Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – August 2020
VI Semester

Class : III UG **Time : 2 Hours**
Major : Mathematics **Max. Marks : 50**

15BMAC21 Abstract Algebra II

Part A **10 x 1 = 10**

Choose the Correct Answer

1. a_0 is a prime element of $R iff$ ideal $A = (a_0)$ is a ________ of the Euclidean ring R.
   a. maximal ideal b. minimal ideal
   c. no ideal d. none of the above

2. If $a + bi$ is not a unit of $J[i]$, then, $a^2 + b^2$ is
   a. ≤ 1 b. ≥ 1
   c. < 1 d. > 1

3. If V is a vector space over F, then,
   a. α0 = 0 b. α0 ≥ 0
   c. α0 ≤ 0 d. none of the above

4. Any two finite-dimensional vector spaces over F ,having same dimensions are
   a. homomorphic b. isomorphic
   c. endomorphic d. automorphic

5. If $dim_F V = n$,then,$dim_F(Hom(V,V)) =$____
   a. $2n$ b. $2n^2$
   c. n^2 d. n

6. If $dim_F V = m$, then, $dim_F(Hom(V,F)) =$ ____
   a. $2m4$ b. $2m^2$
   c. m^2 d. m

7. If W is a subspace of V then the annihilator of W, $A(W) =$
   a. $\{f \in \hat{V} | f(w) = 0\ all\ w \in W\}$ b. $\{f \in \hat{V} | f(w) < 0\ all\ w \in W\}$
   c. $\{f \in \hat{V} | f(w) > 0\ all\ w \in W\}$ d. none of the above

8. $\|v\| =$
   a. $(v,v)^2$ b. (v,v)
   c. $\sqrt{(v,v)}$ d. none of the above

9. $W \cap W^\perp =$
   a. 0 b. W
   c. $W^\perp$ d. none of the above

10. Any finite number of elements in a subset S of vector space V is
    a. linearly dependent b. linearly independent
    c. finite-dimensional d. None of these

Part B $3 \times 6 = 18$

Answer any **Three** questions

Each answer should not exceed 400 words or two pages

11. Let R be a Euclidean ring. Suppose that for $a, b, c \in R, a|bc$ but $(a, b) = 1$. then,Show that $a|c$.

12. If p is a prime number of the form $4n + 1$, then show that the equation $x^2 \equiv -1(mod\ p)$ has solution.

13. Show that $L(S)$is a sub-space of V, for, $S \leq V$ a Vectorspace.

14. If V is a vector space over F, then, show that,
    i. α 0 = 0 for $\propto \in F$.
    ii. $0v$ = 0 for $v \in V$.
    iii. $(-\propto)v = -(\propto v)$ for $\propto \in F, v \in V$.
    iv. If $v \neq 0, then\ \propto v = 0\ implies\ that\ \propto = 0$

15. If $v_1, \ldots, v_n$ are in V, then prove that either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1, \ldots, v_{k-1}$.

16. If A and B are finite-dimensional subspaces of a vector space V, then show that, $(A + B)$ is finite-dimensional and $\dim(A + B) = \dim(A) + \dim(B) - \dim(A \cap B)$.

17. If V is finite dimensional inner product space and W is a subspace of V then prove that $(W^{\perp})^{\perp} = W$.

18. Prove that $\|\propto u\| = |\propto|\|u\|$, for $u \in V$ and $\propto$ is a scalar.

19. Prove that $A(W)$ is a subapace of $\hat{V}$, for a subspace W of a vector space V.

20. Prove that $A(A(W)) = W$.

Part C $2 \times 11 = 22$

Answer any **Two** questions

Each answer should not exceed 800 words or four pages

21. State and Prove Fermat's Theorem.

22. State and Prove Unique Factorization Theorem.

23. If $v_1, \ldots, v_n$ is a basis of V over F and if $w_1, \ldots, w_m$ in V are linearly independent over F, then $m \leq n$.

24. If V is the internal direct sum of $U_1, \ldots, U_n$, then $V$is isomorphic to the external direct sum of $U_1, \ldots, U_n$.

25. If V is finite-dimensional vector space and if W is a subspace of V, then show that,
    i.W is finite-dimensional
    ii.$\dim W \leq \dim V$ and
    iii.$\dim V/W = \dim V - \dim W$

26. i) If V is finite-dimensional vector space over F then show that, any two bases of V have the same number of elements.

ii)show that $F^{(n)}$ is isomorphic $F^{(m)}$ if and only if $n = m$.

27. If V is finite-dimensional vector space and $v \neq 0 \in V$, then there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

28. If V and W are of dimensions m and n, respectively, over F, then show that $Hom(V, W)$ is of dimension mn over F.

29. If V is a finite-dimensional inner product space and if W is a subspace of V, then prove that $V = W + W^{\perp}$. More particularly, V is the direct sum of W and $W^{\perp}$.

30. State and Prove Schwarz Inequality.

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