On $\pi\beta$ Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces

By

Nandhitha K

(20PMA009)

Supervisor

Dr. S. Prema

Thesis submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore – 641 043

In Partial Fulfilment of the Requirement for the Degree of

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N. Balamani
18-05-2022

Signature of the Head of the Department

S. Prema 18/5/22

Signature of the Supervisor

DECLARATION

I do hereby declare that the thesis entitled " **On Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces** " submitted to the Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore for the award of the degree of **Master of Science** in Mathematics is a record of original word done by me during the period from December 2021 to May 2022 under the guidance and supervision of **Dr. S.PREMA,** Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and it has not previously formed the basis for the award of any degree, diploma, fellowship or associateship or any other similar title of any candidate of any university.

K. Nandhitha.

Signature of the Candidate

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CONTENT

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CHAPTER 1

1.1 INTRODUCTION

Topology

Topology is a collection of data suitable for mathematical models for mathematizing not only quantitative data but also qualitative ones.

Topology is a branch of mathematics which studies properties of the spaces that are invariant under transformation. The word topology derived from a Greek word TOPOS meaning “location” and LOGOS meaning “study”. Topology is an extension of geometry. It is used in nearly all branches of mathematics in one form or another. It is the mathematical study of the properties that are preserved through deformations , twisting and stretching of objects. Topology is implemented recently to understand diverse topics such as cell biology ,superconductors, robot motion and also suitable in the fields of computer graphics, pattern recognition, artificial intelligence, data mining, information systems, rough set theory, quantum physics etc.

The first work on topology came into existence due to Euler, when he published the solution to Konigsberg bridge problem.

Fuzzy Topology

Fuzzy mathematics forms a branch of mathematics related to fuzzy set theory and fuzzy logic. The notion of fuzzy set theory has caused great interest among both pure and applied mathematicians. The first publication in fuzzy set theory was introduced by Zadeh in the year (1965) and Chang in (1968) gave the topological structure to these fuzzy sets called fuzzy topological spaces and studied the topological properties. Through fuzzy sets we can speak only about membership values and it does not give a correct answer for non-membership values. Fuzzy sets are designed to handle a particular kind of uncertainity, namely degree vagueness, which results a property that can be possessed by objects to varying degrees.

Intutionistic Fuzzy Topology

Atanassov (1986), a Bulgarian mathematician, created an idea about the non-membership value and he introduced a new set which includes the non-membership value and coined the set as Intuitionisitic fuzzy set where the degree of membership is denoted by ($\mu_A(x)$) ∈ [0,1] of each element x∈ X to a set A and non membership function is denoted by ($\nu_A(x)$)∈ [0,1] .Intuitionistic fuzzy set allows one to address the positive and negative side of an imprecise concept separately.

Intuitionistic fuzzy sets have been found in diverse applied areas of science and technology and have been applied to logic programming, medical diagnosis micro electronic fault analysis, decision making problems, image processing, soft computing and many other areas. Using the notion of intuitionistic fuzzy sets, Coker (1997) has constructed the basic concepts of intuitionistic fuzzy topological spaces. After giving the fundamental definitions and the necessary examples he introduced the definitions of intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, intuitionistic fuzzy connectedness and obtained several preservation properties and some characterizations concerning intuitionistic fuzzy connectedness.

Szmidt and Kacprzyk described a geometrical representation of an intuitionistic fuzzy set is a point of departure for our proposal of distances between intuitionistic fuzzy sets.

Different types of closed sets in intuitionistic fuzzy topological spaces namely intuitionistic fuzzy regular closed set, intuitionistic fuzzy closed set, intuitionistic fuzzy semi closed set, intuitionistic fuzzy pre closed set, intuitionistic fuzzy α -closed set and their generalized closed sets have been introduced and studied by many authors .

In this thesis, a new generalization of closed set called intuitionistic fuzzy $\pi\beta$ generalized closed set in intuitionistic fuzzy topological spaces is introduced. Further the corresponding open sets and continuous mapping are being introduced and studied. Some interesting results are discussed with necessary counter examples.
For this, the thesis is organized into three chapters, as given below:

In Chapter 1, the first section begins with the introduction for topology, fuzzy topology and intuitionistc fuzzy topology. Second section deals with review of literature for topology, fuzzy topology and intutionistic fuzzy topology. In third section, the preliminary definitions, results, and lemma required for intuitionistic fuzzy $\pi\beta$ closed sets are given in detail.

In Chapter 2, in the first section we have introduced and studied a new class of sets called intuitionistic fuzzy $\pi\beta$ generalized closed sets in intuitionistic fuzzy topological spaces. This section deals with the interrelation of our newly introduced sets with other type of intuitionistic fuzzy closed sets such as intuitionistic fuzzy regular closed sets, intuitionistic fuzzy semi closed sets, intuitionistic fuzzy α closed sets, intuitionistic fuzzy pre closed sets, intuitionistic fuzzy π closed sets, intuitionistic fuzzy β closed sets, intuitionistic fuzzy generalized closed sets, intuitionistic fuzzy generalized pre closed sets and intuitionistic fuzzy generalized semi closed sets. Also it is shown that the converses are not true in general and they are proved with necessary counter examples. In the second section, intuitionistic fuzzy genera $\pi\beta$ generalized open sets are introduced and their interrelations with other intuitionistic fuzzy open sets are established. Also some of their properties are studied.

In Chapter 3, a new type of intuitionistic fuzzy continuous mapping called intuitionistic fuzzy $\pi\beta$ generalized continuous mapping has been introduced. In the first section, we give a short introduction to intuitionistic fuzzy continuous mappings and we have introduced intuitionistic fuzzy $\pi\beta$ generalized continuous mappings and analyzed the interrelations between intuitionistic fuzzy $\pi\beta$ generalized continuous mappings with other existing continuous mappings. Some fascinating theorems concerning intuitionistic fuzzy $\pi\beta$ generalized continuous mappings are discussed.

Throughout the thesis, the following notations are used.

- (X , τ) and (Y, σ) denote non empty intuitionistic fuzzy topological spaces on which no separation axioms are mentioned unless it is stated specifically.
- The closure and interior of a subset A of an intuitionistic fuzzy topological space is denoted by cl(A) and int(A) respectively.
- The extension of the notation of union, intersection, contained in and contain, in intutionistic fuzzy topological spaces are denoted as $\cup, \cap, \leq, \geq$.

In all the diagrams A⟶B represents A implies B but not conversely.

1.2. REVIEW OF LITERATURE

A review of literature of recent developments on generalized notions of closed and open sets, continuous mappings in topological spaces, fuzzy topological spaces and intuitionistic fuzzy topological spaces are given below.

Topological Spaces

Closedness is the basic concept for the study and investigation in topological spaces. In the study of topological spaces many concepts of topology have been generalized by considering the concepts of semi open sets by Levine (1963) instead of open sets. Bourbaki(1966) have introduced about the general topology. Levine (1970) introduced the concept of generalized closed sets in topological spaces. Using this concept and Levine's idea, many researchers have introduced and studied various types of generalized closed sets. Abd El-Monsef et al. (1983) have introduced β open, β closure and β interior in topological spaces.

Palaniappan and Rao (1993) have introduced regular generalized closed sets in topological spaces. Maki et al. (1993) have introduced generalized α closed sets and α generalized closed sets in topological spaces. Fukutake, Nasef and El-Maghrabi (2003) introduced γ generalized closed sets in topological spaces. Kannan and Nagaveni (2012) have introduced $\widehat{\beta}$ generalized closed sets and open sets in topological spaces.

Continuity plays a vital role in the study of topological spaces. In addition to continuity, separation axioms also provide a wide area on the study of topological spaces. Noiri (1984) introduced α-continuous functions in topology. Reilly and Vamanamurthy (1985) have introduced α continuity in topological spaces. Yuksel and Noiri (1996) have defined the notion of $\beta*$ set and established the decomposition of continuity. Caldas and Navalagi (2003) have investigated weak forms of β -open and β-closed functions. Ali M. Mubarki et al. (2014) introduced and studied the notion of β * open sets and β * continuous functions in topological spaces. Tahiliani (2006) have introduced generalized β -closed functions.

Fuzzy Topological Spaces

The notion of fuzzy set theory has caused great interest among both pure and applied mathematicians. It has also raised enthusiasm among some engineers, biologists, psychologists, economists and experts in other areas who use mathematical ideas and methods in their research. Many sets defined in topology take their newform in fuzzy topology which was determined by Chang (1968).

Lowen (1982) has introduced fuzzy neighbourhood spaces.Sostak (1985) introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and fuzzy topology, in the sense that not only the objects are fuzzified, but also the axiomatics. Nanda (1986) studied fuzzy almost open mappings. Ganguly and Saha (1986) has introduced fuzzy semiopen sets in fuzzy topological spaces. Singal and Niti Prakash (1991) have introduced fuzzy preopen sets. Thakur and Malviya (1995) have introduced generalized closed sets in fuzzy topology. Maki et al. (1998) has introduced generalized closed sets in fuzzy topological spaces.

Saraf and Meena Khanna (2003) have introduced gs closed sets in fuzzy topology. Bayaz Daraby and Nimse (2007) have discussed fuzzy generalized alpha closed set and its applications. Fuzzy α sets and α continuous mappings were introduced by Singal and Niti Rajvanshi (1992). Balasubramanian and Sundaram (1997) have investigated some generalizations of fuzzy continuous functions. Abd EI-Hakeim (1999) has introduced generalized semi continuous mappings in fuzzy topological space.

