

**15BPHI01 DSE – I Mathematics - I**

**Part A**

**Choose the correct Answer**

1. When  $|x| < 1$ ,  $1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \dots$  is equal to -----  
 (a)  $(1+x)^n$  (b)  $(1-x)^{-n}$  (c)  $(1-x)^n$  (d)  $(1+x)^{-n}$
2. The Coefficient of  $x^5$  in the expansion of  $(1+x+x^2+x^3+x^4)^6$  is equal to -----  
 (a) 240 (b) 242 (c) 244 (d) 246
3.  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{nx}$  is equal to -----  
 (a) 1 (b)  $e$  (c)  $e^x$  (d) 0
4. If  $|x| < 1$  then  $\log\left(\frac{1+x}{1-x}\right) =$  -----  
 (a)  $2\left[x - \frac{x^2}{2} + \frac{x^3}{3} + \dots\right]$  (b)  $2\left[x + \frac{x^2}{2} + \frac{x^3}{3} + \dots\right]$  (c)  $2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \dots\right]$   
 (d)  $2\left[x - \frac{x^3}{3} + \frac{x^5}{5} + \dots\right]$
5.  $\Delta^2 y_k =$  -----  
 (a)  $\Delta y_{k+1} - \Delta y_{k-1}$  (b)  $\Delta y_{k+1} - \Delta y_k$  (c)  $\Delta y_{k-1} - \Delta y_{k+1}$  (d)  $\Delta y_{k-1} - \Delta y_k$
6. The process of computing intermediate values of a function from a set of given tabular values of that function is called -----  
 (a) Extrapolation (b) Integration (c) Interpolation (d) Computation
7. If  $x^n = \cos n\theta + i \sin n\theta$ , then  $2i \sin n\theta =$  -----  
 (a)  $x^n + \frac{1}{x^n}$  (b)  $x^n - \frac{1}{x^n}$  (c)  $x + \frac{1}{x}$  (d)  $x - \frac{1}{x}$
8. If  $\tan \theta =$  -----  
 (a)  $1 - \frac{\theta^2}{2} + \frac{\theta^4}{24} + \dots$  (b)  $1 + \frac{\theta^2}{2} + \frac{\theta^4}{24} + \dots$  (c)  $1 - \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \dots$  (d)  $1 + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \dots$
9. The  $n$ th derivative of  $\log(ax+b)$  is -----  
 (a)  $\frac{a^n(-1)^{n-1}(n-1)!}{(ax+b)^n}$  (b)  $\frac{a^n(-1)^{n-1}n!}{(ax+b)^n}$  (c)  $\frac{a^n(-1)^{n-1}(n+1)!}{(ax+b)^n}$  (d)  $\frac{a^n(-1)^{n+1}(n-1)!}{(ax+b)^n}$
10. If  $u$  and  $v$  are functions of  $x$ , if  $y = uv$  then  $(uv)_n = u_n v + n C_1 u_{n-1} v_1 + \dots + u v_n$  is called -----  
 (a) Euler's (b) Newton's Backward (c) Leibnitz (d) Maclaurin's

**PART B**

**5 × 6 = 30**

**Answer the following**

11. (a) Show that  $\left(\frac{1+x}{1-x}\right)^n = 1 + n\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{1.2}\left(\frac{2x}{1+x}\right)^2 + \dots$   
 (Or)  
 (b) Sum the Series  $1 + \frac{1}{3} + \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \dots$
12. (a) Find the coefficient of  $x^n$  in the expansion of  $\frac{1+x+x^2}{e^x}$ .  
 (Or)  
 (b) Show that  $\lim_{x \rightarrow 0} \left(\frac{1+x}{1-x}\right)^{\frac{1}{x}} = e^2$
13. (a) Find the Cubic Polynomial which takes the following values

$x$	0	1	2	3
$y$	1	2	1	10

**(Or)**

- (b) Find the Polynomial to the data which takes the following values using Lagrange's Interpolation Formula

$x$	0	1	3	4
$y$	-12	0	6	12

14. (a) Prove that  $\frac{\sin 5\theta}{\sin \theta} = 16\sin^4 \theta - 20\sin^2 \theta + 5$

(Or)

(b) Evaluate:  $\lim_{x \rightarrow 0} \frac{\tan 2x - 2x \cos x}{x^3}$

15. (a) If  $y = a \cos mx + b \sin mx$  then prove that  $y_2 + m^2 y = 0$

(Or)

(b) Find the  $n$ th derivative of  $(ax + b)$ .

PART C

5 × 12 = 60

Answer the following

16. (a) Evaluate:  $\lim_{x \rightarrow a} \frac{\sqrt{2x+a} - \sqrt{3x}}{\sqrt{x+3a} - 2\sqrt{x}}$

(Or)

(b) Sum the Series  $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots$

17. (a) Show that  $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$

(Or)

(b) Prove that  $\log \sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right) \frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right) \frac{1}{4^2} + \dots$

18. (a) Prepare a Forward & Backward Difference Table and express  $y$  as a function of  $x$  and also find  $y$  when  $x = 43$  &  $x = 84$

$x$	40	50	60	70	80	90
$y$	184	204	226	250	276	304

(Or)

- (b) From the following table, find  $\log_{10} 301$ .

$x$	300	304	305	307
$y = \log_{10} x$	2.4771	2.4829	2.4843	2.4871

19. (a) Prove that  $\cos^5 \theta \sin^3 \theta = -(1/2^7)[\sin 8\theta + 2\sin 6\theta - 2\sin 4\theta - 6\sin 2\theta]$

(Or)

(b) (i) If  $\lim_{\theta \rightarrow 0} \frac{\theta(a+b\cos\theta) - c\sin\theta}{\theta^5} = 1$  then find the values of  $a, b, c$

(ii) If  $\cos \theta = \frac{1681}{1682}$  then show that  $\theta$  is approximately equal to  $2^\circ$ .

20. (a) Find the  $n$ th derivative of  $e^{ax}(\cos bx + c)$

(Or)

(b) If  $y = a \cos(\log x) + b \sin(\log x)$  then prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$