Chapter V

CHAPTER V

GENERALISED FUZZY SOFT SETS

Definition: 5.1

Let U = $\{x_1, x_2, \ldots\ldots\ldots\ldots, x_n\}$ be the universal set of elements and E = $\{e_1, e_2, \ldots\ldots\ldots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to F(U)$ and μ be a fuzzy subset of E, *i.e.* $\mu : E \to [0, 1]$, where $F(U)$ is the collection of all fuzzy subsets of U. Let F_μ be the mapping $F_\mu : E \to F(U) \times [0, 1]$ be a function defined as follows: $F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in F(U)$. Then F_μ is called a **Generalized Fuzzy Soft Set (GFSS)** over the soft universe (U, E).

Here for each parameter e_i, $F_\mu(e_i) = (F(e_i), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

Example: 5.2

Let U = $\{x_1, x_2, x_3\}$ be a set of three shirts under consideration. Let E = $\{e_1, e_2, e_3\}$ be a set of qualities where e_1 = bright, e_2 = cheap, e_3 = colorful. Let $\mu : E \to [0, 1]$ be defined as follows: $\mu(e_1) = 0.1$, $\mu(e_2) = 0.4$, $\mu(e_3) = 0.6$.

We define a function $F_\mu : E \to F(U) \times [0, 1]$ be defined as follows:

$F_\mu(e_1) = (\{x_1/0.7, x_2/0.4, x_3/0.3\}, 0.1),$

$F_\mu(e_2) = (\{x_1/0.1, x_2/0.2, x_3/0.9\}, 0.4)$

$F_\mu(e_3) = (\{x_1/0.8, x_2/0.5, x_3/0.2\}, 0.6)$

Then F_μ is a GFSS over (U, E).

In matrix form this can be represented as $F_\mu = \begin{pmatrix} 0.7 & 0.4 & 0.3 & 0.1 \\ 0.1 & 0.2 & 0.9 & 0.4 \\ 0.8 & 0.5 & 0.2 & 0.6 \end{pmatrix}$, where the ith row vector represent $F_\mu(e_i)$, ith column vector represent x_i, the last column represents the value of μ and it will be called membership matrix of F_μ.

Definition: 5.3

Let F_μ and G_δ be two GFSS over (U, E). Now F_μ is said to be a **Generalized Fuzzy Soft Subset** of G_δ if

1) μ is a fuzzy subset of δ
2) $F(e)$ is also a fuzzy subset of $G(e)$, $\forall e \in E$.

In this case we write $F_\mu \subseteq G_\delta$.

Example: 5.4

Consider the GFSS F_μ over (U, E) given in Example 5.2. Let G_δ be another GFSS over (U, E) defined as follows:

$G_\delta(e_1) = (\{x_1/0.2, x_2/0.3, x_3/0.1\}, 0.1)$,

$G_\delta(e_2) = (\{x_1/0.0, x_2/0.1, x_3/0.7\}, 0.3)$,

$G_\delta(e_3) = (\{x_1/0.7, x_2/0.3, x_3/0.1\}, 0.5)$, where $\delta \in F(E)$ be defined as above.

Then G_δ is a generalized fuzzy soft subset of F_μ.

Note: 5.5

Let c be an involutive fuzzy complement and g be an increasing generator of c. Let $*$ and $\circ$ be two binary operations on [0, 1] defined as follows:

$a * b = g^{-1}(g(a)+g(b)-g(1))$ and $a \circ b = g^{-1}(g(a)+g(b))$.

Then $*$ is a t-norm and is a t-conorm. Moreover $(*, \circ, c)$ becomes a triple. Henceforth in the rest of the paper we will take such an involutive dual triple to consider the general case.

Definition: 5.6

Let F_μ be a GFSS over (U, E). Then the **Complement** of F_μ, denoted by $F_\mu^C = G_\delta$, where $\delta(e) = \mu^C(e)$ and $G(e) = F^C(e)$, $\forall e \in E$.

Note: 5.7

Obviously $(F_\mu^C)^C = F_\mu$ as the fuzzy complement c is involutive in nature.

Definition: 5.8

Union of two GFSS F_μ and G_δ, denoted by $F_\mu \tilde{\cup} G_\delta$, is a GFSS H_υ, defined as $H_\upsilon : E \to F(U) \times [0, 1]$ such that $H_\upsilon(e) = (H(e), \upsilon(e))$, where $H(e) = F(e) \circ G(e)$ and $\upsilon(e) = \mu(e) \circ \delta(e)$.

Definition: 5.9

Intersection of two GFSS F_μ and G_δ, denoted by $F_\mu \tilde{\cap} G_\delta$, is a GFSS H_υ, defined as $H_\upsilon : E \to F(U) \times [0, 1]$ such that $H_\upsilon(e) = (H(e), \upsilon(e))$, where $H(e) = F(e) * G(e)$ and $\upsilon(e) = \mu(e) * \delta(e)$.