Intuitionistic Fuzzy Topological Spaces

Atanassov (1986, 1989) has introduced intuitionistic fuzzy sets and also he gave new results in intuitionistic fuzzy sets and operations. Intuitionistic fuzzy points are introduced by Coker and Demirci (1995). Intuitionistic fuzzy open sets and intuitionistic fuzzy closed sets are introduced by Coker (1997). Intuitionistic semi closed sets, intuitionistic fuzzy pre closed sets, intuitionistic fuzzy α closed sets and intuitionistic fuzzy β closed sets were introduced by Gurkay, Coker and Haydar (1997). Thakur and Rekha Chaturvedi (2008) have introduced generalized closed sets in intuitionistic fuzzy topological spaces.

Santhi and Arun Prakash (2010) have introduced intuitionistic fuzzy semi-generalized closed sets and their applications. Dhavaseelan, Roja and Uma (2010) have introduced generalized intuitionistic fuzzy closed sets and advances in fuzzy mathematics.

Rajarajeshwari and Senthil Kumar (2011) have introduced generalized pre-closed sets in intuitionistic fuzzy topological spaces. Arun prakash and Santhi(2012) have introuduced intuitionistic fuzzy semi generalized closed mappings. Intuitionistic fuzzy generalized beta closed sets are introduced by Jayanthi (2014a). The concept nowhere dense in intuitionistic fuzzy topological space is introduced by Thakur and Dhavaseelan (2015).

Saranya and Jayanthi (2016a) have introduced intuitionistic fuzzy β-generalized closed sets in intuitionistic fuzzy topological spaces. Venkatachalam, Kannan and Ramesh have introduced closed sets in intuitionistic fuzzy topological spaces. Andal and Thiripurasundari(2019) have introduced the concept of fuzzy π closed and fuzzy generalized π closed sets in a fuzzy topological space.

Hur and Jun (2003) have studied intuitionistic fuzzy alpha continuous mappings. Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity are introduced by Joung Kon Jeon et.al (2005). Thakur S.S and Rekha Chaturvedi (2006) have introduced generalized continuity in intuitionistic fuzzy topological spaces.

Generalized beta continuous mappings in intuitionistic fuzzy topological spaces are introduced by Jayanthi (2014b). Beta generalized continuous mappings in intuitionistic fuzzy topological spaces are introduced by Saranya and Jayanthi (2016).

Sakthivel and Manikandan have introduced the concept of $\pi g\gamma*$ closed sets and $\pi g\gamma*$ open sets in intutionistic fuzzy topological spaces.

In my research work, a new generalization of closed set called $\pi\beta$ generalized closed set in intuitionistic fuzzy topological space is introduced. Their basic properties, preservation theorems, interrelations, continuity are established with necessary counter examples.

Some of the research articles which I refer for the thesis are given below:

1. GENERAL TOPOLOGY

[Williard, S., 1970]

In this book, the author encompasses two broad areas of topology : continuous topology, represented by sections on convergence, compactness, metrization and complete metric spaces and function spaces.

2. FUZZY SETS

[Zadeh, L.A., 1965]

In this article, the author has introduced a new class of sets namely fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Further the author has provided the notions of inclusion, union, intersection, complement etc., with respect to the fuzzy sets.

3. FUZZY TOPOLOGICAL SPACES

[Chang, C.L., 1968]

In this article, the author has introduced fuzzy topological spaces. This concept is considered to be the generalization of general topological spaces. In brief, the basic concepts such as fuzzy open set, fuzzy closed set, fuzzy neighbourhood, fuzzy continuity etc., are discussed in depth.

4. ON FUZZY GENERALIZED ALPHA CLOSED SET AND ITS APPLICATIONS

[Bayaz Daraby and Nimse, S.B., 2007]

In this article they have defined and studied fuzzy generalized alpha closed sets and r open sets, fuzzy alpha continuous functions and their applications.

5. INTUITIONISTIC FUZZY SETS

[Atanassov, K., 1986]

In this article, the author has provided the notion of intuitionistic fuzzy sets.

This is considered to be the generalization on fuzzy sets. The highlight of this particular article is that some relations and operations concerning classical sets are extended to intuitionistic fuzzy sets.

6. MORE ON INTUITIONISTIC FUZZY SETS

[Atanassov, K., 1989]

In this article, author has introduced new results on intuitionistic fuzzy sets. Two news operators on intuitionistic fuzzy sets are defined and their basic properties are studied.

7. AN INTRODUCTION TO INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Coker, D., 1997]

In this article, the author has introduced intuitionistic fuzzy topological space. The notions of intuitionistic fuzzy interior and intuitionistic fuzzy closure are being provided and this is followed by the discussion of some important properties concerning them. Furthermore, the notion of intuitionistic fuzzy continuity is provided.

8. ON INTUITIONISTIC FUZZY β-GENERALIZED CLOSED SETS

[Saranya, M. and Jayanthi, D., 2016a]

This article consists of the notion of intuitionistic fuzzy β-generalized closed sets. The authors have analyzed some of their properties and obtained some interesting theorems. Also, the relationship between this new class of sets and some of the previously existing sets are discussed.

9. NOWHERE DENSE SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Thakur, S.S. and Dhavaseelan, R., 2015]

In this article, the authors have introduced the concept of nowhere dense subsets and investigated the characterizations of intuitionistic fuzzy nowhere dense sets.

10. ON FUZZY CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Gurcay, H., Coker, D. and Haydar, Es.A., 1997]

In this article, the authors have introduced some definitions and features releated to fuzzy continuity, fuzzy membership functions and fuzzy continuous functions in intuitionistic fuzzy topological spaces. Using these definitions and properties, some theorems about fuzzy continuous functions have been proved.

11. INTUITIONISTIC FUZZY ALPHA-CONTUINUITY AND INTUITIONISTIC FUZZY PRECONTUINUITY

[Joung Kon Jeon, Young Bae Jun and Jin Han Park, 2005]

In this article, the authors have defined the notion of semi open mappings, pre open mappings and alpha open mappings and investigated the relation among them. They gave a characterization of intuitionistic fuzzy α open set, intuitionistic fuzzy α continuous mappings and intuitionistic fuzzy pre continuous mappings and provided conditions for a mapping of intuitionistic fuzzy topological spaces to be an intuitionistic fuzzy α continuous mapping.

12.INTUTIONISTIC FUZZY GENERALIZED SEMI CONTINUOUS MAPPINGS

[Santhi .R, Sakthivel.K,2009a]

In this article ,the authors have introduced intuitionistic fuzzy generalized semi continuous mappings.They also gave advances in theoretical and applied mathematics.

13. GENERALIZED CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

[Thakur, S.S. and Rekha Chaturvedi, 2006b]

In this article, the authors have introduced and studied the concept of generalized continuous mappings in intuitionistic fuzzy topological space.

14. ON INTUITIONISTIC FUZZY β GENERALIZED OPEN SETS

[Saranya,M. and Jayanthi,D.,2016]

In this paper , we have discussed and investigated some of the properties and some of the characterization of intuitionistic fuzzy β generalized open sets in intuitionitic topological spaces.

15. ON βGENERALIZED CLOSED SETS IN INTUITIONISTC FUZZY TOPOLOGICAL SPACES**

[Sudha, S.M., and Jayanthi, D,2020]

In this paper the author has introduced the concepts of intutionistic fuzzy β^{**}generalized topological spaces. Also the author have obtained some interesting theorems.

16. ON INTUITIONISTIC FUZZY $\pi g\beta$ CLOSED SETS

[Jenitha premalatha,T., and Jothimani ,S,2014]

A new class of functions called intuitionistic fuzzy π-generalized π-closed sets is introduced. Basic properties of intuitionistic fuzzy π-generalized π-closed sets are studied.

17. FUZZY GENERALIZED π CLOSED SET IN FUZZY TOPOLOGICAL SPACES

[Andal. M, Thiripurasundari. V, 2019]

In this paper, the author had introduced the concept of fuzzy π closed and fuzzy generalized π closed sets in a fuzzy topological space. Some characterizations, examples, and properties are discussed by the author.

Notations

IFS - Intuitionistic fuzzy set

IFSs - Intuitionistic fuzzy sets

IFT - Intuitionistic fuzzy topology

IFTS - Intuitionistic fuzzy topological space

A^c - The complement of A

Int(A) - Interior of A

cl(A) - Closure of A

IFC(X) - The family of all intuitionistic fuzzy closed sets of X

IFSC(X) - The family of all intuitionistic fuzzy semi closed sets of X

IFPC(X) - The family of all intuitionistic fuzzy pre closed sets of X

IFαC(X) - The family of all intuitionistic fuzzy α closed sets of X

IFRC(X) - The family of all intuitionistic fuzzy regular closed sets of X

IFβC(X) - The family of all intuitionistic fuzzy β closed sets of X

IFβGC(X) - The family of all intuitionistic fuzzy β generalized closed sets of X

IFO(X) - The family of all intuitionistic fuzzy open sets of X

IFSO(X) - The family of all intuitionistic fuzzy semi open sets of X

IFPO(X) - The family of all intuitionistic fuzzy pre open sets of X

IFαO(X) - The family of all intuitionistic fuzzy α open sets of X

IFRO(X) - The family of all intuitionistic fuzzy regular open sets of X

IFβO(X) - The family of all intuitionistic fuzzy β open sets of X

IFβGO(X) - The family of all β generalized open sets of X

IFβGCSs - Intuitionistic fuzzy β generalized closed sets

IFGOSs-Intuitionistic fuzzy β generalized open sets

IFGSCSs- Intuitionistic fuzzy generalized semi closed sets

IFGPCSs- Intuitionistic fuzzy generalized pre closed sets

IF$\pi\beta$GCSs - Intuitionistic fuzzy $\pi\beta$ generalized closed sets

IF$\pi\beta$GOSs - Intuitionistic fuzzy $\pi\beta$ generalized open sets

1.3. PRELIMINARIES

In this section, the basic definitions of intuitionistic fuzzy sets, intuitionistic fuzzy continuous mappings and some results in intuitionistic fuzzy topological spaces that are used to accomplish the present study are given in detail.

