Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with A+ Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test II –October 2024

Semester V

Class : III UG **Time : 2 Hours**
Branch : Mathematics / Special Education and Mathematics **Max. Marks : 60**

21BMAC13/ 21BSMC13 - Abstract Algebra I

Course Outcomes:

CO1: Analyse the properties of groups and rings.

CO2: Apply Lagrange's Theorem to analyze the cyclic subgroups of a group.

CO3: Explore the concepts of isomorphism and homomorphism for groups and rings.

CO4: Prove standard theorems in groups and rings.

CO5: Demonstrate examples for ideals and quotient rings.

Part – A **6 x 1 = 6**

Choose the Correct Answer

1. Every finite group having more than .... elements has a non trivial automorphism. **CO3K3**

a) 6 b) 2 c) 24 d) 120

2. Which of the following is an example of field? **CO4K2**

a) (Z, +, .) b) (Z^+, +, .) c) (Z, $+_6$, $.6$) d) ring of integers modulo p,(p prime)

3. D is an integral domain with finite characteristic then characteristic of D is a/an ..... number. **CO4K3**

a) Prime b) composite c) even d) odd

4. If R is ring, and $R = R'$ and define $\varphi(a) = 0, \forall a \in R$, then kernel of ϕ is **CO4K2**

a) 0 b) R c) {1} d) {e}

5. If U is an ideal of R and $1 \in U$, then **CO5K2**

a) $U \neq R$ b) $U = R$ c) $U \subseteq R$ d) $R \subseteq U$

6. If I is an ideal of the ring R then unit element of R/I is **CO5K2**

a) 0 b) 1 c) R d) 1 + I

Part - B **3 x 6 = 18**

7.a) Prove that if G is a finite group, and $H \neq G$ is a subgroup of G such that $o(G) \dagger i(H)!$ then H must contain a non trivial normal subgroup of G. In particular G must be simple. **CO3K4**

(or)

7. b) Let G be a group and φ be an automorphism of G. If a ϵ G is of order o(a) > 0, then o(φ(a)) = o(a). **CO3K4**

8. a) Define ring with example **CO4K5**

(or)

8. b) Prove that a finite integral domain is a field. **CO4K5**

9. a) Let R be a commutative ring with unit element whose only ideals are $\{0\}$ and R itself. Then prove that R is a field. **CO5K4**

(or)

9. b) Prove that if U is an ideal of the ring R, then R/U is a ring and is a homomorphic image of R. **CO5K4**

Part - C **3 x 12 = 36**

10.a) State and prove Cayley's theorem **CO3K4**

(or)

10.b) If G is a group, H is a subgroup of G and S is the set of all right cosets of H in G, then show that there is a homomorphism θ of G into A(S) and the kernel of θ is the largest normal subgroup of G which is contained in H . **CO3K3**

11.a) The homomorphism of Φ of R into R` is an isomorphism if and only if I(Φ) = (0). **CO4K4**

(or)

11. b) If R is the set of integers mod 7 under the addition and multiplication mod 7 then show that R is a commutative ring with unit element. **CO4K3**

12. a) Prove that R is a commutative ring with unit element and M is an ideal of R, then M is a maximal ideal of R if and only if R/M is a field. **CO5 K4**

(or)

12. b) Prove that every integral domain can be imbedded in a field. **CO5K4**

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