Chapter IV

CHAPTER IV

Supra Regular Generalized Closed (Open) Soft Sets in Supra Soft Topological Spaces

Section 4.1

Supra Regular Generalized Closed (Open) Soft Sets

Definition: 4.1.1

Let (X, μ, E) be a supra soft topological space. A soft set (F, E) is called a **supra regular open soft (supra regular closed soft)** in X if $(F, E) = int^s(cl^s(F, E))$ ($(F, E) = cl^s(int^s(F, E))$).

Remark: 4.1.2

Every supra regular closed soft set in supra soft topological space (X, μ, E) is supra closed soft set but the converse does not hold.

Example: 4.1.3

Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and

$\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where

$F_1(e_1) = \{x_1, x_2\}$, $F_1(e_2) = \{x_1, x_4\}$,

$F_2(e_1) = \{x_2\}$, $F_2(e_2) = \{x_4\}$,

$F_3(e_1) = \{x_3, x_4\}$, $F_3(e_2) = \{x_2, x_3\}$,

$F_4(e_1) = \{x_2, x_3, x_4\}$, $F_4(e_2) = \{x_2, x_3, x_4\}$.

Then (X, μ, E) is a supra soft topological space over X. Clearly $(F_4, E)^c$ is a supra closed soft set in (X, μ, E), but it is not a supra regular closed soft set.

Definition: 4.1.4

Let (X, μ, E) be a supra soft topological space. A soft set (F, E) is called a **supra regular generalized closed soft set** (supra rg-closed soft) in X if

$cl^s(F,E) \widetilde{\subseteq} (G,E)$ whenever $(F,E) \widetilde{\subseteq} (G,E)$ and (G,E) is a supra regular open soft set in X.

Theorem: 4.1.5

Every supra g-closed soft set in a supra soft topological space (X, μ, E) is a supra rg-closed soft set.

Proof:

Suppose that $(F,E) \widetilde{\subseteq} (G,E)$, where (G,E) is supra regular open soft. If (G,E) is supra regular open soft, then (G,E) is supra open soft. Thus $(F,E) \widetilde{\subseteq} (G,E)$ and (G,E) is supra open soft. Since (F,E) is a supra g-closed soft, then $cl^s(F,E) \widetilde{\subseteq} (G,E)$. Therefore (F,E) is a supra rg-closed soft set.

Example: 4.1.6

The supra soft topological space (X, μ, E) is the same as in Example 4.1.3. Let (H,E) be a soft set over X such that $H(e_1) = \{x_2, x_3\}$, $H(e_2) = \{x_4\}$. Clearly (H,E) is supra rg-closed soft set in (X, μ, E), but it is not supra g-closed soft.

Remark: 4.1.7

The intersection (resp. union) of two supra rg-closed soft sets is generally not a supra rg- closed soft set.

Example: 4.1.8

Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $E = \{e_1, e_2\}$ and

$\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_9,E), (F_{10},E), (F_{11},E)\}$ where

$F_1(e_1) = \{x_2, x_5, x_6\}$, $F_1(e_2) = \{x_4, x_5, x_6\}$,

$F_2(e_1) = \{x_5\}$, $F_2(e_2) = \{x_5\}$,

$F_3(e_1) = \{x_1, x_2, x_3\}$, $F_3(e_2) = \{x_1\}$,

$F_4(e_1) = \{x_2\}$, $F_4(e_2) = \{x_4, x_6\}$,

$F_5(e_1) = \{x_1, x_2, x_3, x_5, , x_6\}$, $F_5(e_2) = \{x_1, x_4, x_5, x_6\}$,

$F_6(e_1) = \{x_1, x_2, x_3, x_5\}$, $F_6(e_2) = \{x_1, x_5\}$,

$F_7(e_1) = \{x_2, x_5\}$, $F_7(e_2) = \{x_4, x_5, x_6\}$,

$F_8(e_1) = \{x_1, x_2, x_3\}$, $F_8(e_2) = \{x_1, x_4, x_6\}$,

$F_9(e_1) = \{x_1, x_2, x_3, x_5\}$, $F_9(e_2) = \{x_1, x_4, x_5, x_6\}$,

$F_{10}(e_1) = X$, $F_{10}(e_2) = \{x_1, x_2, x_3, x_4, x_6\}$,

$F_{11}(e_1) = \{x_1, x_2, x_3, x_4, x_6\}$, $F_{11}(e_2) = X$.