Definition 1.3.1:

Let X be a non empty fixed set. An *intuitionistic fuzzy set*(IFS in short) A in X is an object having the form A = $\{< x, \mu_A(x), \nu_A(x) >/x \in X\}$where the functions $\mu_A(x)$:X → [0,1] and $\nu_A(x): X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non – membership (namely $\upsilon_A(x)$) of each element x $\in X$ to the set A, respectively, and $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ for each x $\in X$. Denote by IFS(X), the set of all intuitionstic fuzzy sets in X.

Definition 1.3.2:

Let A and B be IFSs of the form A= $\{< x, \mu_A(x), \nu_A(x) >/x \in X\}$ and

B= $\{< x, \mu_B(x), \nu_B(x) >/x \in X\}$. Then

a) A ⊆ B if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all x $\in X$
b) A=B if and only if A ⊆ B and B ⊆ A
c) $A^c = \{< x, \nu_A(x), \mu_A(x) >/x \in X\}$
d) A ∩B = $\{< x, \mu_A(x) \wedge \mu_B(x),\ \nu_A(x) \vee \nu_B(x) >/x \in X\}$
e) A ∪ B = $\{< x, \mu_A(x) \vee \mu_B(x),\ \nu_A(x) \wedge \nu_B(x) >/x \in X\}$

For the sake of simplicity, we shall use the notation A= $<x, (\mu_A, \mu_B), (\nu_A, \nu_B) >$ instead of A= $<x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)>$.

The intuitionistic fuzzy sets 0~ = $\{< x, 0,1) >/x \in X\}$ and 1~ = $\{< x, 0,1) >/x \in X\}$ are respectively the empty set and the whole set of X.

Definition 1.3.3:

An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

i. 0~, 1~ ϵ τ
ii. G1 ∩ G2 ϵ τ, for any G1, G2 ϵ τ
iii. ∪ G_iϵ τ for any family {G_i / i ϵ J} ⊆ τ.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 1.3.4:

Let A, B and C be intuitionistic fuzzy sets in X. Then

i. (A ⊆ B) and (C ⊆ D) ⇒ (A ∪ C) ⊆ (B ∪ D) and (A ∩ C) ⊆ (B ∩ D)
ii. A ⊆ B and A ⊆ C ⇒ A ⊆ (B ∩ C)
iii. A ⊆ C and B ⊆ C ⇒ (A ∪ B) ⊆ C
iv. A ⊆ B and B ⊆ C ⇒ A ⊆ C
v. $(A \cup B)^c = A^c \cap B^c$
vi. $(A \cap B)^c = A^c \cup B^c$
vii. $A \subseteq B \Rightarrow B^c \subseteq A^c$
viii. $(A^c)^c = A$
ix. $(0_\sim)^c = 1_\sim$
x. $(1_\sim) = 0_\sim$

Definition 1.3.5:

Let (X, τ) be an IFTS and A $=< x,, \mu_A, \nu_A >$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

int(A) = ∪ { G / G is an IFOS in X and G ⊆ A },

cl(A) = ∩ { K / K is an IFCS in X and A ⊆ K }.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 1.3.6:

Let (X, τ) be an IFTS and A, B be IFSs in X. Then the following properties hold:

i. int(A) ⊆ A
ii. A ⊆ cl(A)
iii. A⊆ B ⟹cl(A) ⊆ cl(B)
iv. A⊆ B ⟹ int(A) ⊆ cl(B)
v. int(int(A)) = int(A)
vi. cl(cl(A)) = cl(A)
vii. int(A∩B) = int(A) ∩ int(B)
viii. cl(A∪B) = cl(A) ∪ cl(B)
ix. int(1~) = 1~
x. cl(0~) = 0~

Definition 1.3.7:

An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy regular open set* (IFROS in short) if A = int(cl(A)),

(ii) *intuitionistic fuzzy regular closed set* (IFRCS in short) if A = cl(int(A)).

Definition 1.3.8:

An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy semi open set* (IFSOS in short) if A ⊆ cl(int(A),

(ii) *intuitionistic fuzzy semi closed set* (IFSCS in short) if int(cl(A)) ⊆ A.

Definition 1.3.9:

An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy α-open set* (IFαOS in short) if A ⊆ int(cl(int(A))),

(ii) *intuitionistic fuzzy α-closed set* (IFαCS in short) if cl(int(cl(A)) ⊆ A.

Definition 1.3.10:

An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy pre open set* (IFPOS in short) if A ⊆ int(cl(A)),

(ii) *intuitionistic fuzzy pre closed set* (IFPCS in short) if cl(int(A)) ⊆ A.

Definition 1.3.11:

The union of IFROSs is called *intuitionistic fuzzy π-open set* (IFπOS in short) of an IFTS (X, τ). The complement of IFπOS is called *intuitionistic fuzzy π-closed set* (IFπCS in short).

Definition 1.3.12:

An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy β-open set* (IFβOS in short) if $A \subseteq cl(int(cl(A)))$.

(ii) *intuitionistic fuzzy β-closed set* (IFβCS in short) if $int(cl(int(A))) \subseteq A$.

Definition 1.3.13:

Let A be an IFS in an IFTS in (X,τ). Then the intuitionistic fuzzy β- interior and intuitionistic fuzzy β-closure of A are defined by

i. $\beta int(A) = \cup \{G/G$ is an IFβOS in X and $G \subseteq A\}$,

ii. $\beta cl(A) = \cap \{K/K$ is an IFβCS in X and $A \subseteq K\}$.

Note that for any IFS A in (X,τ), we have $\beta\, cl(A^c) = (\beta\, int(A))^c$ and $\beta int(A^c) = (\beta cl(A))^c$.

Definition 1.3.14:

Let A be an IFS in (X,τ), then

i. $\beta cl(A) \supseteq A \cup int(cl(int(A)))$

ii. $\beta int(A) \subseteq A \cap cl(int(cl(A)))$

Definition 1.3.15:

An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized closed set* (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X. The complement of IFGCS is called ***intuitionistic fuzzy generalized open set*** (IFGOS in short).

Definition 1.3.16:

An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized open set* (IFGOS in short) if A^c is an IFGCS in X.

Definition 1.3.17:

An IFS A is said to be an ***intuitionistic fuzzy generalized pre-closed set*** (IFGPCS in short) in (X,τ) if pcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X. The family of all IFGPCSs of an IFTS (X,τ) is denoted by IFGPC(X).

Definition 1.3.18:

An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy generalized β closed sets(IFGβCS for short) if βcl(A)⊆U and U is an IFOS in (X,τ). The familyof all IFGβCSs of an IFTS (X,τ) is denoted by IFGβC(X).

Definition 1.3.19:

An IFS A is an IFTS (X,τ) is said to be an intuitionistic fuzzy β generalized closed set (IFβGCS for short) if βcl(A) ⊆ U whenever A ⊆ U and U is an IFβOS in (X,τ). The complement A^c of an IFβGCS A is an IFTS (X, τ) is called intuitionistic fuzzy β generalized open set (IFβGOS in short) in X.

Definition 1.3.20:

An A in (X,τ) is called an *intuitionitic fuzzy nowhere dense set* if there exist no IFOS U such that U ⊆ $cl(A)$. That is int(cl(A))=$0_{\sim}$

Definition 1.3.21:

An intuitonistic fuzzy point (IFP in short) written as $p_{(\alpha,\beta)}$ is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}= \begin{cases} (\alpha,\beta) & \text{if } x = p \\ (0,1) & otherwsie \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to as a set A if $\alpha \leq \mu_A$ and $\beta \geq \vartheta_A$.

Definition 1.3.22:

Two IFSs are said to be q-coincident (AqB in short) if and only if there exists an element x ϵ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 1.3.23:

For any two IFSs A and B are said to be not q- coincident $(A_{\bar{q}}\ B)$ if and only if $A \subseteq B^c$.

Definition 1.3.24:

Let A, $A_i (i \in J)$ be an intuitionistic fuzzy sets in X and B, $B_j (j \in K)$ be an intuitionistic fuzzy sets in Y and f: X → Y be a mapping. Then

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
c) $A \subseteq f^{-1}(f(A))$ [If f is injective, then $A = f^{-1}(f(A))$]
d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective, then $B = f(f^{-1}(B))$]
e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
g) $f^{-1}(0_\sim) = 0_\sim$
h) $f^{-1}(1_\sim) = 1_\sim$
i) $f^{-1}(B^c) = (f^{-1}(B))^c$.

Definition 1.3.25:

Let X and Y be two non empty sets and f: X → Y be a mapping. If $A = \{\langle x, (\mu_A(x), \nu_A(x) / x \in X\rangle\}$ is an IFS in X, then the ***image*** of A under f, denoted by f(A), is the IFS in Y defined by

$$f(A) = \{\langle y, f(\mu_A)(y), f_-(\nu_A)(y) / y \in Y\rangle\},$$

where $f_-(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 1.3.26:

Let X and Y be two non empty sets and f: X → Y be a mapping. If $B = \{\langle y, (\mu_B(y), \nu_B(y) / y \in Y\rangle\}$ is an IFS in Y, then the ***preimage*** of B under f is denoted and defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) / x \in X\rangle\}$$

where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every $x \in X$.

Intuitionistic fuzzy continuous mappings

Definition 1.3.27:

Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy continuous (IF continuous) mapping if f^{-1} (V) is an IFCS in(X,τ) for every IFCS V of (Y, σ)

Definition 1.3.28:

Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an

i. ***intuitionistic fuzzy semi continuous*** (IFS continuous) ***mapping*** if f^{-1} (V) is an IFSCS in (X, τ) for every IFCS V of (Y, σ),

ii. ***intuitionistic fuzzy*** α **continuous** (IFα continuous) ***mapping*** if f^{-1} (V) is an IFαCS in (X, τ) for every IFCS V of (Y, σ),

iii. ***intuitionistic fuzzy pre continuous*** (IFP continuous) ***mapping*** if f^{-1} (V) is an IFPCS in (X, τ) for every IFCS V of (Y, σ).

