Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test II- April 2025

II Semester

Class : I PG **Time : 2 Hours**

Branch : Mathematics **Max.Marks : 60**

23MMAC07 – Advanced Algebra II

Course Outcomes:

CO1: Demonstrate knowledge of conjugacy relation and class equation

CO2: Apply Sylow theorem to determine the nature of subgroups

CO3: Identify the irreducibility of polynomials

CO4: Develop the concepts of extension fields

CO5: Find the splitting field for a given polynomial.

Part A **6 x 1 = 6**

Choose the Correct Answer

1. Two nilpotent transformations are similar if and only if they have the ...... invariants.
   a. same b. different c. invertible d. primitive CO3K1
2. If the matrix of dimension m, is cyclic with respect to T then the dimension of MT^k is
   a. m-k b. m+k c. m-n d. m+n CO3K1
3. If dim(V) = n then the characteristic polynomial of T is the ..... of its elementary divisor. CO4K2
   a. multiplicity b. product c. additive d. both (a) and (b)
4. If the multiplicity of each characteristic root of T is 1 and if all the characteristic roots of T are in F then T is .... over F. CO4K3
   a. diagonalizable b. Canonical c. commute d. not commute
5. If the field F has p^m elements then F is the splitting field of the polynomial .....
   a. $x^{p^m} - x$ b. $x^{p^m} + x$ c. $x^{p^m} - m$ d. $x^{p^m} + m$ CO5K2
6. The multiplicative group of non zero elements of a finite field is........ CO5K1
   a. cyclic b.isomorphic c. field d. ring

Part B **3 x 6 = 18**

Answer ALL questions

7.a. Prove that there exists a subspace W of V which is invariant under T such that $V = V_1 \oplus W$.

(or)

7.b. If T ϵ A(V) is nilpotent then prove that $\alpha_0 + \alpha_1 T + \alpha_2 T^2 + ... + \alpha_m T^m$ is invertible if $\alpha_0 \neq 0$.

8.a. If $V = V_1 \oplus V_2$, where V_1 and V_2 are subspaces of V invariants under T. Let T_1 and T_2 be the linear transformations induced by T on V_1 and V_2 respectively. If the minimal polynomial of T_1 over F is $P_1(x)$ while that of T_2 is $P_2(x)$ then show that the minimal polynomial for T over F is the least common multiple of $P_1(x)$ and $P_2(x)$.

(or)

8. b. Prove that the elements S and T in $A_F(V)$ are similar if and only if they have the same elementary divisors.

9.a. For every prime number P and for every positive integer m, prove that there exists a field having P^m elements.

(or)

9. b. Let G be a finite abelian group satisfying the relation $x^n = e$ is solved by atmost n elements of G for every integer n then prove that G is a cyclic group.

Part C **3 x 12 = 36**

Answer ALL questions

10.a. Prove that the two nilpotent linear transformations are similar if and only if they have the same invariants.

(or)

10.b. If T ϵ A(V) is nilpotent is of index of nilpotence n_1 then show that a basis of V can formed such that the matrix of T in this basis has the form $\begin{bmatrix} M_{n1} & 0 \dots\dots & 0 \\ 0 & M_{n2} & 0 \\ 0 & 0 \dots\dots\dots & M_{nr} \end{bmatrix}$

where $n_1 \geq n_2 \geq \dots n_r$ and $n_1 + n_2 + \dots + n_r = \dim V$

11.a.Let V and W be two vector spaces over F and suppose that ψ is a vector space isomorphism of V onto W. Suppose that S ϵ $A_F(V)$ and T ϵ $A_F(W)$ are such that for any v ϵ V, $(vs)\psi = (v\psi)T$ then show that S and T have same elementary divisors.

(or)

11.b. Let $V_i \neq (O)$ and $V = V_1 \oplus V_2 \oplus \dots\dots \oplus V_k$ then prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$.

12.a. Summarize and prove Jacobson theorem.

(or)

12. b. State and prove Wedderburn's theorem.

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