



# *Chapter I*

**CHAPTER-1**  
**GENERALIZED LOCALLY CLOSED SETS AND**  
**GLC-CONTINUOUS FUNCTION**

In this chapter Generalized locally closed sets and GLC continuous functions due to Balachandran, Sundaram and Maki [5] are discussed. Properties, characterizations of these concepts are studied.

**SECTION: 1.1**  
**GENERALIZED LOCALLY CLOSED SETS.**

**Definition: 1.1.1**

A subset  $S$  of  $(X, \tau)$  is called **g-closed** if  $\text{cl}(S) \subset G$  whenever  $S \subset G$  and  $G$  is open in  $(X, \tau)$ .

**Definition: 1.1.2**

A subset  $S$  of  $(X, \tau)$  is called **g-open** if its complement  $X - S$  is g-closed.

**Proposition: 1.1.3**

Every closed set is a g-closed.

**Remark: 1.1.4**

The converse of proposition 1.1.3 need not be true.

**Proposition: 1.1.5**

Every open set is a g-open.

**Remark: 1.1.6**

The converse of proposition 1.1.5 need not be true.

**Definition: 1.1.7**

A subset  $S$  of  $(X, \tau)$  is called **locally closed** if  $S = G \cap F$  where  $G \in \tau$  and  $F$  is closed in  $(X, \tau)$ .

**Proposition: 1.1.8**

A subset  $S$  of  $(X, \tau)$  is locally closed if and only if its complement  $X - S$  is the union of an open set and a closed set.

**Proposition: 1.1.9**

Every open subset of  $X$  is locally closed.

**Proposition: 1.1.10**

Every closed subset of  $X$  is locally closed.

**Proposition: 1.1.11**

The complement of a locally closed set need not be locally closed.

**Definition: 1.1.12**

A subset  $S$  of  $(X, \tau)$  is called **generalized locally closed (glc)** if  $S = G \cap F$  where  $G$  is  $g$ -open in  $(X, \tau)$  and  $F$  is  $g$ -closed in  $(X, \tau)$ .

**Remark: 1.1.13**

- i) Every  $g$ -closed set is generalized locally closed.
- ii) Every  $g$ -open set is generalized locally closed.

**Notation: 1.1.14**

i) The collection of all locally closed sets of  $(X, \tau)$  will be denoted by  $LC(X, \tau)$ .

ii) The collection of all generalized locally closed sets of  $(X, \tau)$  will be denoted by  $GLC(X, \tau)$ .

**Definition: 1.1.15**

Let  $S$  be a subset of  $(X, \tau)$ . Then  $S \in GLC^*(X, \tau)$  if there exist a  $g$ -open set  $G$  and a closed set  $F$  of  $(X, \tau)$ , such that  $S = G \cap F$ .

**Definition: 1.1.16**

Let  $S$  be a subset of  $(X, \tau)$ . Then  $S \in GLC^{**}(X, \tau)$  if there exist an open set  $G$  and a  $g$ -closed set  $F$  of  $(X, \tau)$ , such that  $S = G \cap F$ .

**Proposition: 1.1.17**

Let  $S$  be a subset of  $(X, \tau)$

i) If  $S$  is locally closed, then  $S \in GLC^*(X, \tau)$  and  $S \in GLC^{**}(X, \tau)$ , however not conversely.

ii) If  $S \in GLC^*(X, \tau)$  or  $S \in GLC^{**}(X, \tau)$  then  $S$  is  $GLC$ .

**Proof:**

i) As  $S$  is locally closed,  $S = G \cap F$  where  $G \in \tau$  and  $F$  is closed in  $(X, \tau)$ . As every open set is  $g$ -open set,  $G$  is  $g$ -open and hence  $S \in GLC^*(X, \tau)$ .

As every closed set is  $g$ -closed,  $F$  is  $g$ -closed and  $G$  is open hence  $S \in GLC^{**}(X, \tau)$ .

ii) Let  $S \in GLC^*(X, \tau)$ . Then there exists a  $g$ -open set  $G$  and a closed set  $F$  of  $(X, \tau)$  such that  $S = G \cap F$ . Since every closed set is  $g$ -closed set,  $F$  is a  $g$ -closed set of  $(X, \tau)$ . Hence there exists a  $g$ -open set  $G$  and a  $g$ -closed set  $F$  of  $(X, \tau)$  such that  $S = G \cap F$ . Therefore  $S$  is  $GLC$ .

Let  $S \in GLC^{**}(X, \tau)$ . Then there exists an open set  $G$  and a  $g$ -closed set  $F$  of  $(X, \tau)$  such that  $S = G \cap F$ . Since every open set is  $g$ -open,  $G$  is  $g$ -open.

Hence there exists a g-open set  $G$  and a g-closed set  $F$  of  $(X, \tau)$  such that  $S = G \cap F$ . Therefore  $S$  is GLC.