Definition 1.3.29:

Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an ***intuitionistic fuzzy β continuous*** (IFβ continuous) mapping if f^{-1} (V) is an IFβCS in (X, τ) for every IFCS V of (Y, σ).

Definition 1.3.30:

Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an ***intuitionistic fuzzy β generalized continuous*** (IFβG continuous) mapping if f^{-1} (V) is an IFβGCS in (X, τ) for every IFCS V of (Y, σ).

CHAPTER 2

2.1 Intuitionistic Fuzzy $\pi\beta$ Generalized Closed Sets

In this section we introduced intuitionistic fuzzy $\pi\beta$ generalized closed sets and studied some of its properties. Also we have extablished the relationship between basic intutionistic fuzzy sets and intuitionistic fuzzy $\pi\beta$ generalized closed and open sets. Also we have analyzed some properties of $\pi\beta$ generalized closed sets and open sets in intuitionistic fuzzy topological spaces.

Definition 2.1.1:

An IFS A in (X,τ) is said to be an intuitionistic fuzzy πβ *generalized closed sets* (IFπβGCS in short) if βcl(A)⊆ U whenever A ⊆ U and U is an $IF\pi OS$ in (X, τ).

Example 2.1.2:

Let X ={a,b} and let τ ={$0\sim, G_1, G_2, 1\sim$ } is an IFT on X, where G_1 =<$x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi$βGCS in (X, τ).

Theorem 2.1.3:

Every intuitionistic fuzzy closed set (IFCS in short) in (X, τ) is an IF$\pi\beta$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\beta cl(A) \subseteq cl(A)$ = A ⊆ U. We have $\beta cl(A) \subseteq U$. Therefore A is an IFπβGCS.

Example 2.1.4:

Let X ={a, b} and let τ ={$0\sim, G_1, G_2, 1\sim$ } is an IFT on X, where G_1 =<$x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi$βGCS in (X, τ) but not an IFCS in X as cl(A)=$G_1^c \neq$ A.

Theorem 2.1.5:

Every intuitionistic fuzzy regular closed set (IFRCS in short) in (X, τ) is an IF$\pi\beta$GCS in (X, τ) but not conversely.

Proof:

Since every IFRCS is an IFCS.Hence A is an IF$\pi\beta$GCS in (X, τ).

Example 2.1.6:

Let X ={a,b} and let τ ={$0\sim, G_1, G_2, 1\sim$ } is an IFT on X, where $G_1 =<x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)>$ and $G_2 =<x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)>$. Then the IFS A $=< x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)>$ is an IF$\pi\beta$GCS in (X, τ) but not an IFRCS in X as cl(int(A))=cl(G_1)= $G_1^c \neq$ A.

Theorem 2.1.7:

Every intuitionistic fuzzy semi closed set (IFSCS in short) in (X, τ) is an IF$\pi\beta$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFSCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\beta cl(A) \subseteq Scl(A)$ = A $\subseteq U$, by hypothesis. Hence $\beta cl(A) \subseteq U$. Therefore A is an IF$\pi\beta$GCS in (X, τ).

Example 2.1.8:

Let X ={a,b} and let τ ={$0\sim, G_1, G_2, 1\sim$ } is an IFT on X, where $G_1 =<x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)>$ and $G_2 =<x, (0.2_a, 0.2_b), (0.8_a, 0.8_b)>$. Then the IFS A $=< x, (0.4_a, 0.6_b), (0.6_a, 0.4_b)>$ is an IF$\pi\beta$GCS in (X, τ) but not an IFSCS in X as int(cl(A)=int(G_2^c)= $G_1 \nsubseteq$ A.

Theorem 2.1.9:

Every intuitionistic fuzzy α closed set (IFαCS in short) in (X, τ) is an IF$\pi\beta$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFαCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\beta cl(A) \subseteq \alpha cl(A)$ = A⊆ U. By hypothesis $\beta cl(A) \subseteq U$. Therefore A is an IFπβGCS in (X, τ).

Example 2.1.10:

Let X ={a,b} and let τ ={0 ~, G_1, G_2, 1 ~ } is an IFT on X, where G_1 =<$x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi\beta$GCS in (X, τ) but not an IFαCS in X as cl(int(cl(A)))=cl(int(G_1^c)=cl(G_1)= $G_1^c \nsubseteq$A.

Theorem 2.1.11:

Every intuitionistic fuzzy pre closed set (IFPCS in short) in (X,τ) is an IF$\pi\beta$GCS in (X,τ) but not conversely.

Proof:

Let A be an IFPCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\beta cl(A)$ ⊆ pcl(A)= A⊆ U. By hypothesis $\beta cl(A)$ ⊆ U. Therefore A is an IF$\pi\beta$GCS in (X, τ).

Example 2.1.12:

Let X ={a,b} and τ ={ $0_\sim, G_1, G_2, 1_\sim$} is an IFT on X, where G_1 =<$x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi\beta$GCS in (X, τ) but not an IFPCS in (X, τ) as cl(int(A))=cl(G_1)= $G_1^c \nsubseteq$ A .

Theorem 2.1.13:

Every intuitionistic fuzzy π closed set (IFπCS in short) in (X, τ) is an IFπβGCS in (X, τ) but not conversely

Proof:

let A be an IFπCS in (X, τ) and let A ⊆ U.Since every IFπCS is an IFCS. Therefore A is an IFπβGCS in (X, τ).

Example 2.1.14:

Let X ={a,b} and let τ ={ $0_{\sim}, G_1, G_2, 1_{\sim}$} is an IFT on X, where G_1 =<$x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi\beta$GCS in (X, τ) but not an IFπCS in (X τ) as cl(int(A)) = cl(G_1) = $G_1^c \neq$ A.

Theorem 2.1.15:

Every Intuitionistic fuzzy β closed set (IF βCS *in short*) in (X, τ) is an IFπβGCS in (X, τ) but not conversely.

Proof:

Let A be an IFβCS in (X, τ) and let A ⊆ U and U be an IFπOS in (X, τ) . As $\beta cl(A) = A \subseteq$ U, by hypothesis. Therefore A is an $IF\pi\beta GCS$ in (X, τ)

Example 2.1.16:

Let X ={a,b} and let τ ={ $0_{\sim}, G_1, G_2, 1_{\sim}$} is an IFT on X, where G_1 =<x, $(0.4_a, 0.3_b), (0.6_a, 0.7_b)$> and G_2 =<$x, (0.5_a, 0.4_b), (0.5, 0.6_b)$> . Then the IFS A =< $x, (0.4_a, 0.6_b), (0.6_a, 0.4_b)$> is an IF$\pi\beta$GCS in (X, τ) but not an IFβCS in (X,τ) as int(cl(int(A)))= int(cl(G_1)) = int(G_2^c)= $G_2 \nsubseteq$ A .

Theorem 2.1.17:

Every intuitionistic fuzzy generalized closed set (IFGCS in short) in (X, τ) is an IFπβGCS in (X, τ) but not conversely.

Proof:

Let A be an IFGCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\beta cl(A) \subseteq cl(A) \subseteq U$. We have $\beta cl(A) \subseteq U$. Therefore A is an IFπβGCS in (X, τ).

Example 2.1.18:

Let X ={a,b} and let τ ={0 ~, G_1, G_2, 1 ~ } is an IFT on X, where G_1 =<x, $(0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<x, $(0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< x, $(0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi$βGCS in (X, τ) but not an IFGCS in X as cl(A)=$G_1^c \nsubseteq G_1, G_2$ where A ⊆ G_1, G_2.

Theorem 2.1.19:

Every intuitionistic fuzzy generalized pre closed set (IFGPCS in short) in (X, τ) is an IF$\pi\beta$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFGPCS and let A ⊆ U and U be an IFπOS in (X, τ). Now βcl(A) ⊆ $pcl(A)$ ⊆ U, by hypothesis, which implies β(cl(A)) ⊆ U. Therefore A is an IF$\pi\beta$GCS in (X, τ).

Example 2.1.20:

Let X ={a,b} and let τ ={ $0_\sim, G_1, G_2, 1_\sim$} is an IFT on X, where G_1 =<x, $(0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<x, $(0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi$βGCS in (X,$\tau$) but not an IFGPCS in (X, τ) .

Theorem 2.1.21:

Every *intuitionistc fuzzy generalized semi closed set* (IFGSCS in short) in (X, τ) is an IF$\pi\beta$GCS in (X, τ) but not conversely.