Then (X, μ, E) is a supra soft topological space over X. Let (H, E) and (G, E) be two sets over X such that $H(e_1) = \emptyset$, $H(e_2) = \{x_5\}$ and $G(e_1) = \{x_5\}$, $G(e_2) = \emptyset$. Clearly, (H, E) and (G, E) are supra rg-closed soft sets in (X, μ, E) but $(H, E) \cup (G, E)$ is not a supra rg-closed soft set.

Example: 4.1.9

The supra soft topological space (X, μ, E) is the same as in Example 4.1.8. Let (H, E) and (G, E) be two soft sets over X such that $H(e_1) = \{x_1, x_2, x_3, x_4\}$, $H(e_2) = \{x_1, x_4, x_6\}$ and $G(e_1) = \{x_1, x_2, x_3\}$, $G(e_2) = \{x_1, x_4, x_5, x_6\}$. Clearly, (H, E) and (G, E) are supra rg-closed soft sets in (X, μ, E) but $(H, E) \cap (G, E)$ is not a supra rg-closed soft set.

Theorem: 4.1.10

Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X. If a soft set (F, E) is a supra rg-closed soft set, then $cl^s(F, E)\backslash(F, E)$ contains only null supra regular closed soft set.

Proof:

Suppose that (F, E) is a supra rg-closed soft set. Let (H, E) be a supra regular closed soft subset of $cl^s(F, E)\backslash(F, E)$. Then $(H, E) \subseteq cl^s(F, E) \cap (F, E)^c$ so $(F, E) \subseteq (H, E)^c$. But (F, E) is a supra rg-closed soft set. Therefore $cl^s(F, E) \subseteq (H, E)^c$.

Consequently $(H, E) \subseteq (cl^s(F, E))^c$ (1)

We have already $\quad (H,E) \subseteq cl^s(F,E) \quad$ (2)

From (1) and (2) $\quad (H,E) \subseteq cl^s(F,E) \cap (cl^s(F,E))^c = \emptyset$

Thus $(H,E) = \emptyset$. Therefore $cl^s(F,E)\backslash(F,E)$ contains only null supra regular closed soft set.

Remark: 4.1.11

The converse of Theorem 4.1.10 is not true in general and is shown in following example.

Example: 4.1.12

The supra soft topological space (X,μ,E) is the same in Example 4.1.8. $cl^s(F_8,E)\backslash(F_8,E)$ contains only null supra regular closed soft set. But (F_8,E) is not a supra rg-closed soft set in (X,μ,E).

Corollary: 4.1.13

Let (X,μ,E) be a supra soft topological space and (F,E) be a supra rg-closed soft set in X. (F,E) is supra regular closed soft if and only if $cl^s(int^s(F,E))\backslash(F,E)$ is supra regular closed soft.

Proof:

Let (F,E) be a supra rg-closed soft set. If (F,G) is supra regular closed soft, then $cl^s(int^s(F,E))\backslash(F,E) = \emptyset$. Since $\emptyset$ is always supra regular closed soft, then $cl^s(int^s(F,E))\backslash(F,E)$ is supra regular closed soft.

Conversely, suppose that $cl^s(int^s(F,E))\backslash(F,E)$ is regular closed soft. Since (F,E) is supra rg-closed soft and $cl^s(F,E)\backslash(F,E)$ contains the supra regular closed soft set $cl^s(int^s(F,E))\backslash(F,E)$, then $cl^s(int^s(F,E))\backslash(F,E) = \emptyset$ by Theorem 4.1.10. Hence $cl^s\big(int^s(F,E)\big) = (F,E)$. Therefore (F,E) is supra regular closed soft.