**Example: 1.1.18**

Let  $X = \{a,b,c\}$  and  $\tau = \{ \phi, \{a\}, X \}$  then  $LC(X, \tau) = \{ \phi, \{a\}, \{b,c\}, X \}$  and  $GLC^*(X, \tau) = GLC^{**}(X, \tau) = GLC(X, \tau) = P(X)$  because  $\phi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}$  and  $\{b,c\}$  are the g-closed sets of  $(X, \tau)$ . Then  $LC(X, \tau)$  is a proper subset of  $GLC(X, \tau)$ .

**Theorem: 1.1.19**

For a subset  $S$  of  $(X, \tau)$ , the following are equivalent

- i)  $S \in GLC^*(X, \tau)$ .
- ii)  $S = P \cap cl(S)$  for some g-open set  $P$ .
- iii)  $cl(S) - S$  is g-closed.
- iv)  $S \cup (X - cl(S))$  is g-open.

**Proof:**

i)  $\Rightarrow$  ii)

Let  $S$  be a subset of  $X$  such that  $S \in GLC^*(X, \tau)$ . Since  $S \in GLC^*(X, \tau)$  there exist a g-open set and closed set  $F$  such that  $S = P \cap F$ . since  $S \subset P$  and  $S \subset cl(S)$ ,  $S \subset P \cap cl(S)$ . Conversely, Since  $cl(S) \subset F$ ,

$$S = P \cap F \supset P \cap cl(S).$$

Therefore  $S = P \cap cl(S)$ .

ii)  $\Rightarrow$  i)

Let  $S = P \cap cl(S)$  for some g-open set  $P$ . Since  $P$  is g-open and  $cl(S)$  is closed,  $S \in GLC^*(X, \tau)$ .

ii)  $\Rightarrow$  iii)

Let  $S = P \cap \text{cl}(S)$  for some  $g$ -open set  $P$ .  $\text{cl}(S) - S = \text{cl}(S) \cap (X - P)$  is  $g$ -closed.

iii)  $\Rightarrow$  ii)

Let  $\text{cl}(S) - S$  be  $g$ -closed. Let  $U = X - (\text{cl}(S) - S)$ . Then  $U$  is  $g$ -open and by set theory  $S = U \cap \text{cl}(S)$ . Hence the result.

iii)  $\Rightarrow$  iv)

Let  $\text{cl}(S) - S$  be  $g$ -closed. Let  $F = \text{cl}(S) - S$ . Then  $X - F$  is  $g$ -open and  $X - F = S \cup (X - \text{cl}(S))$

$\Rightarrow S \cup (X - \text{cl}(S))$  is  $g$ -open.

iv)  $\Rightarrow$  iii)

Let  $S \cup (X - \text{cl}(S))$  be  $g$ -open.  $X - U = \text{cl}(S) - S$ . Since  $X - U$  is  $g$ -closed,  $\text{cl}(S) - S$  is  $g$ -closed.

**Remark: 1.1.20**

As can be seen from example 1.1.18 it is not true that  $S \in \text{GLC}^*(X, \tau)$  if and only if  $S \subset \text{int}(S \cup (X - \text{cl}(S)))$ .

**Definition: 1.1.21**

A topological space  $(X, \tau)$  is called  **$g$ -submaximal** if and only if every dense subset is  $g$ -open.

**Proposition: 1.1.22**

A topological space  $(X, \tau)$  is  $g$ -submaximal if and only if  $P(X) = \text{GLC}^*(X, \tau)$ .

**Proof:**

**Necessity:**

Let  $(X, \tau)$  be  $g$ -submaximal. Let  $S \in P(X)$  and let  $U = S \cup (X - \text{cl}(S))$  then  $\text{cl}(U) = \text{cl}(S) \cup \text{cl}(X - \text{cl}(S)) \Rightarrow X = \text{cl}(U)$  (i.e)  $U$  is a dense subset of  $(X, \tau)$ . By assumption  $U$  is  $g$ -open therefore by Theorem 2.10,  $S \in \text{GLC}^*(X, \tau)$ . Hence  $P(X) = \text{GLC}^*(X, \tau)$ .

**Sufficiency:**

Let  $P(X) = \text{GLC}^*(X, \tau)$  and let  $S$  be a dense subset of  $(X, \tau)$ . Since  $P(X) = \text{GLC}^*(X, \tau)$  and  $S \cup (X - \text{cl}(S))$  is  $g$ -open. As  $S \cup (X - \text{cl}(S)) = S$ ,  $S \in \text{GLC}^*(X, \tau)$  and  $S$  is  $g$ -open and hence  $(X, \tau)$  is  $g$ -submaximal.

**Remark: 1.1.23**

It follows from definitions that every submaximal space is  $g$ -submaximal. The converse need not be true i.e every  $g$ -submaximal space need not be submaximal. The topological space  $(X, \tau)$  of example 1.1.19 is  $g$ -submaximal since  $P(X) = \text{GLC}^*(X, \tau)$  holds. But  $(X, \tau)$  is not submaximal, since  $\text{LC}(X, \tau) \neq P(X)$ .