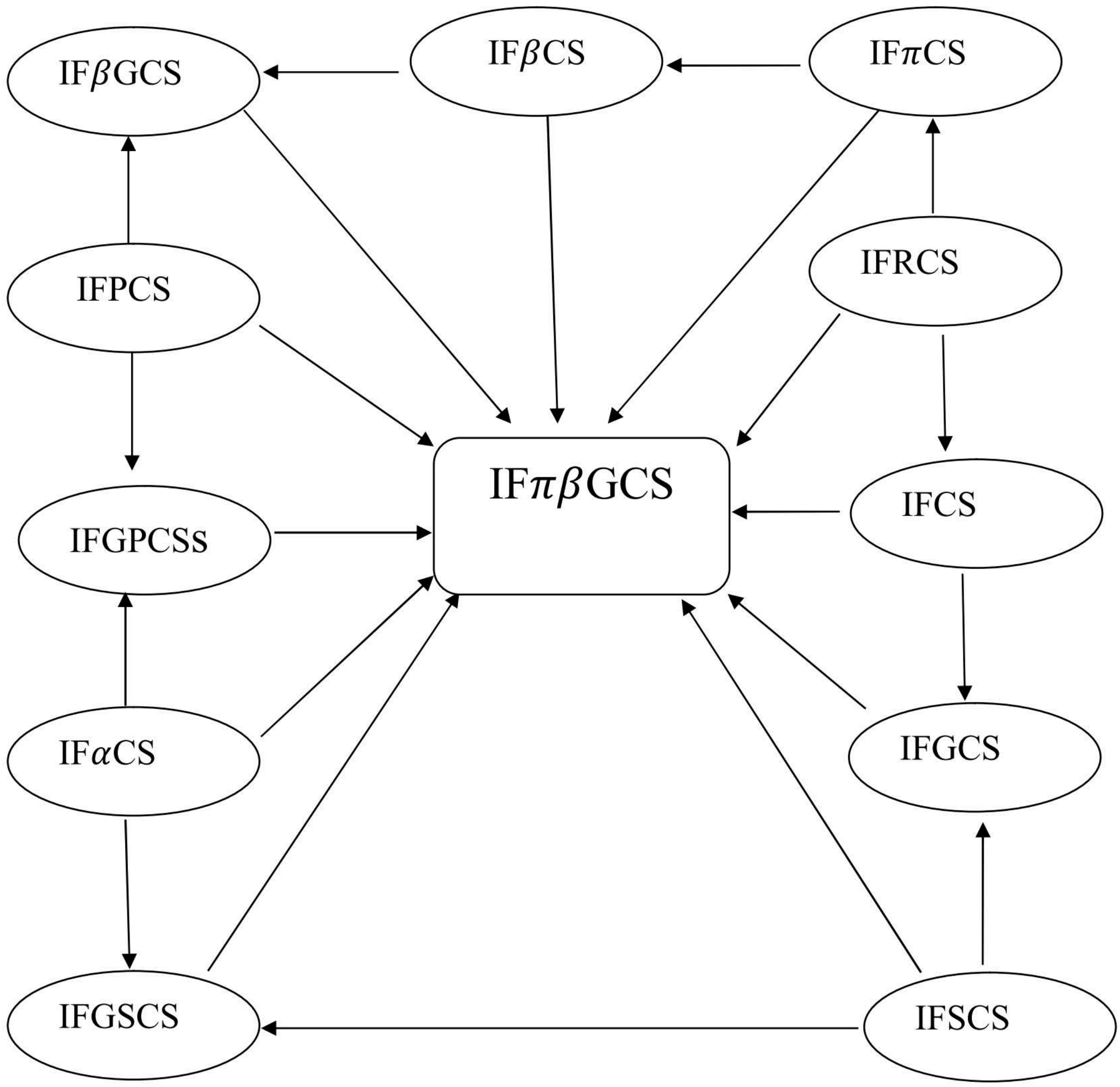
Proof:

Let A be an IFGSCS in X. Let A ⊆ U and U be an IFπOS in (X, τ). Therefore scl(A) = A ∪ int(cl(A)) ⊆ U, by hypothesis. This implies int(cl(A)) ⊆ U. Now int(cl(int(A)) ⊆ cl(int(A)) ∩ U ⊆ cl(A) ∩ U ⊆ cl(U) ∩ U ⊆ U. Hence A is an IF$\pi\beta$GCS in (X, τ).

Example 2.1.22:

Let X ={a,b} and let τ ={ $0_\sim, G_1, G_2, 1_\sim$} is an IFT on X, where G_1 =<$x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)$> and G_2 =<$x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)$> . Then the IFS A =< $x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)$> is an IF$\pi\beta$GCS in (X,$\tau$) but not an IFGSCS in (X, τ) as scl(A)= A∪ int(cl(A)) =A∪ G_1= G_1 ⊈ G_2, but A ⊆ G_2.

In the following diagram, we have provided relationship between various types of intuitionistic fuzzy closed sets.



Remark 2.1.23:

The intersection of any two IF$\pi\beta$GCS need not be an IF$\pi\beta$GCS in (X,τ) in general.

Example 2.1.24:

Let X={a,b}, G_1=<x,(0.4,0.2),(0.6,0.8)> and G_2 =<x, (0.4,0.4),(0.5,0.5)>. Then τ = {$0_\sim$,G_1, G_2, $1_\sim$} is an IFT on X. Here the IFSs A=<x,(0.4,0.5),(0.5,0.4)> and B=<x, (0.5,0.2),(04,0.6)> are IF $\pi\beta$ GCS in (X, τ) but A ∩ B =<x,(0.4,0.2),(0.5,0.6)> is not an IF$\pi\beta$GCS in (X,τ).

Theorem 2.1.25:

Let (X, τ) be an IFTS. Then for every A ϵ IF$\pi\beta$GC(X) and for every B ϵ IFS(X), A ⊆B ⊆ $\beta cl(A)$ implies B ϵ IF$\pi\beta$GC(X)

Proof:

Let B ⊆ U and U be an IFπOS. Since A ⊆ B, A ⊆ U, by hypothesis B⊆ $\beta cl(A)$. Therefore $\beta cl(B) \subseteq \beta cl(\beta cl(A)) = \beta cl(A)$ ⊆ U. since A is an IF$\pi\beta$GCS. Hence B ϵ IF$\pi\beta$GC(X).

Theorem 2.1.26:

If A is an IFβOS and IF$\pi\beta$GCS in (X, τ) then A is an IFβCS in (X, τ).

Proof:

Since A⊆ A and A is an IFβOS, by hypothesis β(cl(A)) ⊆ A. But A ⊆ β(cl(A). Therefore β(cl(A))= A. Hence A is an IFβCS in (X,τ).

Theorem 2.1.27:

Let F ⊆ A ⊆ X where A is an IFβOS and an IF$\pi\beta$GCS in X. Then F is an IF$\pi\beta$GCS in A if and only if F is an IF$\pi\beta$GCS in (X,τ).

Proof:

Necessity: Let U be an IFπOS in X and F ⊆ U. Also let F be an IF$\pi\beta$GCS in A. Then clearly F ⊆ A ∩ U and A ∩ U is an IFπOS in A. Hence $\beta(cl_A(F)) \subseteq A \cap U$ and by theorem 2.1.24 , A is an IFβCS. Therefore β(cl(A))= A. Now βcl(F)⊆ βcl(F)∩ βcl(A)= βcl(F) ∩ A = βcl_A(F) ⊆ A ∩ U.That is βcl(F)⊆ U, whenever F ⊆ U. Hence F is an IF$\pi\beta$GCS in (X,τ).

Sufficiency:

Let V be an IFβOS in A such that F⊆V. Since A is an IFβOS in X, V is an IF β OS in X. Therefore β cl(F) ⊆ V as F is an IF $\pi\beta$ GCS in (X, τ). Thus, $\beta cl_A(F) = \beta cl(F) \cap A \subseteq V \cap A \subseteq V$. Hence F is an IF$\beta$GCS in A.

Theorem 2.1.28:

Let A ⊆Y ⊆X and suppose that A is an IF$\pi\beta$GCS in X then A is an IF$\pi\beta$GCS relative to Y.

Proof:

Given that A ⊆Y ⊆ X and A is an IF$\pi\beta$GCS in X. Now let A ⊆Y ∩ U where U is an IFπOS in X. Since A is an IF$\pi\gamma$GCS in X, A ⊆ U implies $\beta_y cl(A) \subseteq U$. It follows that Y∩ βcl(A)= βcl(A) ⊆ Y ∩ U = U. Thus A is an IF$\pi\beta$GCS relative to Y.

Theorem 2.1.29:

If an IFS A of an IFTS (X, τ) is an intuitionistic fuzzy nowhere dense then A is an IF$\pi\beta$GCS in X.

Proof:

If A is an intuitionistic fuzzy nowhere dense, then by definition int(cl(A))= $0_\sim$. Let A ⊆ U where U is an IFπOS in X. The βcl(A) =$0_\sim$ ⊆ U and hence A is an IF$\pi\beta$GCS in X.

Theorem 2.1.30:

For an IFS A, the following conditions are equivalent:

(i) A is an IFOS and an IF$\pi\beta$GCS

(ii) A is an IFROS.

Proof:

(i) $\Longrightarrow$ (ii) Let A be an IFOS and an IF$\pi\beta$GCS. Then βcl(A) ⊆ A and A ⊆ βcl(A). This implies that βcl(A)= A. Therefore A is an IFβCS, Since int(cl(int(A))) ⊆ A. Since A is an IFOS , int(A)=A. Therefore int(cl(A))=A .Since A is an IFOS and IFPOS. Hence A ⊆ int(cl(A)). Therefore A=int(cl(A)). Hence A is an IFROS.

(ii) $\Longrightarrow$ (i) Let a be an IFROS .Therefore A=int(cl(A)). Since every IFROS is an IFOS and A⊆ A . This implies int(cl(A)) ⊆ A. That is int(cl(int(A))) ⊆ A. Therefore A is an IFβCS. Hence A is an IF$\pi\beta$GCS.

Theorem 2.1.31:

If A is both an IFαOS and an IF$\pi\beta$GCS in (X,τ). Then A is an IFβCS in (X,τ).

Proof:

Let A be an IFαOS .Then A is an IFβOS. As A⊆ A, by hypothesis βcl(A) ⊆ A ⊆ βcl(A), A is an IFβCS in (X,τ).

2.2 Intuitionistic Fuzzy $\pi\beta$ Generalized Open Sets

In this section we have introduced intuitionistic fuzzy βgeneralized open sets and studied some of the properties.

Definition 2.2.1:

An IFS A is said to be an intuitionistic fuzzy $\pi\beta$generalized open sets (IF$\pi\beta G$OS in short) in (X, τ) if the complement A^c is an IF$\pi\beta$GOS in X.

The family of all IF$\pi\beta$GOSs of an IFTS (X, τ) is denoted by IF$\pi\beta$GO(X).

