Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University), Coimbatore-641 043

Bachelor's Degree Examination- November 2018

I-Semester

Class : I UG | Time: 3 hours

Major: Mathematics | Max. Marks: 100

18BMAC02 – Classical Algebra and Theory of numbers

Part-A **10 x 1=10**

Circle the correct answer

1. Every equation f(x) = 0 of the nth degree has __________

   (a) n roots (b) n roots and no more
   (c) n-1 roots (d) n roots and more

2. If the roots of the equation $x^3 + \frac{x^2}{4} - \frac{x}{16} + \frac{1}{72} = 0$ are multiplied by ______________

   Then the fractional coefficients of the equation will be removed.

   (a)24 (b) 12 (c) 72 (d) 16

3. The inverse of an orthogonal matrix is ____________________

   (a) Orthogonal (b) not orthogonal
   (c) unit matrix (d) null matrix

4. If $|A|$ =0, then the matrix A is called __________________

   (a) null matrix (b) Singular matrix
   (c) non-singular matrix (d) unit matrix

5. By Euler's function ϕ(N), the value of ϕ(5) = ___________

   (a) 1 (b) 2 (c) 3 (d) 4

6. The integral part of $\left[\frac{2}{5}\right]$ = ______________

   (a) 0 (b) 1 (c) 3 (d) 4

7. If $a \equiv b(\bmod\ m);\ a_1 \equiv b_1(\bmod\ m)$, then

   (a) $a + a_1 \equiv b1(\bmod\ m)$ (b) $aa_1 \equiv bb1(\bmod\ m)$

   (c) $a \equiv bb1(\bmod\ m)$ (d) $aa_1 \equiv b(\bmod\ m)$

8. The least positive integer having remainders 2,3,2 when divided by 3,5,7 is ___________

   (a) 24 (b) 124 (c) 28 (d) 128

9. If p is an odd prime and a is prime to p, then _______________ is divisible by p.

   (a) a^{p-1} -1 (b) a^{p-1} +1 (c) $a^{½ (p-1)} \pm 1$ (d) $a^{(p-1)} \pm 1$

10. The p is a prime greater than 3, then A_{p-2} is a multiple of ______________

    (a) P (b) P^2 (c) 3 (d) greater than 3

Part B **(5 X 6=30)**

Answer all the questions

11.a Solve the equation $x^4+3x^3-3x-1=0$

(or)

11.b. Determine completely the nature of the roots of the equation
$x^5-6x^2-4x+5 = 0$

12.a. Show that

$$A = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$$ is orthogonal

(or)

12.b. Calculate A^4 when $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

13. a. Find the smallest number with 18 divisors

(or)

13. b. Find the number of integers less than n and prime to it when n=729 and 720

14.a Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

(or)

14.b. Show that $x^5 - x$ is divisible by 30

15.a .Show that the 8^{th} power of any number is of the form 17m (or) $17m \pm 1$.

(or)

15. b. Show that (18)!+1 is divisible by 437.

Part C **5 x 12=60**

Answer all the questions

16.a Solve the equation $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$

(or)

16. b. Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve the equation

17.a. Diagonalise the matrix

$$\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & +3 & -1 \end{pmatrix}$$

(or)

17. b) Find the characteristic equation of the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$ and hence determine its inverse.

18.a. If $d_1, d_2, d_3 \ldots\ldots d_r$ (including 1 and N) are the divisors of N, then show that $\phi(d_1) + \phi(d_2) + \ldots\ldots + \phi(d_r) = N$

(or)

b. (i) Find the highest power of 3 dividing 1000!
(ii) Show that n(2n+1) (n+1) is divisible by 6

19.a. (i) Find the remainder obtained in dividing 2^{46} by 47
(ii) Find the remainder when 2^{1000} is divisible by 17

(or)

19. b. (i) Show that every integer which is a perfect cube is of the form 7P (or) $7P \pm 1$
(ii) If x,y,z be three consecutive integers, show that $(\Sigma x)^3 - 3\Sigma x^3$ is divisible by 108.

20.a. State and prove Fermat's theorem

(or)

b. State and prove Lagrange's theorem

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1