

Class : I PG

Max. Marks : 60

Majors : Mathematics

Time : 3 Hours

17MMAC01 Mathematical Physics-I

Part – A

Choose the Correct Answer

1. If p is a prime number and $p \mid o(G)$, then G has an element of order -----
(a) p^2 (b) p (c) $p+1$ (d) $2p$
2. If $o(G) = p^2$ where p is a prime number, then G is -----
(a) Abelian (b) normal (c) Quotient (d) Permutation group
3. If $p^m \mid o(G)$, $p^{m+1} \nmid o(G)$, then G has a subgroup of order -----
(a) p (b) p^2 (c) p^m (d) p^{m+1}
4. Let G be a group, A, B subgroups of G . If $x, y \in G$, then $x \vee y$ is defined as A -----
for some $a \in A$ and $b \in B$.
(a) $Y = a \chi b$ (b) $\chi = a y b$ (c) $a = x b y$ (d) $b = x a y$
5. $F[\chi]$ is a ----- of polynomials.
(a) Field (b) Group (c) Set (d) Ring
6. Any polynomial in $F[\chi]$ can be written in a unique manner as a ----- of irreducible polynomials in $F[\chi]$.
(a) Sum (b) Difference (c) Product (d) L.C.M
7. The ----- of K over F is the dimension of K as a vector space over F .
a. Index b. Order c. Size d. Degree
8. A complex number is said to be an algebraic number if it is algebraic over the field of ----- numbers.
a. Real b. Rational c. Irrational d. Natural
9. If $p(\chi) \in F[\chi]$, then an element a lying in some extension field of F is called a root of $p(\)$ if -----.
a. $P(\chi) = 0$ (b) $p(\chi) = a$ (c) $p(a) = 0$ (d) $p(a) = \chi$
10. If $a \in K$ is a root of $p(\chi) \in F[\chi]$, where $F \subset K$, then in $K[\chi]$, -----.
(a) $(\chi - a) \mid p(\chi)$ (b) $p(\chi) \mid (\chi - a)$ (c) $\chi \mid p(\chi)$ (d) $a \mid p(\chi)$

Part B
Answer All Questions

5x4=20

11. a. If G is abelian of order $O(G)$ and $P^\alpha \mid O(G)$, $P^{\alpha+1} \nmid O(G)$, prove that there is a unique subgroup of G of order p^α . (Or)
- b. If $O(G) = p^n$ where p is a prime number, prove that $Z(G) \neq \{e\}$.
12. a. Prove that $n(K) = 1 + p + \dots + p^{R-1}$. (Or)
- b. Define internal direct product and give an example.
13. a. State and prove the division algorithm. (Or)
- b. State and prove the Eisenstein criterion.
14. a. If a and b are algebraic over F of degrees m and n , respectively, prove that $a \pm b$, ab and a/b (if $b \neq 0$) are algebraic over F of degree at most mn . (Or)
- b. If L is an algebraic extension of K and if K is an algebraic extension of F , prove that L is an algebraic extension of F .
15. a. If $p(x) \in F[x]$ and if k is an extension of F , prove that for any element $b \in k$, $p(x) = (x-b)q(x) + p(b)$ where $q(x) \in F[x]$ and where $\deg q(x) = \deg p(x) - 1$. (Or)
- b. Let $f(x) \in F[x]$ be of degree ≥ 1 . Prove that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n roots.

Part C
Answer All Questions

(5x7=35)

16. a. State and prove Cauchy's theorem for abelian groups. (Or)
- b. Let Φ be a homomorphism of G onto \bar{G} with kernel K , and let \bar{N} be a normal subgroup of \bar{G} , $N = \{x \in G \mid \Phi(x) \in \bar{N}\}$. Prove that $G/N \cong \bar{G}/\bar{N}$. That is, $(G/N) \cong (G/K)/(N/K)$.
17. a. State and prove the third part of Sylow's theorem. (Or)
- b. Let G be a finite group and suppose that G is a subgroup of the finite group M , M has a p -Sylow subgroup Q . Prove that G has a p -Sylow subgroup P , and $P = G \cap x Q x^{-1}$ for some $x \in M$.
18. a. State and prove Gauss's lemma. (Or)
- b. If $f(x), g(x)$ are two nonzero elements of $F[x]$, prove that $\deg(f(x)g(x)) = \deg f(x) + \deg g(x)$.
19. a. Prove that $a \in k$ is algebraic over F if and only if $F(a)$ is a finite extension of F . (Or)
- b. If L is a finite extension of K and if K is a finite extension of F , prove that L is a finite extension of F . Also, prove that $[L:F] = [L:K][K:F]$.

20. a. Prove that a polynomial of degree n over a field can have at most n roots in any extension field. (Or)

b. If $P(x)$ is irreducible in $F[x]$ and if θ is a root of $P(x)$, prove that $F(\theta)$ is isomorphic to $F(w)$ where w is a root of $P^1(t)$; moreover, this isomorphism σ can so be chosen that
(i) $\theta\sigma = w$ (ii) $\alpha\sigma = \alpha$ for every $\alpha \in F$.