Example 2.2.2:

Let X={a,b} and τ = {0 ~ ,G_1,G_2,1~ } where G_1=<x,(0.4,0.2),(0.6,0.8)> G_2= <x.(0.7,0.8),(0.3,0.2)>.Then IFS A= <x,(0.5,0.5),(0.5,0.5)> is an IF$\pi\beta$GOS in(X , τ).

Theorem 2.2.3:

For any IFTS (X, τ) ,we have the following:

- Every IFOS in IF$\pi\beta$GOS in (X, τ) .
- Every IFαOS in IF$\pi\beta$GOS in (X, τ).
- Every IFROS in IF$\pi\beta$GOS in (X, τ).
- Every IFPOS in IF$\pi\beta$GOS in (X, τ).
- Every IFβOS in IF$\pi\beta$GOS in (X, τ).
- Every IF$\pi\beta$OS in IF$\pi\beta$GOSin (X, τ). But the converse are not true in general.

Proof: Straight forward.

Example 2.2.4:

Let X={a,b} and τ = {0 ~ ,G_1,G_2,1~ } where G_1=<x,(0.4,0.2),(0.6,0.8)> G_2= < x.(0.7_a,0.8_b),(0.3_a,0.2_b) > .Then IFS A= < x,(0.5_a,0.5_b),(0.5_a,0.5_b) > is an IF$\pi\beta$GOS in (X , τ),but not an IFOS in (X, τ) as cl(A) = $G_1 \neq$ A.

Example 2.2.5:

Let X={a,b} and τ = {0 ~ ,G_1,G_2,1~ } where G_1=<x,(0.4,0.2),(0.6,0.8)> G_2= < x.(0.7_a,0.8_b),(0.3_a,0.2_b) > .Then IFS A= < x,(0.5_a,0.5_b),(0.5_a,0.5_b) > is an IF $\pi\beta$ GOS in (X, τ), but not an IF α OS in (X, τ) as int(cl(int(A)))=int(cl(G_1))=int(G_1^c)=G_1 , A $\nsubseteq$ G_1.

Example 2.2.6:

Let X={a,b} and τ = {0 ~ ,G_1,G_2,1~ } where G_1=<x,(0.4,0.2),(0.6,0.8)> G_2= < x.(0.7_a,0.8_b),(0.3_a,0.2_b) > .Then IFS A= < x,(0.5_a,0.5_b),(0.5_a,0.5_b) > is an IF$\pi\beta$GOS in (X , τ),but not an IFROS in (X, τ) as int(cl(A))=int(G_1^c)=G_1,≠A.

Example 2.2.7:

Let X={a,b} and τ = {0 ~ ,G_1,G_2,1~ } where G_1=<x,(0.4,0.2),(0.6,0.8)> G_2= < x.(0.7_a,0.8_b),(0.3_a,0.2_b) > .Then IFS A= < x.(0.5_a,0.5_b),(0.5_a,0.5_b) > is an IF$\pi\beta$GOS in (X , τ),but not an IFPOS in (X, τ) as int(cl(A))=int(G_1^c)=G_1 ,A $\nsubseteq$ G_1.

Example 2.2.8:

Let X={a,b} and τ = {0 ~ ,G_1,G_2,1~ } where G_1=<x,(0.4,0.2),(0.6,0.8)> G_2= < x.(0.7_a,0.8_b),(0.3_a,0.2_b) > .Then IFS A= < x,(0.5_a,0.5_b),(0.5_a,0.5_b) > is an IF$\pi\beta$GOS in (X,τ),but not an IFβOS in (X,τ) as cl(int(cl(A)))=cl(int(G_1^c)=cl(G_1) =G_1^c ,A $\nsubseteq$ G_1.

Example 2.2.9:

Let X={a,b} and τ = {0 ~ ,G_1,G_2,1~ } where G_1=<x,(0.4,0.2),(0.6,0.8)> G_2= < x.(0.7_a,0.8_b),(0.3_a,0.2_b) > .Then IFS A= < x,(0.5_a,0.5_b),(0.5_a,0.5_b) > is an IF$\pi\beta$GOS in (X , τ),but not an IFπOS in (X, τ) as int(cl(A))=int(G_1^c)=G_1, A≠G_1.

Theorem 2.2.10:

Let (X, τ) be an IFTS. Then for every A ϵ IF$\pi\beta$GO(X) and for every B ϵ IFS(X), $\beta\ int\ (A) \subseteq$ B $\subseteq$A $\Rightarrow$ B ϵ IF$\pi\beta$GO(X).

Proof:

Let A be an IF$\pi\beta$GOS of X and B be any IFS on X. Let $\beta\ int\ (A) \subseteq$B $\subseteq$A. Then A^c is an IFπβGCS and $A^c \subseteq B^c \subseteq\ \beta\ cl(A^c)$. Therefore B^c is an IFπβGCS which implies B is an IFπβGOS in X. Hence B ϵ IFπβGO(X).

Theorem 2.2.11:

If A is an IFRCS and B is an IFβOS, then A∪B is an IFπβGOS in (X, τ).

Proof:

Let B be an IFβOS and A be an IFRCS. Then B $\subseteq$ $cl(int(cl(B)))$ and $cl(int(A))$= A .Therefore A∪B ⊆ A ⊆ cl(int(cl(B)))= cl(int(A)) ∪ cl(int(cl(B))) ⊆ cl(int(cl(A))) ∪ cl(int(cl(B)))=cl(int(cl(A)) ∪ int(cl(B))) ⊆ cl(int(cl(A) ∪ cl(B)) . Therefore A∪B is an IFβOS and hence by theorem 2.2.3, A∪B is an IF$\pi\beta$GOS in X.

Theorem 2.2.12:

If an IFS A of an IFTS in both an IFCS and an IFGOS, then A is an IF$\pi\beta$GOS in (X, τ).

Proof:

Suppose A is both an IFCS and IFGOS. Then as A ⊆ A, by hypothesis A ⊆ int(A). But int(A) ⊆ A. Therefore int(A) = A. We have A is an IFπOS, since every IFπOS is an IF$\pi\beta$GOS. Hence A is an IF$\pi\beta$GOS in X.

Theorem 2.2.13:

Let (X, τ) be an IFTS. Then for every $A \epsilon$ IFS(X) and for every $B \epsilon$ IFβO(X), $B \subseteq A \subseteq int(cl(int(B)))$ ⟹ $A\epsilon$ IFπβGO(X).

Proof:

Let B be an IF β OS. Then $B \subseteq cl(int(cl(B)))$. By hypothesis $A \subseteq int(cl(int(B))) \subseteq int(cl(int(cl(int(cl(B))))) \subseteq int(cl(cl(int(cl(B))))) = int(cl(int(cl(B)))) \subseteq int(cl(cl(A))) \subseteq int(cl(A))$ as $B \subseteq A$. Therefore A is an IFPOS. By theorem 2.2.3, A is an IFπβGOS . Hence $A \epsilon$IFπβGO(X).

Theorem 2.2.14:

If A is an IFβCS and an IF$\pi\beta$GOS in (X, τ), then A is an IFβOS in (X, τ).

Proof:

As $A \supseteq A$,by hypothesis $\beta int(A) \supseteq A$. But we have $A \supseteq \beta int(A)$. This impiles $A = \beta int(A)$. Hence A is an IFβOS in (X, τ).

CHAPTER 3

3.1 Intuitionistic Fuzzy $\pi\beta$ Generalized Continuous Mappings

In this section we have introduced a new type of intuitionistic fuzzy $\pi\beta$ generalized continuous mapping and established the relationship between basic intuitionistic fuzzy continuous mappings and newly introduced intuitionistic fuzzy $\pi\beta$ generalized continuous mappings. Also, we have analysed some properties of $\pi\beta$ generalized continuous mappings in intuitionistic fuzzy topological spaces.

Definition 3.1.1:

A mapping f: $(X,\tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy $\pi\beta$ generalized continuous (IF$\pi\beta$G continuous for short)mappings if $f^{-1}(V)$ is an IF$\pi\beta$GCS in (X,τ) for every IFCS V of (Y, σ).

we shall use the notation A= $< x, (\mu_A, \mu_B), (\nu_A, \nu_B) >$ instead of A= $<x, (a/\mu_A, b/\mu_B), (a/\nu_A, b/\nu_B)>$ in the following examples. Similarly we shall use the notation B= $< y, (\mu_u, \mu_v), (\nu_u, \nu_v) >$ instead of B= $<y, (a/\mu_u, b/\mu_v), (a/\nu_u, b/\nu_v)>$ in the following examples.

The intuitionistic fuzzy sets $0\sim = \{< x, 0,1) >/x \in X\}$ and $1\sim = \{< x, 0,1) >/x \in X\}$ are respectively the empty set and the whole set of X.

Example 3.1.2:

Let X={a,b} , Y={u,v} and G_1 = $<x, (0.4_a,0.2_b),(0.6_a,0.8_b)>$, $G_2=<x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$,$G_3=<y,(0.5_u,0.6_v),(0.5_u,0.4_v)>$.Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{ 0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS $G_3^c = <y,(0.5_u,0.4_v),(0.5_u,0.6_v)>$ is an IFCS in Y. Then $f^{-1}(G_3^c) = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$ is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ). Therefore f is an IF$\pi\beta$G continuous mapping.

Theorem 3.1.3:

Every IF continuous mapping is an IF$\pi\beta$G continuous mapping in (X,τ) but not conversely in general.

