then cl(A) = G_2^c ⊆ 1~. Hence A is an IFGCS in X but cl(A) = $G_2^c \neq A$, A is not an IFCS in X. Therefore (X, τ) is not an $IFT_{1/2}$ space.
Theorem 3.9: Every $IFP\gamma T_{1/2}$ space is an $IF\gamma T_{1/2}$ space but not conversely.
Proof: Let (X, τ) be an $IFP\gamma T_{1/2}$ space and let A be an IFGγCS in X. By hypothesis, A is an IFPCS in X. Since every IFPCS is an IFγCS[2], A is an IFγCS in X. Hence (X, τ) is an $IF\gamma T_{1/2}$ space.
Example 3.10: Let X={a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then (X, τ) is an IFTS.
Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 \leq \mu_b + \nu_b \leq 1$}.
Since every IFGγCS is an IFγCS in (X, τ), it is an $IF\gamma T_{1/2}$ space. Let $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ be an IFS in X. Now since cl(int(A)) = cl(G_2) = $G_1^c \nsubseteq A$, A is not an IFPCS in X. Therefore (X, τ) is not an $IFP\gamma T_{1/2}$.
Theorem 3.11: Every $IF\alpha\gamma T_{1/2}$ space is an $IF\gamma T_{1/2}$ space but not conversely.
Proof: Let (X, τ) be an $IF\alpha\gamma T_{1/2}$ space and let A be an IFGγCS in X. By hypothesis, A is an IFαCS in X. Since every IFαCS is an IFγCS[2], A is an IFγCS in X. Hence (X, τ) is an $IF\gamma T_{1/2}$ space.
Example 3.12: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then (X, τ) is an IFTS.
Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 \leq \mu_b + \nu_b \leq 1$}.
Since every IFGγCS is an IFγCS in (X, τ), it is an $IF\gamma T_{1/2}$ space. Let $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ be an IFS in X. Now since cl(int(cl(A))) = cl(int(G_1^c)) = cl(G_1) = $G_1^c \nsubseteq A$, A is not an IFαCS in X. Therefore (X, τ) is not an $IF\alpha\gamma T_{1/2}$.
Theorem 3.13: Every $IFS\gamma T_{1/2}$ space is an $IF\gamma T_{1/2}$ space but not conversely.
Proof: Let (X, τ) be an $IFS\gamma T_{1/2}$ space and let A be an IFGγCS in X. By hypothesis, A is an IFSCS in X. Since every IFSCS is an IFγCS[2], A is an IFγCS in X. Hence (X, τ) is an $IF\gamma T_{1/2}$ space.
Example 3.14: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then (X, τ) is an IFTS.
Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 \leq \mu_b + \nu_b \leq 1$}.
Since every IFGγCS is an IFγCS in (X, τ), it is an $IF\gamma T_{1/2}$ space. Let $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ be an IFS in X. Now since int(cl(A)) = int(G_1^c) = $G_1 \nsubseteq A$, A is not an IFSCS in X. Therefore (X, τ) is not an $IFS\gamma T_{1/2}$.
Theorem 3.15: An IFTS (X, τ) is an IFγ $T_{1/2}$ space if and only if IFGγO(X) = IFγO(X)

Proof: Necessity: Let A be an IFGγOS in (X, τ), then A^c is an IFGγCS in X. By hypothesis A^c is an IFγCS in (X, τ). Hence A is an IFγOS in (X, τ). Then IFGγO(X) = IFγO(X).

Sufficiency: Let A be an IFGγCS in (X, τ). Then A^c is an IFGγOS in (X, τ). By hypothesis A^c is an IFγOS in (X, τ). Therefore A is an IFγCS in (X, τ). Hence (X, τ) is an IFγ $T_{1/2}$ space.

Theorem 3.16: An IFTS (X, τ) is an IFγ_g $T_{1/2}$ space if and only if IFGγO(X) = IFO(X)

Proof: Necessity: Let A be an IFGγOS in (X, τ), then A^c is an IFGγCS in X. By hypothesis A^c is an IFCS in (X, τ). Hence A is an IFOS in (X, τ). Then IFGγO(X) = IFO(X).

Sufficiency: Let A be an IFGγCS in (X, τ). Then A^c is an IFGγOS in (X, τ). By hypothesis A^c is an IFOS in (X, τ). Therefore A is an IFCS in (X, τ). Hence (X, τ) is an IFγ_g $T_{1/2}$ space.

Theorem 3.17: Let (X, τ) is an IFγ_g $T_{1/2}$ space. Then

(i) Any union of IFGγCS is an IFGγCS

(ii) Any intersection of IFGγOS is an IFGγOS

Proof: (i) Let $\{A_i\}_{i \in J}$ be any collection of IFGγCSs. Since (X, τ) is an IFγ_g $T_{1/2}$ space, every IFGγCS is an IFCS. But any union of IFCSs is an IFCS, $\bigcup_{i \in J} A_i$ is also an IFCS in (X, τ). Therefore $\bigcup_{i \in J} A_i$ is an IFGγCS in X, as every IFCS is an IFGγCS.

(ii) It can be proved that taking complement in (i).

Theorem 3.19: Let (X, τ) be an IFγ_g $T_{1/2}$ space. Then A is an IFGγOS in X if and only if for every IFP $p_{(\alpha, \beta)} \in A$, there exists an IFGγOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Proof: Necessity: If A is an IFGγOS in X, then we can take B = A, so that $p_{(\alpha, \beta)} \in B \subseteq A$ for every IFP $p_{(\alpha, \beta)} \in A$.

Sufficiency: Let A be an IFS in (X, τ) and assume that there exists an IFGγOS in X such that $p_{(\alpha, \beta)} \in B \subseteq A$. Since X is an IF$\gamma_g$ $T_{1/2}$ space, B is an IFOS in X. Then $A = \cup_{p(\alpha, \beta) \in A} p_{(\alpha, \beta)} \subseteq \cup_{p(\alpha, \beta) \in A} B \subseteq A$. Therefore $A = \cup_{p(\alpha, \beta) \in A} B$ which is an IFOS. Then A is an IFGγOS in X.[9]

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