**Definition: 1.1.24**

Intersection of all  $g$ -closed set containing  $S$  is called  $\text{cl}^*(S)$ .

**Proposition: 1.1.25**

For a subset  $S$  of  $(X, \tau)$ , if  $S \in \text{GLC}^{**}(X, \tau)$  then there exists an open set  $P$  Such that  $S = P \cap \text{cl}^*(s)$

**Proof:**

Let  $S$  be a subset of  $(X, \tau)$  and  $S \in \text{GLC}^{**}(X, \tau)$ . Then there exist an open set  $P$  and a  $g$ -closed set  $F$  such that  $S = P \cap F$ . Since  $S \subseteq P$  and

$S \subseteq \text{cl}^*(S)$ ,  $S \subseteq P \cap \text{cl}^*(S)$ . Conversely, since  $\text{cl}^*(S) \subseteq F (= \text{cl}^*(F))$  holds,  $S = P \cap F \supseteq P \cap \text{cl}^*(S)$  and hence  $S = P \cap \text{cl}^*(S)$ .

**Lemma: 1.1.26**

If  $P$  and  $Q$  are  $g$ -open sets, then  $P \cap Q$  is a  $g$ -open set.

**Proof:**

Let  $P$  and  $Q$  be  $g$ -open sets. Let  $A = X - P$  and  $B = X - Q$ . Then  $A$  and  $B$  are  $g$ -closed sets and  $A \cup B$  is  $g$ -closed

$$\begin{aligned} P \cap Q &= (X - A) \cap (X - B) \\ &= (X - (A \cup B)) \end{aligned}$$

Therefore  $P \cap Q$  is  $g$ -open.

**Corollary: 1.1.27**

Let  $A$  be a  $g$ -closed set and  $F$  is closed set, then  $A \cap F$  is a  $g$ -closed set.

**Proposition: 1.1.28**

Let  $A$  and  $B$  be subsets of  $(X, \tau)$

- i) If  $A \in \text{GLC}^*(X, \tau)$  and  $B \in \text{GLC}^*(X, \tau)$  then  $A \cap B \in \text{GLC}^*(X, \tau)$ .
- ii) If  $A \in \text{GLC}^{**}(X, \tau)$  and  $B$  is closed or open then  $A \cap B \in \text{GLC}^{**}(X, \tau)$ .
- iii) If  $A \in \text{GLC}(X, \tau)$  and  $B$  is  $g$ -open or closed then  $A \cap B \in \text{GLC}(X, \tau)$ .

**Proof:**

- i) Let  $A \in \text{GLC}^*(X, \tau)$  and  $B \in \text{GLC}^*(X, \tau)$  it follows from theorem 1.1.20 (ii) then there exists an g-open sets  $P$  and  $Q$  such that  $A = P \cap \text{cl}(A)$  and  $B = Q \cap \text{cl}(B)$  .

$$\begin{aligned} A \cap B &= (P \cap \text{cl}(A)) \cap (Q \cap \text{cl}(B)) \\ &= (P \cap Q) \cap (\text{cl}(A) \cap \text{cl}(B)) \end{aligned}$$

Since  $P \cap Q$  is g-open by lemma 1.1.26 and  $\text{cl}(A) \cap \text{cl}(B)$  is closed then  $A \cap B \in \text{GLC}^*(X, \tau)$ .

- ii) Let  $A \in \text{GLC}^{**}(X, \tau)$  then there exists an open set  $G$  and g-closed set  $F$  such that  $A = G \cap F$ . First suppose that  $B$  is open. Then  $A \cap B = G \cap F \cap B = (G \cap B) \cap F \in \text{GLC}^{**}(X, \tau)$ . Next, suppose that  $B$  is closed, by corollary 1.1.27  $F \cap B$  is g-closed and therefore  $A \cap B \in \text{GLC}^{**}(X, \tau)$ .
- iii) Let  $A \in \text{GLC}(X, \tau)$ , then there exists a g-open set  $G$  and a g-closed set  $F$  such that  $A = (G \cap F)$ . First suppose that  $B$  is g-open, then  $A \cap B = (G \cap F) \cap B$ . By lemma 1.1.26,  $G \cap B$  is g-open and therefore  $A \cap B \in \text{GLC}^{**}(X, \tau)$ . Next suppose that  $B$  is closed, by corollary 1.1.27  $F \cap B$  is g-closed and so  $A \cap B = G \cap (B \cap F)$  is in  $\text{GLC}(X, \tau)$ .

**Notation: 1.1.29**

Let  $Z$  be a subset of  $(X, \tau)$ , then the subspace topology of  $Z$  is denoted by  $\tau|Z$ .

**Proposition: 1.1.30**

Let  $A$  and  $Z$  be subsets of  $(X, \tau)$  and let  $A \subseteq Z$ .