Theorem: 4.1.14

Let (X,μ,E) be a supra soft topological space, (F,E) and (G,E) soft sets over X. If (F,E) is supra rg-closed soft and $(F,E) \subseteq (G,E) \subseteq cl^s(F,E)$, then $cl^s(G,E)\backslash(G,E)$ contains only null supra regular closed soft set.

Proof:

If $(F,E) \subseteq (G,E)$, then $(G,E)^c \subseteq (F,E)^c$. (1)

If $(G,E) \subseteq cl^s(F,E)$, then $cl^s(G,E) \subseteq cl^s(cl^s(F,E)) = cl^s(F,E)$ (2)

That is $cl^s(G,E) \subseteq cl^s(F,E)$.

From (1) and (2) $(cl^s(G,E) \cap (G,E)^c) \subseteq (cl^s(F,E) \cap (F,E)^c)$

which implies $(cl^s(G,E) \backslash (G,E)) \subseteq (cl^s(F,E) \backslash (F,E))$

Now (F,E) is supra rg-closed soft. Hence $cl^s(F,E) \backslash (F,E)$ contains only null supra regular closed soft, neither does $cl^s(G,E) \backslash (G,E)$.

Definition: 4.1.15

Let (X, μ, E) be a supra soft topological space. A soft set (F,E) is called supra regular generalized open soft (supra rg-open soft) in X if $(F,E)^c$ is supra rg-closed soft.

Theorem: 4.1.16

A soft set (F,E) is supra rg-open soft in a supra soft topological space (X, μ, E) if and only if $(H,E) \subseteq int^s(F,E)$ whenever $(H,E)$is supra regular closed soft in X and $(H,E) \subseteq (F,E)$.

Proof:

Let $(H,E) \subseteq int^s(F,E)$ whenever (H,E) is supra regular closed soft in X, $(H,E) \subseteq (F,E)$ and $(K,E) = (F,E)^c$. Suppose that $(K,E) \subseteq (G,E)$ where (G,E) is supra regular open soft.

Now $(F,E)^c \subseteq (G,E)$ implies $(H,E) = (G,E)^c \subseteq (F,E)$ and (H,E) is supra regular closed soft which implies $(H,E) \subseteq int^s(F,E)$. Also $(G,E) \subseteq int^s(F,E)$ implies $cl^s(int^s(F,E)) \subseteq cl^s(H,E) = (G,E)$. This in turn implies $(int^s((K,E)^c))^c \subseteq (G,E)$. or equivalently $cl^s(K,E) \subseteq (G,E)$. Thus (K,E) is supra rg-closed soft. Hence we obtain (F,E) is supra rg-open soft.

Conversely, suppose that (F,E) is supra rg-open soft, $(H,E) \subseteq (F,E)$ and (H,E) is supra regular closed soft. Then $(F,E)^c \subseteq (H,E)^c$. Since $(F,E)^c$ is supra rg-closed soft, $cl^s((F,E)^c) \subseteq (H,E)^c$.

Therefore $(H,E) \subseteq \left(cl^s((F,E)^c)\right)^c = cl^s(F,E)$.

Remark: 4.1.17

The intersection (resp. union) of two supra rg-open soft sets is generally not a supra rg- open soft set.

Section 4.2

Supra Regular Generalized Closed (Open) Soft Sets with respect to a Soft Ideal

Definition: 4.2.1

A soft set (F,E) is called a **supra regular generalized closed soft sets with respect to a soft ideal I (supra I-rg closed soft)** in a supra soft topological space (X,μ,E) if $cl^s(F,E)\backslash(G,E) \in I$ whenever $(F,E) \subseteq (G,E)$ and (G,E) is supra regular open soft in (X,μ,E).

Example: 4.2.2

Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1,E), (F_2,E), (F_3,E)\}$ where

$F_1(e_1) = \{x_3\}$, $F_1(e_2) = \{x_3\}$,

$F_2(e_1) = \{x_2, x_3\}$, $F_2(e_2) = \{x_3\}$,

$F_3(e_1) = \{x_2, x_3\}$, $F_3(e_2) = \emptyset$.