Example: 5.10

Let us consider the generalized fuzzy soft sets F_μ and G_δ defined in Example 5.2 and 5.4 respectively. Let us defined the t-norm $*$ on [0, 1] as follows: $a * b = a.b$ and t-conorm $\circ$ on [0, 1] as follows: $a \circ b = a + b - a.b$. Let us also take c as fuzzy complement *i.e.* $a^C = 1 - a$. Then $(*, \circ, c)$ forms a involutive dual triple.

Then $F_\mu \tilde{\cup} G_\delta = \begin{pmatrix} 0.76 & 0.58 & 0.37 & 0.19 \\ 0.1 & 0.28 & 0.97 & 0.58 \\ 0.98 & 0.65 & 0.28 & 0.80 \end{pmatrix}$,

$F_\mu \tilde{\cap} G_\delta = \begin{pmatrix} 0.14 & 0.12 & 0.03 & 0.01 \\ 0 & 0.02 & 0.63 & 0.12 \\ 0.56 & 0.15 & 0.02 & 0.3 \end{pmatrix}$ and $G_\mu^C = \begin{pmatrix} 0.8 & 0.7 & 0.9 & 0.9 \\ 1 & 0.9 & 0.3 & 0.7 \\ 0.3 & 0.7 & 0.9 & 0.5 \end{pmatrix}$.

Definition: 5.11

A GFSS is said to be a **Generalized Null Fuzzy Soft Set**, denoted by Φ_θ, if $\Phi_\theta : E \to F(U) \times [0, 1]$ such that $\Phi_\theta(e) = (F(e), \theta(e))$, where $F(e) = \overline{0}\ \forall e \in E$ and $\theta(e) = 0\ \forall e \in E$.

Definition: 5.12

A GFSS is said to be a **Generalized Absolute Fuzzy Soft Set**, denoted by $\tilde{A}_\alpha$, if $A_\alpha : E \to F(U) \times [0, 1]$ where $\tilde{A}_\alpha(e) = (A(e), \alpha(e))$ is defined by $A(e) = \overline{1}\ \forall e \in E$ and $\alpha(e) = 1\ \forall e \in E$.

Theorem: 5.13

Let F_μ be a GFSS over (U, E), then the following holds:

1) F_μ is a GF soft subset of $F_\mu \tilde{\cup} F_\mu$.
2) $F_\mu \tilde{\cap} F_\mu$ is a GF soft subset of F_μ.
3) $F_\mu \tilde{\cup} \Phi_\theta = F_\mu$
4) $F_\mu \tilde{\cap} \Phi_\theta = \Phi_\theta$
5) $F_\mu \tilde{\cup} \tilde{A}_\alpha = \tilde{A}_\alpha$
6) $F_\mu \tilde{\cap} \tilde{A}_\alpha = F_\mu$

Proof: The results trivially follow the definition.

Note: 5.14

Instead of taking any dual triple as described in Note 5.5, if we take standard fuzzy operations (i.e. max, min and standard complement) then we get equality relation in (1) and (2) above.

Theorem: 5.15

The following laws also holds here:

1) $F_\mu \tilde{\cup} F_\mu^C = \tilde{A}_\alpha$

2) $F_\mu \tilde{\cap} F_\mu^C = \Phi_\theta$

Theorem: 5.16

Let F_μ, G_δ and H_λ be any three GFSS over (U, E), then the following holds:

1) $F_\mu \tilde{\cup} G_\delta = G_\delta \tilde{\cup} F_\mu$

2) $F_\mu \tilde{\cap} G_\delta = G_\delta \tilde{\cap} F_\mu$

3) $F_\mu \tilde{\cup} (G_\delta \tilde{\cup} H_\lambda) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cup} H_\lambda$

4) $F_\mu \tilde{\cap} (G_\delta \tilde{\cap} H_\lambda) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cap} H_\lambda$

Proof: The properties follow from definition.

Note: 5.17

The following does not here:

1) $F_\mu \tilde{\cap} (G_\delta \tilde{\cup} H_\lambda) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cup} (F_\mu \tilde{\cap} H_\lambda)$

2) $F_\mu \tilde{\cup} (G_\delta \tilde{\cap} H_\lambda) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cap} (F_\mu \tilde{\cup} H_\lambda)$

But if we take standard fuzzy operations then distributive property holds.

Theorem: 5.18

Let F_μ and G_δ be any two GFSS over (U, E), then the following holds:

1) $(F_\mu \tilde{\cap} G_\delta)^C = (F_\mu^C \tilde{\cup} G_\delta^C)$

2) $(F_\mu \tilde{\cup} G_\delta)^C = (F_\mu^C \tilde{\cap} G_\delta^C)$

Proof: The properties follow from definition.