- i) If  $Z$  is  $g$ -open in  $(X, \tau)$  and  $A \in \text{GLC}^*(Z, \tau|Z)$ , then  $A \in \text{GLC}^*(X, \tau)$ .
- ii) If  $Z$  is  $g$ -closed in  $(X, \tau)$  and  $A \in \text{GLC}^{**}(Z, \tau|Z)$ , then  $A \in \text{GLC}^{**}(X, \tau)$ .
- iii) If  $Z$  is  $g$ -closed and  $g$ -open in  $(X, \tau)$  and  $A \in \text{GLC}(Z, \tau|Z)$ , then  $A \in \text{GLC}(X, \tau)$ .

**Proof:**

- i) Let  $Z$  be  $g$ -open in  $(X, \tau)$  and  $A \in \text{GLC}^*(Z, \tau|Z)$ . Then there exists a  $g$ -open set  $G$  of  $(Z, \tau|Z)$  such that  $A = G \cap \text{cl}_Z(A)$ . As  $\text{cl}_Z(A) = Z \cap \text{cl}(A)$ ,  $A = (Z \cap G) \cap \text{cl}(A)$  and hence  $A \in \text{GLC}^*(X, \tau)$ .
- ii) Let  $Z$  be  $g$ -closed in  $(X, \tau)$  and  $A \in \text{GLC}^{**}(Z, \tau|Z)$ , then there exist an open set  $G$  of  $(Z, \tau|Z)$  and  $g$ -closed set  $F$  of  $(Z, \tau|Z)$  such that  $A = G \cap F$ . By result  $F$  is  $g$ -closed in  $(X, \tau)$ . Since  $G = B \cap Z$  for some open set  $B$  of  $(X, \tau)$ ,  $A = (Z \cap B) \cap F = F \cap B$ . Hence  $A \in \text{GLC}^{**}(X, \tau)$ .
- iii) Let  $Z$  be  $g$ -closed and  $g$ -open in  $(X, \tau)$  and  $A \in \text{GLC}(Z, \tau|Z)$  then there exists a  $g$ -open set  $G$  of  $(Z, \tau|Z)$  and  $g$ -closed set  $F$  of  $(Z, \tau|Z)$  such that  $A = G \cap F$ . Hence  $A \in \text{GLC}(X, \tau)$ .

**Remark: 1.1.31**

The following example shows that the  $g$ -openness of  $Z$  in proposition 1.1.30(i) is essential.

**Example: 1.1.32**

Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ . Let  $\mathcal{V}$  denote the collection of all g-open sets of  $(X, \tau)$ . Then  $\mathcal{V} = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ . Let  $Z = A = \{a, c\}$ . Then  $Z$  is not g-open and  $A \in \text{GLC}^*(Z, \tau|_Z)$ .  $A \notin \text{GLC}^*(X, \tau)$  Since  $\text{GLC}^*(X, \tau) = P(X) - \{\{a, c\}\}$ ,  $A \notin \text{GLC}^*(X, \tau)$ .

**Proposition: 1.1.33**

Suppose that the collection of all g-open sets of  $(X, \tau)$  is closed under finite union. Let  $A \in \text{GLC}^*(X, \tau)$  and  $B \in \text{GLC}^*(X, \tau)$ . If  $A$  and  $B$  are separated. i.e  $A \cap \text{cl}(B) = \phi$  and  $B \cap \text{cl}(A) = \phi$  then  $A \cup B \in \text{GLC}^*(X, \tau)$

**Proof:**

Let  $A \in \text{GLC}^*(X, \tau)$  and  $B \in \text{GLC}^*(X, \tau)$  then there exist g-open sets  $G$  and  $S$  of  $(X, \tau)$  such that  $A = G \cap \text{cl}(A)$  and  $B = S \cap \text{cl}(B)$ . Let  $U = G \cap (X - \text{cl}(B))$  and  $V = S \cap (X - \text{cl}(A))$ . Then  $A = U \cap \text{cl}(A)$ ,  $B = V \cap \text{cl}(B)$  and  $U \cap \text{cl}(B) = \phi$ ,  $V \cap \text{cl}(A) = \phi$ . Then  $U$  and  $V$  are g-open sets of  $(X, \tau)$  and Lemma 1.1.26  $U \cup V$  is g-open As  $A \cup B = (U \cup V) \cap (\text{cl}(A \cup B))$ . Hence,  $A \cup B \in \text{GLC}^*(X, \tau)$ .

**Remark: 1.1.34**

Example 1.1.32 shows that  $A$  and  $B$  are separated is essential in Proposition 1.1.33.  $\mathcal{V}$  is closed under finite unions, and  $\{a\} \in \text{GLC}^*(X, \tau)$  and  $\{c\} \in \text{GLC}^*(X, \tau)$ . However,  $\{a\}$  and  $\{c\}$  are not separated and  $\{a, c\} \notin \text{GLC}^*(X, \tau)$ .

