Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test II – April 2025
VI Semester

Class : III UG **Time : 2 Hours**
Branch : Mathematics/ Special Education & Mathematics **Max.Marks : 60**

21BMAC21/21BSMC17 - Abstract Algebra II

Course Outcomes:

CO1: Apply the Euclidean ring concepts to the Gaussian integers.
CO2: Analyze finite dimensional vector spaces and subspaces over a field and their properties, including the basis structure of vector spaces.
CO3: Utilize the dimension of annihilator concepts to study the system of linear homogeneous equations.
CO4: Prove standard theorems regarding Euclidean rings, vector spaces and inner produc spaces.
CO5: Compute inner products and determine orthogonality on vector spaces.

Part A **6 x 1 = 6**

Choose the Correct Answer

1. If V is finite-dimensional over F then any two bases of V have the---------. CO3 K2
   a. same elements b. same number of elements
   c. finite number of elements d. finite elements
2. If dim(A)=10, dim(B)=7 and dim(A+B)=12 then dim(A∩B)= CO3 K1
   a. 3 b. 5 c. 4 d. 7
3. If $\dim_F V = m$ then $\dim_F Hom(V,V) =$ CO4 K1
   a. m b. m^2 c. m.n d. n^2
4. A(A(W)) = CO4 K2
   a. A(W) b. V c. W d. V/W
5. Inner product $(u, \alpha v + \beta w) =$ CO5 K3
   a. $\alpha(u,v)+\beta(u,w)$ b. $\overline{\alpha}(u,v)+\beta(u,w)$ c. $\overline{\alpha}(u,v)+\overline{\beta}(u,w)$ d. $\alpha(u,v)+\overline{\beta}(u,w)$
6. If $u, v \in$ V and u is orthogonal to v then CO5K1
   a. (u,v) =1 b. $(u,u)=1$ c. (u,v) =0 d. (v,v) =0

Part B **3 x 6 = 18**

Answer ALL questions

7. a. Prove that if $v_1, v_2, ...v_n$ is a basis of V over F and if $w_1, w_2, ...w_m$ in V are linearly independent over F, prove that $m \le n$. CO3K3

(or)

7. b. Show that the annihilator of W, A(W) is a subspace of $\hat{V}$. CO3K3
8. a. If V is finite-dimensional and $v \ne 0 \in V$ then show that there is an element $f \in \hat{V}$ such that $f(v) \ne 0$. CO4K4

(or)

8. b. If $u, v \in V$ and then prove that CO4K4
   $(\alpha u + \beta v, \alpha u + \beta v) = \alpha\overline{\alpha}(u,u) + \alpha\overline{\beta}(u,v) + \overline{\alpha}\beta(v,u) + \beta\overline{\beta}(v,v)$
9. a. Prove that if a,b,c are real numbers such that $a > 0$ and $a\lambda^2 + 2b\lambda + c \ge 0$ for all real numbers λ, then $b^2 \le ac$. CO5K4

(or)

9. b. Define orthogonal complement of W and show that $W^{\perp}$ is a subspace of V. CO5 K3

Part C **3 x 12 = 36**

Answer ALL questions

10. a. Show that if V is finite dimensional and W is a subspace of V then dim(V/W) = dim(V) - dim(W) CO3K4

(or)

10. b.Prove that if V is a finite dimensional inner product space then show that V has an orthonormal set as the basis. CO5K5

11. a. Show that $\hat{W}$ is isomorphic to $\hat{V} / A(W)$ and $\dim A(W) = \dim V - \dim W$ if V is finite-dimensional and W is a subspace of V. CO4K4

(or)

11. b. Prove that if *V* and *W* are of dimensions m and n, respectively, over F, then Hom (*V, W*) is of dimension m.n over F. CO4K4
12. a. State and prove Schwartz Inequality. CO5K4

(or)

12. b. Show that a finite dimensional inner product space V is the direct sum of W and $W^{\perp}$. CO5K5

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