Definition: 5.19

Let F_μ and G_δ be any two GFSS over the parameterized universe (U, E) and $C \subseteq E^2$. Then a **Fuzzy Soft Relation** R from F_μ to G_δ is a function $R : C \to F(U) \times [0, 1]$, defined as follows:

$$R(e, f) = F_\mu(e) \tilde{\cap} G_\delta(f) \text{ for all } (e, f) \in C$$

A Generalisation of this may be:

Definition: 5.20

Let $F = \{F_{\mu_i}^i, i \in \Delta\}$, where Δ is the index set, be any collection of GFSS over (U, E) and $C \subseteq E^n$. Then an n-ary **Generalised Fuzzy Soft Relation** R on F is the mapping $R : C \to F(U) \times [0, 1]$, defined by $R(e_{i_1}, e_{i_2}, ..., e_{i_n}) = \cap_{j=1}^n F_{\mu_j}^{i_j}(e_{i_j})$, where $(e_{i_1}, e_{i_2}, ..., e_{i_n}) \in C$.

Example: 5.21

An application of this Generalised fuzzy soft relation in a decision making problem is shown below:

Suppose the universe consists of four machines x_1, x_2, x_3, x_4, *i.e.* $U = \{x_1, x_2, x_3, x_4\}$ and there are three parameters $e_i, i = 1,2,3$ which describe their performances according certain specific task. Hence $E = \{e_1, e_2, e_3\}$. Suppose a firm wants to buy one such machine depending on any two of the parameters only. Let there be two observations F_μ and G_δ by two experts A, B repectively.

Let their corresponding membership matrices be as follows:

$$F_\mu = (F, \mu) = \begin{pmatrix} 0.4 & 0.2 & 0.1 & 0.6 & 0.5 \\ 0.7 & 0.8 & 0.5 & 0.4 & 0.6 \\ 0.6 & 0.4 & 0.5 & 0.6 & 0.8 \end{pmatrix} \text{ and } G_\delta = (G, \delta) = \begin{pmatrix} 0.4 & 0.6 & 0.5 & 0.3 & 0.5 \\ 0.8 & 0.4 & 0.9 & 0.6 & 0.7 \\ 0.1 & 0.2 & 0.1 & 0.4 & 0.3 \end{pmatrix}$$

Let $R : C \to F(U) \times [0, 1]$, be the generalised fuzzy soft relation between F_μ and G_δ, defined as follows:

$$\left(\begin{array}{cccccc} R & x_1 & x_2 & x_3 & x_4 & \lambda \\ (e_1, e_1) & (0.4) & 0.2 & 0.1 & 0.3 & 0.5 \\ (e_1, e_2) & 0.1 & 0.2 & 0.1 & (0.6) & 0.5 \\ (e_1, e_3) & 0.1 & 0.2 & 0.1 & (0.4) & 0.3 \\ (e_2, e_1) & 0.4 & (0.6) & 0.5 & 0.3 & 0.5 \\ (e_2, e_2) & (0.7) & 0.4 & 0.5 & 0.4 & 0.6 \\ (e_2, e_3) & 0.1 & 0.2 & 0.1 & (0.4) & 0.3 \\ (e_3, e_1) & 0.4 & 0.4 & (0.5) & 0.3 & 0.5 \\ (e_3, e_2) & 0.6 & 0.4 & (0.5) & 0.6 & 0.7 \\ (e_3, e_3) & 0.1 & 0.2 & 0.1 & (0.4) & 0.3 \end{array}\right)$$

Now to determine best machine we first mark the highest numerical grade (indicated in parenthesis) in each row excluding the last column which is the grade of belongingness of a machine against each pair of parameters. Now the score of each of such machines is calculated by taking the sum of the products of these numerical grades with the corresponding values of λ. The machine with the highest score is the desired machine. We do not consider the numerical grades of the machine against the pairs (e_i, e_i), $i = 1, 2, 3$ as both the parameters are same.

Grade table

$$\left(\begin{array}{cccccccccc}
R & (e_1,e_1) & (e_1,e_2) & (e_1,e_3) & (e_2,e_1) & (e_2,e_2) & (e_2,e_3) & (e_3,e_1) & (e_3,e_2) & (e_3,e_3) \\
x_i & x_1 & x_4 & x_4 & x_2 & x_1 & x_4 & x_3 & x_3 & x_4 \\
h & \times & 0.6 & 0.4 & 0.6 & \times & 0.4 & 0.5 & 0.5 & \times \\
\lambda & & 0.5 & 0.3 & 0.5 & & 0.3 & 0.5 & 0.7 &
\end{array}\right)$$

h - highest numerical grade

Score $(x_1) = 0$,

Score $(x_2) = 0.6 \times 0.5 = 0.30$,

Score $(x_3) = 0.5 \times 0.5 + 0.5 \times 0.7 = 0.60$ and

Score $(x_4) = 0.6 \times 0.5 + 0.4 \times 0.3 + 0.4 \times 0.3 = 0.54$

Then the firm will select the machine with highest score. Hence they will buy machine x_3.