Then (X,μ,E) is a supra soft topological space. Let $I = \{\emptyset, (I_1,E), (I_2,E), (I_3,E), \dots, (I_7,E)\}$ be a soft ideal on X, where

$I_1(e_1) = \emptyset$, $I_1(e_2) = \{x_2\}$,

$I_2(e_1) = \emptyset$, $I_2(e_2) = \{x_2, x_3\}$,

$I_3(e_1) = \{x_1\}$, $I_3(e_2) = \emptyset$,

$I_4(e_1) = \{x_1\}$, $I_4(e_2) = \{x_1, x_2\}$,

$I_5(e_1) = \{x_1\}$, $I_5(e_2) = \{x_1\}$,

$I_6(e_1) = \emptyset$, $I_6(e_2) = \{x_1\}$,

$I_7(e_1) = \{x_1\}$, $I_7(e_2) = \{x_2\}$.

Clearly, (F_3, E) is supra I-rg closed soft. In fact, $(F_3, E) \subseteq (F_3, E)$ and (F_3, E) is supra regular open soft. Hence w obtain $cl^s(F_3, E) \backslash (F_3, E) \in I$.

Theorem: 4.2.3

Every supra rg-closed soft set is supra I-rg-closed soft.

Proof:

Let (F, E) be a supra rg-closed soft set in a supra soft topological space (X, μ, E). Let $(F, E) \subseteq (G, E)$ and (G, E) is supra regular open soft in(X, μ, E). Since (F, E) is supra rg-closed soft, then $cl^s(F, E) \subseteq (G, E)$ and hence $cl^s(F, E) \subseteq (G, E) = \emptyset \in I$. Consequently (F, E) is a supra I-rg-closed soft set.

The converse of this theorem is not true in general.

Example: 4.2.4

Let us take the supra soft topology μ on X in Example 4.2.2. Let (H, E) be a soft set over X such that $H(e_1) = \{x_3\}$, $H(e_2) = \emptyset$. Clearly (H, E) is supra I-rg-closed soft but it is not supra rg-closed soft.

Theorem: 4.2.5

Every supra I-g-closed soft set is supra I-rg-closed soft.

Proof:

Let (F, E) be a supra I-rg-closed soft set in a supra soft topological space (X, μ, E). We show that (F, E) is supra I-rg-closed soft. Suppose that $(F, E) \subseteq (G, E)$, where (G, E) is supra regular open soft. If (G, E) is supra regular open soft, then (G, E) is supra open soft. Thus $(F, E) \subseteq (G, E)$ and (G, E) is supra open soft. Since (F, E) is supra I-g-closed soft, then $cl^s(F, E) \backslash (G, E) \in I$. Therefore (F, E) is a supra I-rg-closed soft set.

Example 4.2.6

Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E)\}$ where

$F_1(e_1) = \{x_1, x_2, x_3\}$, $F_1(e_2) = X$,

$F_2(e_1) = \{x_1\}$, $F_2(e_2) = \{x_2\}$,

$F_3(e_1) = \{x_1, x_2\}$, $F_3(e_2) = \{x_1, x_2, x_3\}$.

Then (X, μ, E) is a supra soft topological space. Let $I = \{\emptyset, (I_1, E), (I_2, E), (I_3, E)\}$ be a soft ideal on X, where

$I_1(e_1) = \{x_2\}$, $I_1(e_2) = \emptyset$,

$I_2(e_1) = \emptyset$, $I_2(e_2) = \{x_1\}$,

$I_3(e_1) = \{x_2\}$, $I_3(e_2) = \{x_1\}$.

Clearly, (F_1, E) is supra I-rg-closed soft in (X, μ, E), but it is not supra I-g-closed soft.

Remark: 4.2.7

The intersection (resp. union) of two supra I-rg-closed soft sets is generally not a supra I-rg-closed soft set.