Proof:

Let f: (X,τ) → (Y, σ) be an IF continuous mapping. Let V be an IFCS in Y. Then f^{-1}(V) is an IFCS in X. Since every IFCS is an IF$\pi\beta$GCS, f^{-1}(V) is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.4:

Let X={a,b} , Y={u,v} and G_1 = <x, ($0.4_a,0.2_b$),($0.6_a,0.8_b$)>, G_2 =<x,($0.5_a,0.4_b$),($0.5_a,0.6_b$)>,G_3=<y,($0.5_u,0.6_v$),($0.5_u,0.4_v$)>. Then $\tau=\{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma=\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c=<y,($0.5_u,0.4_v$),($0.5_u,0.6_v$)> is an IFCS in Y. Then $f^{-1}(G_3^c)$=<x,($0.5_a,0.4_b$),($0.5_a,0.6_b$)> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ). Therefore f is an IF$\pi\beta$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = G_2^c \neq f^{-1}(G_3^c)$, f is not an IF continuous mapping.

Theorem 3.1.5:

Every IFS continuous mapping is an IF$\pi\beta$G continuous mapping in (X,τ) but not conversely in general.

Proof:

Let f: (X,τ) → (Y, σ) be an IFS continuous mapping. Let V be an IFCS in Y. Then f^{-1}(V) is an IFSCS in X. Since every IFSCS is an IF$\pi\beta$GCS, f^{-1}(V) is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.6:

Let $X=\{a,b\}, Y=\{u,v\}$ and $G_1=\langle x,(0.5_a,0.4_b),(0.5_a,0.6_b)\rangle$, $G_2=\langle x,(0.6_a,0.7_b),(0.4_a,0.3_b)\rangle$, $G_3=\langle y,(0.4_u,0.5_v),(0.6_u,0.5_v)\rangle$. Then $\tau=\{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma=\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)= u and f(b) = v. The IFS $G_3^c=\langle y,(0.6_u,0.5_v),(0.4_u,0.5_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c)=\langle x,(0.6_a,0.7_b),(0.4_a,0.3_b)\rangle$ is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ). Therefore f is an IF$\pi\beta$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c))) = int(1\sim) = 1\sim \nsubseteq f^{-1}(G_3^c)$, f is not an IFS continuous mapping.

Theorem 3.1.7:

Every IFP continuous mapping is an IF$\pi\beta$G continuous mapping in (X,τ) but not conversely in general.

Proof:

Let f: $(X,\tau)\to(Y,\sigma)$ be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFβGCS, $f^{-1}(V)$ is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.8:

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1=\langle x,(0.4_a,0.2_b),(0.6_a,0.8_b)\rangle$, $G_2=\langle x,(0.5_a,0.4_b),(0.5_a,0.6_b)\rangle$, $G_3=\langle y,(0.5_u,0.6_v),(0.5_u,0.4_v)\rangle$. Then $\tau=\{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma=\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)= u and f(b) = v. The IFS $G_3^c=\langle y,(0.5_u,0.4_v),(0.5_u,0.6_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c)=\langle x,(0.5_a,0.4_b),(0.5_a,0.6_b)\rangle$ is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ). Therefore f is an IF$\pi\beta$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3^c))) = cl(G_2)=G_2^c \nsubseteq f^{-1}(G_3^c)$, f is not an IFP continuous mapping.

Theorem 3.1.9:

Every IFR continuous mapping is an IF$\pi\beta$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: (X,τ) → (Y,σ) be an IFR continuous mapping. Let V be an IFCS in Y. Then f^{-1}(V) is an IFRCS in X. Since every IFRCS is an IF$\pi\beta$GCS, f^{-1}(V) is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.10:

Let X={a,b} , Y={u,v} and G_1 = <x, $(0.4_a,0.2_b),(0.6_a,0.8_b)$>, G_2=<x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$>,$G_3$=<y,$(0.5_u,0.6_v),(0.5_u,0.4_v)$>. Then $\tau=\{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma=\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y,$(0.5_u,0.4_v),(0.5_u,0.6_v)$> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ).Therefore f is an IF$\pi\beta$G continuous mapping, but not an IFR continuous mapping, since $f^{-1}(G_3^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_3^c)))= cl(G_2)= G_2^c \neq f^{-1}(G_3^c)$, f is not an IFR continuous mapping.

Theorem 3.1.11:

Every IFα continuous mapping is an IF$\pi\beta$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: (X,τ) → (Y,σ) be an IFα continuous mapping. Let V be an IFCS in Y. Then f^{-1}(V) is an IFαCS in X. Since every IFαCS is an IF$\pi\beta$GCS, f^{-1}(V) is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.12:

Let X={a,b} , Y={u,v} and G_1 = <x, $(0.5_a,0.4_b),(0.5_a,0.6_b)$>, G_2=<x,$(0.4_a,0.3_b),(0.6_a,0.7_b)$>,$G_3$=<y,$(0.5_u,0.6_v),(0.5_u,0.4_v)$>. Then τ ={$0_\sim, G_1, G_2, 1_\sim$} and σ ={ $0_\sim, G_3, 1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y,$(0.4_u,0.4_v),(0.5_u,0.6_v)$> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,$(0.4_a,0.4_b),(0.5_a,0.6_b)$> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ).Therefore f is an IF$\pi\beta$G continuous mapping, but not an IFα continuous mapping, since $f^{-1}(G_3^c)$ is not an IFαCS in X, as $cl(int(cl(f^{-1}(G_3^c)))$= $cl(int(G_2^c))$= $cl(G_2)$= $G_2^c \nsubseteq f^{-1}(G_3^c)$.

Theorem 3.1.13:

Every IFπ continuous mapping is an IF$\pi\beta$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: (X,τ) → (Y,σ) be an IFπ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFπCS in X. Since every IFπCS is an IF$\pi\beta$GCS, $f^{-1}(V)$ is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.14:

Let X={a,b} , Y={u,v} and G_1 = <x, $(0.5_a,0.3_b),(0.5_a,0.7_b)$>, G_2=<x,$(0.4_a,0.3_b),(0.6_a,0.7_b)$>, G_3=<y,$(0.7_u,0.8_v),(0.3_u,0.2_v)$>. Then τ ={$0_\sim, G_1, G_2, 1_\sim$} and σ ={ $0_\sim, G_3, 1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y,$(0.3_u,0.2_v),(0.7_u,0.8_v)$> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,$(0.3_a,0.2_b),(0.7_a,0.8_b)$> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ). Therefore f is an IF$\pi\beta$G continuous mapping but not an IFπ continuous mapping, since $f^{-1}(G_3^c)$ is not an IFπCS in X, as $cl(int(f^{-1}(G_3^c))=0\sim \neq f^{-1}(G_3^c)$.

Theorem 3.1.15:

Every IFβ continuous mapping is an IF$\pi\beta$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: (X,τ) → (Y,σ) be an IFβ continuous mapping. Let V be an IFCS in Y. Then f^{-1}(V) is an IFβCS in X. Since every IFβCS is an IF$\pi\beta$GCS, f^{-1}(V) is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.16:

Let X={a,b},Y={u,v} and G_1 = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>, G_2=<x,(0.4_a,0.3_b),(0.6_a,0.7_b)>,G_3=<y,(0.5_u,0.6_v),(0.4_u,0.4_v)>.Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X,τ) → (Y,σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y, (0.4_u,0.4_v),(0.5_u,0.6_v)> is an IFCS in Y. Therefore f is an IF $\pi\beta$ G continuous mapping. Then $f^{-1}(G_3^c)$ = <x,(0.4_a,0.4_b),(0.5_a,0.6_b)> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ). Therefore f is an IF$\pi\beta$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFβCS in X, as int(cl(int($f^{-1}(G_3^c)$) = int(cl(G_2) = $1_\sim$ $\not\subseteq f^{-1}(G_3^c)$, $f^{-1}(G_3^c)$ is not an IFS continuous mapping.

Theorem 3.1.17:

Every IFG continuous mapping is an IF$\pi\beta$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: (X,τ) → (Y,σ) be an IFG continuous mapping. Let V be an IFCS in Y. Then f^{-1}(V) is an IFGCS in X. Since every IFGCS is an IF$\pi\beta$GCS, f^{-1}(V) is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.18:

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1 = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$, $G_2=<x,(0.4_a,0.3_b),(0.6_a,0.7_b)>$,$G_3=<y,(0.5_u,0.6_v),(0.4_u,0.4_v)>$.Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS $G_3^c = <y, (0.4_u,0.4_v),(0.5_u,0.6_v)>$ is an IFCS in Y. Then $f^{-1}(G_3^c)= <x,(0.4_a,0.4_b),(0.5_a,0.6_b)>$ is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ). Therefore f is an IF$\pi\beta$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFGCS in X, as $cl(f^{-1}(G_3^c)) = 1_\sim \nsubseteq G_2$, $f^{-1}(G_3^c)$ is not an IFG continuous mapping.

Theorem 3.1.19:

Every IFGS continuous mapping is an IF$\pi\beta$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be an IFGS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGSCS in X. Since every IFGSCS is an IF$\pi\beta$GCS, $f^{-1}(V)$ is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.20:

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1 = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$, $G_2=<x,(0.4_a,0.3_b),(0.6_a,0.7_b)>$,$G_3=<y,(0.5_u,0.6_v),(0.4_u,0.4_v)>$.Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS $G_3^c = <y, (0.4_u,0.4_v),(0.5_u,0.6_v)>$ is an IFCS in Y. Then $f^{-1}(G_3^c) = <x,(0.4_a,0.4_b),(0.5_a,0.6_b)>$ is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ). Therefore f is an IF$\pi\beta$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFGSCS in X, as $f^{-1}(G_3^c) \cup int(cl(f^{-1}(G_3^c))) = f^{-1}(G_3^c) \cup int(G_1^c) = f^{-1}(G_3^c) \cup G_1 = G_1 \nsubseteq G_2$, $f^{-1}(G_3^c)$ is not an IFGS continuous mapping.

Theorem 3.1.21:

Every IFGP continuous mapping is an IF$\pi\beta$G continous mapping in (X,τ) but not conversely in general.