**Proposition: 1.1.34**

Let  $\{Z_i | i \in \Lambda\}$  be a finite g-closed cover of  $(X, \tau)$ , i.e  $X = \cup \{Z_i | i \in \Lambda\}$ , and let  $A$  be a subset of  $(X, \tau)$ . If  $A \cap Z_i \in \text{GLC}^{**}(Z_i, \tau|_{Z_i})$  for each  $i \in \Lambda$ , then  $A \in \text{GLC}^*(X, \tau)$ .

**Proof:**

Let  $\{Z_i | i \in \Lambda\}$  be a finite g-closed cover of  $(X, \tau)$ , then for each  $i \in \Lambda$  there exists an open set  $U_i \in \tau$  and a g-closed set  $F_i$  of  $(Z_i, \tau|_{Z_i})$  such that  $A \cap Z_i = U_i \cap (Z_i \cap F_i)$ . Then

$$A = \cup \{A \cap Z_i | i \in \Lambda\} = [\cup \{U_i | i \in \Lambda\}] \cap [\cup \{Z_i \cap F_i | i \in \Lambda\}].$$

Hence  $A \in \text{GLC}^{**}(X, \tau)$ .

**Proposition: 1.1.35**

Let  $(X, \tau)$  and  $(Y, \sigma)$  be topological spaces

- i) If  $A \in \text{GLC}(X, \tau)$  and  $B \in \text{GLC}(Y, \sigma)$ , then  
 $A \times B \in \text{GLC}(X \times Y, \tau \times \sigma)$ .
- ii) If  $A \in \text{GLC}^*(X, \tau)$  and  $B \in \text{GLC}^*(Y, \sigma)$ , then  
 $A \times B \in \text{GLC}(X \times Y, \tau \times \sigma)$ .
- iii) If  $A \in \text{GLC}^{**}(X, \tau)$  and  $B \in \text{GLC}^{**}(Y, \sigma)$ , then  
 $A \times B \in \text{GLC}^{**}(X \times Y, \tau \times \sigma)$ .

**Proof**

i) Let  $A \in \text{GLC}(X, \tau)$  and  $B \in \text{GLC}(Y, \sigma)$ . Then there exists g-open sets  $G$  and  $G'$  of  $(X, \tau)$  and  $(Y, \sigma)$  respectively and g-closed sets  $S$  and  $S'$  of  $(X, \tau)$  and  $(Y, \sigma)$  respectively such that  $A = G \cap S$  and  $B = G' \cap S'$ . Then  $A \times B = (G \times G') \cap (S \times S')$ . Hence  $A \times B \in \text{GLC}(X \times Y, \tau \times \sigma)$ .

ii) Let  $A \in \text{GLC}^*(X, \tau)$  and  $B \in \text{GLC}^*(Y, \sigma)$ . Then there exists g-open sets  $G$  and  $G'$  of  $(X, \tau)$  and  $(Y, \sigma)$  respectively and g-closed sets  $S$  and  $S'$  of  $(X, \tau)$  and  $(Y, \sigma)$  respectively such that  $A = G \cap S$  and  $B = G' \cap S'$  then  $A \times B = (G \times G') \cap (S \times S')$ . Hence  $A \times B \in \text{GLC}^*(X \times Y, \tau \times \sigma)$ .

iii) Let  $A \in \text{GLC}^{**}(X, \tau)$  and  $B \in \text{GLC}^{**}(Y, \sigma)$ . Then there exist g-open sets  $G$  and  $G'$  of  $(X, \tau)$  and  $(Y, \sigma)$  respectively and g-closed sets  $S$  and  $S'$  of  $(X, \tau)$  and  $(Y, \sigma)$  respectively such that  $A = G \cap S$  and

$B = G' \cap S'$  Then  $A \times B = (G \times G') \cap (S \times S')$  .Hence  $A \times B \in \text{GLC}^{**}(X \times Y, \tau \times \sigma)$ .

## SECTION: 1.2

### GLC-FUNCTIONS AND SOME OF THEIR PROPERTIES

#### **Definition: 1.2.1**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **LC-irresolute** if  $f^{-1}(V) \in \text{LC}(X, \tau)$  for each  $V \in \text{LC}(Y, \sigma)$ .

#### **Definition: 1.2.2**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **LC\*-irresolute** if  $f^{-1}(V) \in \text{LC}^*(X, \tau)$  for each  $V \in \text{LC}(Y, \sigma)$ .

#### **Definition: 1.2.3**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **LC\*\*-irresolute** if  $f^{-1}(V) \in \text{LC}^{**}(X, \tau)$  for each  $V \in \text{LC}(Y, \sigma)$ .

#### **Definition: 1.2.4**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **LC-continuous** if  $f^{-1}(V) \in \text{LC}(X, \tau)$  for each  $V \in \sigma$ .

#### **Definition: 1.2.5**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **LC\*-continuous** if  $f^{-1}(V) \in \text{LC}^*(X, \tau)$  for each  $V \in \sigma$ .

#### **Definition: 1.2.6**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **LC\*\*-continuous** if  $f^{-1}(V) \in \text{LC}^{**}(X, \tau)$  for each  $V \in \sigma$ .