Example: 4.2.8

The supra soft topological space (X, μ, E) is the same as in Example 4.1.8. Let $I = \{\emptyset, (I_1, E), (I_2, E), (I_3, E)\}$ be a soft ideal on X, where

$I_1(e_1) = \{x_3\}$, $I_1(e_2) = \emptyset$,

$I_2(e_1) = \emptyset$, $I_2(e_2) = \{x_1\}$,

$I_3(e_1) = \{x_3\}$, $I_3(e_2) = \{x_1\}$.

Let (H, E) and (G, E) be two soft sets over X such that $H(e_1) = \emptyset$, $H(e_2) = \{x_5\}$ and $G(e_1) = \{x_5\}$, $G(e_2) = \emptyset$. Clearly, (H, E) and (G, E) are supra I-rg-closed soft sets in (X, μ, E) but $(H, E) \cup (G, E)$ is not a supra I-rg-closed soft set. Also, let (H, E) and (G, E) be two soft sets over X such that

$H(e_1) = \{x_1, x_2, x_3, x_4\}$, $H(e_2) = \{x_1, x_4, x_6\}$ and $G(e_1) = \{x_1, x_2, x_3\}$, $G(e_2) = \{x_1, x_4, x_5, x_6\}$. Clearly, (H, E) and (G, E) are supra I-rg-closed soft sets in (X, μ, E) but $(H, E) \cap (G, E)$ is not a supra I-rg-closed soft set.

Theorem: 4.2.9

Let (X, μ, E) be a supra soft topological space and (F, E) be a soft set over X. If a soft set (F, E) is supra I-rg-closed soft, then $(H, E) \subseteq cl^s(F, E) \setminus (F, E)$ and (H, E) is supra regular closed soft implies $(H, E) \in I$.

Proof:

Suppose that (F, E) is supra I-rg-closed soft. Let (H, E) be a supra regular closed soft subset of $cl^s(F, E) \setminus (F, E)$. Then $(H, E) \subseteq cl^s(F, E) \cap (F, E)^c$ and so $(F, E) \subseteq (H, E)^c$. Since (F, E) is a supra I-rg-closed soft set, then $cl^s(F, E) \setminus (H, E)^c \in I$. But $(H, E) \subseteq cl^s(F, E) \setminus (H, E)^c$, by the properties of soft ideal $(H, E) \in I$.

The converse of this theorem not true in general.

Example: 4.2.10

The supra soft topological space (X, μ, E) and soft ideal I are the same as in Example 4.2.8. $cl^s(F_8, E) \setminus (F_8, E)$ contains soft set (G, E) such that $(G, E) = \emptyset$ is only supra regular closed soft set implies $(G, E) = \emptyset \in I$. But (F_8, E) is not a supra I-rg-closed soft set in (X, μ, E).

Theorem: 4.2.11

Let (X, μ, E) be a supra soft topological space, (F, E) and (G, E) soft sets over X. If (F, E) is supra I-rg-closed soft and $(F, E) \subseteq (G, E) \subseteq cl^s(F, E)$, then (G, E) is supra I-rg-closed soft.

Proof:

Let (F, E) be asupra I-rg-closed soft set and $(F, E) \subseteq (G, E) \subseteq cl^s(F, E)$. Suppose that $(G, E) \subseteq (H, E)$ and (H, E) is supra regular open soft. Then

$(F, E) \subseteq (H, E)$. Since (F, E) is supra I-rg-closed soft, then $cl^s(F, E)\setminus(H, E) \in I$. If $(G, E) \subseteq cl^s(F, E)$, then $cl^s(G, E) \subseteq cl^s(cl^s(F, E) = cl^s(F, E)$. So we have $[cl^s(G, E)\setminus(H, E)] \subseteq [cl^s(F, E)\setminus(H, E)]$ and we obtain $cl^s(G, E)\setminus(H, E) \in I$. That is (G, E) is supra I-rg-closed soft.

Definition: 4.2.12

A soft set (F, E) is called **supra regular generalized open soft set with respect to soft ideal I (supra I-rg-open soft)** in a supra soft topological space (X, μ, E) if and only if $(F, E)^c$ is supra I-rg-closed soft in (X, μ, E).