Proof:

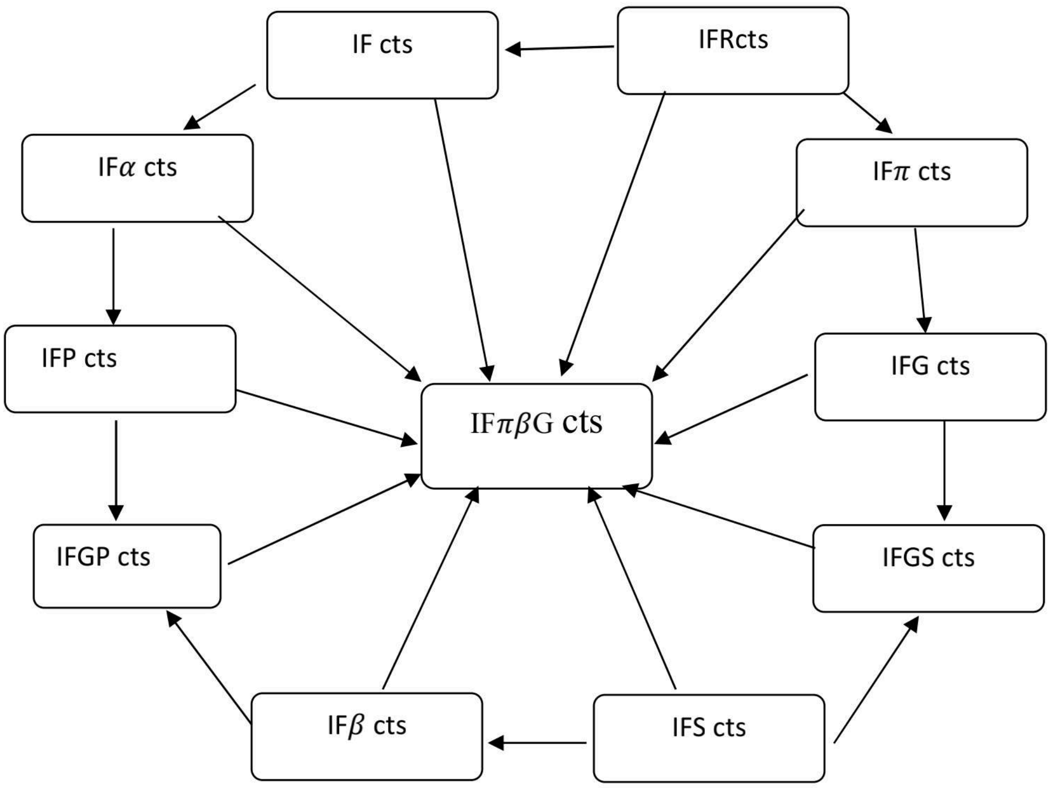
Let f: (X,τ) → (Y,σ) be an IFGP continuous mapping. Let V be an IFCS in Y. Then f^{-1}(V) is an IFGPCS in X. Since every IFGPCS is an IF$\pi\beta$GCS, f^{-1}(V) is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Example 3.1.22:

Let X={a,b} , Y={u,v} and G_1 = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>, G_2=<x,(0.4_a,0.3_b),(0.6_a,0.7_b)>,G_3=<y,(0.5_u,0.6_v),(0.4_u,0.4_v)>.Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y, (0.4_u,0.4_v),(0.5_u,0.6_v)> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,(0.4_a,0.4_b),(0.5_a,0.6_b)> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\beta$GCS in (X,τ). Therefore f is an IF$\pi\beta$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFGPCS in X , $f^{-1}(G_3^c)$ is not an IFGP continuous mapping.

The relationship between various types of intutionistic fuzzy continuity is given in the following figure. In this figure 'cts' means continuous.



Theorem 3.1.23:

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is an IF$\pi\beta$G continuous mapping if and only if the inverse image of each IFπOS in Y is an IF$\pi\beta$GOS in X.

Proof:

Necessity: Let A be an IFπOS in Y. This implies A^c is an IFπCS in Y. Then f^{-1} (A^c) is an IF$\pi\beta$GCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A)^c)$, $f^{-1}(A)$ is an IF$\pi\beta$GOS in X.

Sufficiency: Let A be an IFCS in Y. Then A^c is an IFπOS in Y. By hypothesis $f^{-1}(A^c)$ is an IF$\pi\beta$GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A)^c)$, $(f^{-1}(A)^c)$ is an IF$\pi\beta$GOS in X. Therefore $f^{-1}(A)$ is an IF$\pi\beta$GCS in X. Hence f is an IF$\pi\beta$G continuous mapping.

Theorem 3.1.24:

If f : $(X,\tau) \to (Y,\sigma)$ is an IF$\pi\beta$G continous mapping then for each IFP $p_{(\alpha,\beta)}$ of X and each A ϵ σ such that f($p_{(\alpha,\beta)}$) $_q$ A, there exists an IF$\pi\beta$GOS B of X such that $p_{(\alpha,\beta)}$ $_q$ B and f(B) ⊆ A.

Proof:

Let $p_{(\alpha,\beta)}$ be an IFP of X and Aϵ σ such that f($p_{(\alpha,\beta)}$)$_q$ A. Put B= f^{-1}(A). Then by hypothesis, B is an IFβGOS in X such that $p_{(\alpha,\beta)q}$ B and f(B) =f(f^{-1}(A) ⊆ A.

Theorem 3.1.25:

A mapping f :(X, τ) → (Y, σ) is an $IF\pi\beta G$ continuous mapping if cl(int(cl(f^{-1}(A)))) ⊆ f^{-1}(cl(A)) for every IFS A in Y.

Proof:

Let A be an IFπOS in Y then A^c is an IFCS in Y. By hypothesis, cl(int(cl(f^{-1}(A^c)))) ⊆ f^{-1}(cl(A^c))=f^{-1}(A^c).Now (int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(A^c)))) ⊆ f^{-1}(A^c)=f^{-1}(cl(A))^c.This implies that f^{-1}(A) ⊆ (int(cl(int(f^{-1}(A))))). Hence f^{-1}(A) is an IFαOS and hence it is an IFβGOS. Therefore f is an IF$\pi\beta$G continuous mapping.

Theorem 3.1.26:

Let f: $(X,\tau) \to (Y,\sigma)$ be an IF$\pi\beta$G continuous mapping and g: $(Y,\sigma) \to (Z,\delta)$ is an IF continuous mapping then g ∘ f :$(X,\tau) \to (Z,\delta)$ is an IF$\pi\beta$G continuous mapping.

Proof:

Let V be an IFCS in Z. Then g^{-1}(V) is an IFCS in Y, by hypothesis. Since f is an IF$\pi\beta$G continuous mapping, f^{-1}(g^{-1}(V)) is an IF$\pi\beta$GCS in X. Hence g ∘ f is an IF$\pi\beta$G continuous mapping.

Theorem 3.1.27:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an mapping from an IFTS X into an IFTS Y that satisfies $f^{-1}(int(B)) = int(cl(f^{-1}(B)))$ for every IFS B in Y. Then f is an IF$\pi\beta$G continuous mapping.

Proof:

Let B be an IF π OS in Y. Then int(cl(B))= B, by hypothesis $f^{-1}(B) = int(cl(f^{-1}(B)))$. This implies $f^{-1}(B)$ is an IFROS in X. Therefore it is an IF$\pi\beta$GOS in X. Hence f is an IF$\pi\beta$G continuous mapping.

SUMMARY AND CONCLUSION

SUMMARY AND CONCLUSION

The basic concept of fuzzy set was introduced by L.A. Zadeh in the year 1965. The fuzzy set theory was developed by Zadeh and others have found many applications in the domain mathematics and elsewhere. C.L. Chang in the year 1968 first introduced the concept of fuzzy topological spaces and he used the fuzzy set theory for defining and introducing fuzzy topological spaces. The notion of intutionistic fuzzy sets was introduced by Atanassaov as a generalization of fuzzy sets and in 1997, Coker introduced the concept of intutionistic fuzzy topological spaces.

In this thesis, we have introduced intutionistc fuzzy $\pi\beta$ generalized closed sets(intutionistc fuzzy $\pi\beta$generalized open sets) and compared intutionistc fuzzy $\pi\beta$ generalized closed sets and intutionistc fuzzy $\pi\beta$ generalized open sets with some of the basic intutionistic fuzzy sets such as intuitionistic fuzzy closed sets(intuitionistic fuzzy open sets), intuitionistic fuzzy regular closed sets(intuitionistic fuzzy regular open sets), intuitionistic fuzzy semi closed sets(intuitionistic fuzzy semi open sets), intuitionistic fuzzy α closed sets(intuitionistic fuzzy α open sets), intuitionistic fuzzy pre closed sets(intuitionistic fuzzy pre open sets), intuitionistic fuzzy π closed sets(intuitionistic fuzzy π open sets), intuitionistic fuzzy β closed sets(intuitionistic fuzzy β open sets), intuitionistic fuzzy generalized closed sets,intuitionistic fuzzy generalized semi closed sets and intuitionistic fuzzy generalized pre closed sets. Also, the intersectiom properties of intutionistc fuzzy $\pi\beta$ generalized closed sets and few properties of intutionistc fuzzy $\pi\beta$ generalized closed sets and intutionistc fuzzy $\pi\beta$ generalized open sets are analyzed and discussed with suitable example.

Also, we have introduced a new concept of intutionistc fuzzy $\pi\beta$ generalized continuous mapping and made an attempt to compare with basic intutionistc fuzzy continuous mappings and analyzed some of their properties. The reserve implications which do not hold good are substantiated by suitable examples.

The future research directions based on this research work may be extended as follows:

1. The notion of intuitionistic fuzzy $\pi\beta$ generalized closed sets can be extended to supra and nano topological spaces.

2. The theoretical developments studied in this thesis may be focused on the applications in medical diagnosis, new product marketing, sales analysis and financial services.

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