**Definition: 1.2.7**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **GLC- irresolute** if  $f^{-1}(V) \in \text{GLC}(X, \tau)$  for each  $V \in \text{GLC}(Y, \sigma)$ .

**Definition: 1.2.8**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **GLC\*- irresolute** if  $f^{-1}(V) \in \text{GLC}^*(X, \tau)$  for each  $V \in \text{GLC}^*(Y, \sigma)$ .

**Definition: 1.2.9**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **GLC\*\*- irresolute** if  $f^{-1}(V) \in \text{GLC}^{**}(X, \tau)$  for each  $V \in \text{GLC}^{**}(Y, \sigma)$ .

**Definition: 1.2.10**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **GLC-continuous** if  $f^{-1}(V) \in \text{GLC}(X, \tau)$  for each  $V \in \sigma$ .

**Definition: 1.2.11**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **GLC\*-continuous** if  $f^{-1}(V) \in \text{GLC}^*(X, \tau)$  for each  $V \in \sigma$ .

**Definition: 1.2.12**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **GLC\*\*-continuous** if  $f^{-1}(V) \in \text{GLC}^{**}(X, \tau)$  for each  $V \in \sigma$ .

**Proposition: 1.2.13**

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function

- i) If  $f$  is LC-continuous, then it is GLC\*-continuous and GLC\*\*-continuous.

- ii) If  $f$  is  $GLC^*$ -continuous or  $GLC^{**}$ -continuous, then it is  $GLC$ -continuous.
- iii) If  $f$  is  $GLC$  - irresolute (resp.  $GLC^*$  irresolute, resp.  $GLC^{**}$  irresolute), then it is  $GLC$ -continuous (resp.  $LC^*$  continuous, resp.  $GLC^{**}$ -continuous).
- iv) If  $f$  is continuous and closed, then  $f$  is  $GLC$ - irresolute,  $GLC^*$ - irresolute and  $GLC^{**}$ - irresolute.

**Proof:**

i) Let  $f$  be  $LC$ -continuous and let  $V$  be an open set of  $(Y, \sigma)$ . Then  $f^{-1}(V)$  is locally closed in  $(X, \tau)$ . By proposition 1.1.18,  $f^{-1}(V) \in GLC^*$  and  $GLC^{**}$  continuous. Hence  $f$  is  $GLC^*$  continuous and  $GLC^{**}$  continuous.

The proofs of ii) and iii) are obvious from definitions

iv) Let  $f$  be continuous and closed,  $f: (X, \tau) \rightarrow (Y, \sigma)$ . Let  $V \subseteq Y$  and  $V \in GLC(Y, \sigma)$ . Then  $V = G \cap F$  for some  $g$ -open  $G$  and  $g$ -closed  $F$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = f^{-1}(G \cap F) = f^{-1}(G) \cap f^{-1}(F)$  then  $f^{-1}(G)$  is  $g$ -open and  $f^{-1}(F)$  is  $g$ -closed in  $(X, \tau)$ . Therefore,  $f^{-1}(V) \in GLC(X, \tau)$ . Therefore  $f$  is  $GLC$  - irresolute. In a similar way the result can be proved for  $GLC^*$ - irresolute,  $GLC^{**}$ - irresolute

**Remark: 1.2.14**

The converse of Proposition 1.2.13 need not be true as seen from the following examples.

**Example: 1.2.15**

Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{ \emptyset, \{a\}, X \}$  and  $\sigma = P(Y)$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity

Since  $GLC^*(X, \tau) = GLC^{**}(X, \tau) = GLC(X, \tau) = P(X)$ ,  $LC(X, \tau) = \{\phi, \{a\}, \{b,c\}, X\}$  and  $LC(Y, \sigma) = GLC(Y, \sigma) = GLC^*(Y, \sigma) = GLC^{**}(Y, \sigma) = P(Y)$ , Then  $f$  is not LC-continuous ; it is  $GLC^*$  continuous,  $GLC^{**}$  continuous and  $GLC^*$ -irresolute.

**Example: 1.2.15**

Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, \{a\}, \{a,b\}, X\}$  and  $\sigma = \{\phi, \{a\}, Y\}$   
 Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function defined by  $f(a)=f(b)=a$  and  $f(b)=b$ ..Then  $GLC^{**}(X, \tau) = GLC(X, \tau) = P(X)$ ,  $LC(X, \tau) = GLC^*(X, \tau) = P(X) - \{\{a,c\}\}$  and  $GLC(Y, \sigma) = GLC^{**}(Y, \sigma) = GLC^*(Y, \sigma) = P(Y)$ ,  
 Therefore  $f$  is not  $GLC^*$  continuous, but it is GLC continuous .