Theorem: 4.2.13

A soft set (F, E) is a supra I-rg-open soft set in a supra soft topological space (X, μ, E) if and only if $(G, E)\setminus(H, E) \subseteq int^s(F, E)$ for some $(H, E) \in I$, whenever $(G, E) \subseteq (F, E)$ and (G, E) is supra regular closed soft in (X, μ, E).

Proof:

Suppose that (F, E) is supra I-rg-open soft, $(G, E) \subseteq (F, E)$ and (G, E) is supra regular closed soft. Then $(G, E)^c$ is supra regular closed soft. Then $(F, E)^c \subseteq (G, E)^c$. Since $(F, E)^c$ is supra I-rg-closed soft, $cl^s((F, E)^c)\setminus (G, E)^c \in I$. This implies that $cl^s((F, E)^c)\setminus (G, E)^c = cl^s((F, E)^c) \cap (G, E) = (H, E) \in I$, so $[cl^s((F, E)^c) \cap (G, E)] \cup (G, E)^c = (H, E) \cup (G, E)^c$. Hence $cl^s((F, E)^c) \subseteq (G, E)^c \cup (H, E)$ for some $(H, E) \in I$. So $[(G, E)^c \cup (H, E)]^c \subseteq (cl^s(F, E)^c))^c = int^s(F, E)$ and therefore $(G, E)\setminus (H, E) = (G, E) \cap (H, E)^c \subseteq int^s(F, E)$.

Conversely, assume that $(G, E) \subseteq (F, E)$ and (G, E) is supra regular closed soft (X, μ, E). These imply that $(G, E)\setminus(H, E) \subseteq int^s(F, E)$ for some $(H, E) \in I$. Now we show that $(F, E)^c$ is supra I-rg-closed soft. Let $(F, E)^c \subseteq (K, E)$ and (K, E) is supra regular open soft. Then $(K, E)^c \subseteq (F, E)$. By assumption $(K, E)^c\setminus(H, E) \subseteq int^s(F, E) = (cl^s(F, E)^c))^c$ for some $(H, E) \in I$. This gives that $[(K, E) \cup (H, E)]^c \subseteq (cl^s(F, E)^c))^c$. Then $cl^s((F, E)^c) \subseteq (K, E) \cup (H, E)$ for some $(H, E) \in I$. Thus $cl^s((F, E)^c)\setminus(K, E) \subseteq ((K, E) \cup (H, E))\setminus(K, E) = ((K, E) \cup (H, E)) \cap (K, E)^c$ $=(H, E) \cap (K, E)^c \subseteq (H, E) \in I$. This shows that $cl^s((F, E)^c)\setminus(K, E) \in I$ (from the properties of soft ideal). Hence $(F, E)^c$ is supra I-rg-closed soft and (F, E) is supra I-rg-open soft.

Remark: 4.2.14

The intersection (res. union) of two supra I-rg-open soft sets is generally not supra I-rg-open soft.

Theorem: 4.2.15

Let (X, μ, E) be a super soft topological space and (F, E) be a soft set over X. If a soft set (F, E) is supra I-rg-closed soft, then $cl^s(F, E)\backslash(F, E)$ is supra I-rg-open soft.

Proof:

Suppose that (F, E) is soft I-rg-closed and $(H, E) \subseteq cl^s(F, E)\backslash(F, E)$, where (H, E) is supra regular closed soft. Then $(H, E) \in I$ and hence $(H, E)\backslash(G, E) = \emptyset$ for some $(G, E) \in I$. Clearly $(H, E)\backslash(G, E) \subseteq int^s(cl^s(F, E)\backslash(F, E))$. By Theorem 4.2.13, $cl^s(F, E)\backslash(F, E)$ is supra I-rg-open soft.

The converse of this theorem is not true in general.

Example: 4.2.16

The supra soft topological space (X, μ, E) and soft ideal I are the same in Example 4.2.8. Let (H, E) be a soft set over X such that $H(e_1) = \{x_5\}$, $H(e_2) = \{x_5\}$. $cl^s(H, E)\backslash(H, E)$ is a supra I-rg-open soft set. But (H, E) is not supra I-rg-closed soft in (X, μ, E).