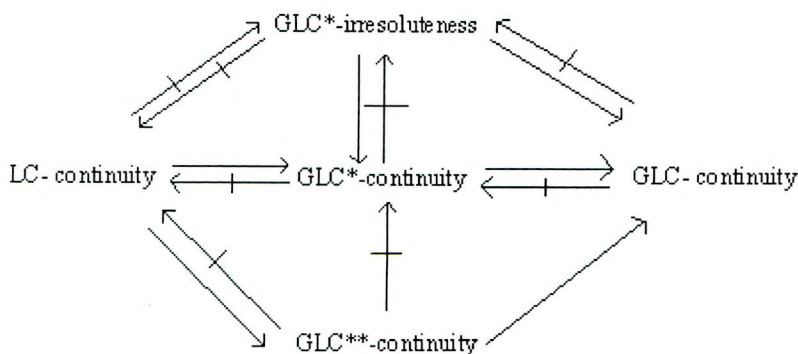
**Example: 1.2.16**

The function of Example 1.2.15 is not  $GLC^*$ -continuous, but it is  $GLC^{**}$ - continuous.

**Example: 1.2.17**

Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, \{a\}, \{a,b\}, X\}$  and  $\sigma = \{\phi, \{a\}, Y\}$ .  
 Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity function. Then,  $f$  is not  $GLC^*$ -irresolute, but it is LC – continuous.

From Proposition 1.2.13 and Example 1.2.15-1.2.17, we have the following diagram :



Where  $A \rightarrow B$  (resp.  $A \not\rightarrow B$ ) represents that A implies B (resp. A does not always imply B)

**Proposition: 1.2.18**

A topological space  $(X, \tau)$  is g-submaximal if and only if every function having  $(X, \tau)$  as its domain is GLC\*- continuous.

**Proof :**

Let  $(X, \tau)$  be a g-submaximal topological space .Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function from  $(X, \tau)$  to a topological space  $(Y, \sigma)$  .Since  $(X, \tau)$  is g-submaximal if  $P(X) = GLC^*(X, \tau)$  ,  $f^{-1}(V) \in GLC^*(X, \tau) = P(X)$  for each open set V of  $(Y, \sigma)$  .Therefore f is GLC\*-continuous.

Conversly, Let every function having  $(X, \tau)$  as its domain be GLC\* continuous. Let  $Y=\{0,1\}$  be Sierpinski space with topology  $\sigma = \{ \phi , \{0\} ,Y\}$  .Let V be a subset of  $(X, \tau)$  and  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function defined by  $f(x)=1$  for every  $x \notin V$  .By assumption f is GLC\*-continuous and hence  $f^{-1}(\{0\}) = V \in GLC^*(X, \tau)$  .Therefore  $P(X) = GLC^*(X, \tau)$ . Hence  $(X, \tau)$  is a g-submaximal space.

**Proposition: 1.2.19**

If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $GLC^{**}$ -continuous and a subset  $B$  is closed in  $(X, \tau)$  Then the restriction of  $f$  to  $B$ , say  $f|_B : (B, \tau|_B) \rightarrow (Y, \sigma)$  is  $GLC^{**}$ -continuous.

**Proof:**

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be  $GLC^{**}$ -continuous and a subset  $B$  is closed in  $(X, \tau)$ . Let  $V$  be an open set of  $(Y, \sigma)$ . Then  $f^{-1}(V) = G \cap F$  for some open set  $G \in \tau$  and  $g$ -closed  $F$  of  $(X, \tau)$ . By using result we get  $(f|_B)^{-1}(V) = (G \cap B) \cap (F \cap B) \in GLC^{**}(B, \tau|_B)$ . This implies that  $f|_B$  is  $GLC^{**}$ -continuous.

**Definition: 1.2.20**

Let  $X = A \cup B$  and  $f: A \rightarrow Y$  and  $h: B \rightarrow Y$  be two functions. Then  $f$  and  $h$  are compatible if  $f|_{A \cap B} = h|_{A \cap B}$ . Then the function  $f \nabla h$  is defined as follows  $f \nabla h: X \rightarrow Y$  as  
 $(f \nabla h)(x) = f(x)$  for every  $x \in A$  and  
 $(f \nabla h)(x) = h(x)$  for every  $x \in B$ . The function  $f \nabla h: X \rightarrow Y$  is called the **Combination** of  $f$  and  $h$ .

**Theorem : 1.2.21**

Let  $X = A \cup B$ , where  $A$  and  $B$  are  $g$ -closed sets of  $(X, \tau)$  and  $f: (A, \tau|_A) \rightarrow (Y, \sigma)$  and  $h: (B, \tau|_B) \rightarrow (Y, \sigma)$  be compatible functions. If  $f$  and  $h$  are  $GLC^{**}$ -continuous then  $f \nabla h: (X, \tau) \rightarrow (Y, \sigma)$  is  $GLC^{**}$ -continuous.

**Proof:**

Let  $V \in \sigma$  then  $(f \nabla h)^{-1}(V) \cap A = f^{-1}(V)$  and  $(f \nabla h)^{-1}(V) \cap B = h^{-1}(V)$ . By assumption,  $(f \nabla h)^{-1}(V) \cap A \in GLC^{**}(A, \tau|_A)$  and

$(f \nabla h)^{-1}(V) \cap B \in \text{GLC}^{**}(B, \tau|B)$ . Therefore from proposition 1.1.34,  $(f \nabla h)^{-1}(V) \in \text{GLC}^{**}(X, \tau)$  and hence  $f \nabla h$  is  $\text{GLC}^{**}$ -continuous.

**Theorem: 1.2.22**

Let  $X = A \cup B$ , where  $A$  and  $B$  are  $g$ -closed sets of  $(X, \tau)$  and  $f: (A, \tau|A) \rightarrow (Y, \sigma)$  and  $h: (B, \tau|B) \rightarrow (Y, \sigma)$  be compatible functions. If  $f$  and  $h$  are  $\text{GLC}^{**}$ -irresolute then  $f \nabla h : (X, \tau) \rightarrow (Y, \sigma)$  is  $\text{GLC}^{**}$ -irresolute.

**Proof:**

Let  $V \in \text{GLC}^{**}(Y, \sigma)$ . Then  $(f \nabla h)^{-1}(V) \cap A = f^{-1}(V)$  and  $(f \nabla h)^{-1}(V) \cap B = h^{-1}(V)$ . By assumption,  $(f \nabla h)^{-1}(V) \cap A \in \text{GLC}^{**}(A, \tau|A)$  and  $(f \nabla h)^{-1}(V) \cap B \in \text{GLC}^{**}(B, \tau|B)$ . Therefore from proposition 1.1.34,  $(f \nabla h)^{-1}(V) \in \text{GLC}^{**}(X, \tau)$  and hence  $f \nabla h$  is  $\text{GLC}^{**}$ -irresolute.

**Proposition: 1.2.23**

The composition of two  $\text{GLC}$ -irresolute function is  $\text{GLC}$ -irresolute.

**Proposition: 1.2.24**

The composition of two  $\text{GLC}^*$ -irresolute function is  $\text{GLC}^*$ -irresolute.

**Proposition: 1.2.25**

The composition  $g \circ f$  of a  $\text{GLC}$ -continuous function  $f$  and a continuous function  $g$  is  $\text{GLC}$ -continuous.

**Proposition: 1.2.26**

The composition  $g \circ f$  of a  $GLC^*$ -continuous function  $f$  and a continuous function  $g$  is  $GLC^*$ -continuous.

$g \circ f$

**Proposition: 1.2.27**

The composition  $g \circ f$  of a  $GLC^{**}$ -continuous function  $f$  and a continuous function  $g$  is  $GLC^{**}$ -continuous.

**Definition: 1.2.28**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called **Sub-  $GLC^{**}$ -continuous** if there is a basis  $\mathcal{B}$  for  $(Y, \sigma)$  such that  $f^{-1}(U) \in GLC^*(X, \tau)$  for each  $U \in \mathcal{B}$

**Remark: 1.2.29**

The following example shows that the converse of proposition is not always true.

**Example: 1.2.30**

Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{ \emptyset, \{a\}, \{b, c\}, X \}$  and  $\sigma = \{ \emptyset, \{a, b\}, Y \}$ . Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity function. Since  $LC(X, \tau) = \{ \emptyset, \{a\}, \{b, c\}, X \}$ ,  $GLC^*(X, \tau) = P(X)$  and the family  $\mathcal{B} = \{ \{a, b\}, Y \}$  is a basis for  $(Y, \sigma)$   $f$  is not sub-LC-continuous ; it is sub- $GLC^*$ -continuous.

**Proposition: 1.2.31**

If  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $h: (X', \tau') \rightarrow (y', \sigma')$  are sub-GLC\*-continuous, then  $fxh: (X \times X', \tau \times \tau') \rightarrow (Y \times y', \sigma \times \sigma')$  is sub-GLC\*-continuous.

**Proof:**

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $h: (X', \tau') \rightarrow (y', \sigma')$  be sub-GLC\*-continuous functions. Then there exists a basis  $\mathcal{B}$  for  $(Y, \sigma)$  and a basis  $\mathcal{B}'$  for  $(y', \sigma')$  such that  $f^{-1}(U) \in \text{GLC}^*(X, \tau)$  for each  $U \in \mathcal{B}$  and  $f^{-1}(V) \in \text{GLC}^*(X', \tau')$  for each  $V \in \mathcal{B}'$  respectively then  $\mathcal{B}'' = \{U \times V \mid U \in \mathcal{B}, V \in \mathcal{B}'\}$  is a basis for the product space  $(X \times X', \tau \times \tau')$ . Then it follows from the proposition 1.1.35 that  $(fxh)^{-1}(U \times V) = f^{-1}(U) \times h^{-1}(V) \in \text{GLC}^*(X \times X', \tau \times \tau')$  for every  $U \times V \in \mathcal{B}''$ . Therefore  $f \times h$  is sub-GLC\*-continuous.