DECLARATION

I hereby declare that the thesis, entitled "**A Descriptive Study on Second Order Bipolar Fuzzy Structures**" submitted to the Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, in partial fulfilment of the requirements for the award of the Degree of **Doctor of Philosophy in Mathematics** is a record of original and independent research work done by me during the period under the supervision and guidance of **Dr. V. M. Vijayalakshmi, M.Sc., M.Phil., Ph.D.,** Assistant Professor, Department of Science and Humanities, School of Engineering, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore and it has not formed the basis for the award of any Degree/ Diploma/ Associateship/ Fellowship or another similar title of any candidate of any other University.

V. M. Vijayalakshmi 13/3/25
Signature of the Supervisor

S. Muthulakshmi 13/3/25
Signature of the Candidate

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Muthamizhselvi. S

NOTATIONS

X, Y	- Nonempty set
I^X	- Family of fuzzy sets on X
x_t	- Fuzzy point
f, g	- Fuzzy sets or first order fuzzy sets
$\mathbf{0}$	- Constant fuzzy set zero
$\mathbf{1}$	- Constant fuzzy set one
$\boldsymbol{\alpha}$	- Constant fuzzy set α where $\alpha \in [0,1]$
Supp f	- Support of a fuzzy set f
f^c	- Complement of a fuzzy set f
τ	- Fuzzy topology
A_{bp}, B_{bp}	- First order bipolar fuzzy sets
BPF(X)	- Set of bipolar fuzzy sets on X
0_{bp}	- Bipolar fuzzy null set
1_{bp}	- Bipolar fuzzy whole set
α_{bp}	- Constant bipolar fuzzy set
$\cup$	- Union
$\cap$	- Intersection
$(A_{bp})^c$	- Complement of A_{bp}
θ	- Mapping from X to Y
$x_{(\alpha,\beta)}$	- Bipolar fuzzy point
$\tau_{\mathfrak{B}}$	- Bipolar fuzzy topology

$\tau_{\mathfrak{B}_1} \hat{\times} \tau_{\mathfrak{B}_2}$	- Bipolar fuzzy product topology
$\hat{f}, \hat{g}$	- Second order fuzzy sets
$(I^I)^X$	- Set of second order fuzzy sets on X
$\hat{0}$	- Constant second order fuzzy set zero
$\hat{1}$	- Constant second order fuzzy set one
$\hat{\alpha}$	- Constant second order fuzzy set
$(\hat{f})_c, (\hat{f})^c$	- Two types of complement of $\hat{f}$
$\hat{x}_r$	- Second order fuzzy point
$S_1(\hat{f}), S_2(\hat{f})$	- Two types of support of $\hat{f}$
$\hat{\tau}$	- Second order fuzzy topology
$\hat{\mathfrak{B}}$	- Base for second order fuzzy topology
$\hat{\mathcal{S}}$	- Subbase for second order fuzzy topology
$\hat{f}_1 \hat{\times} \hat{f}_2$	- Product of second order fuzzy sets $\hat{f}_1$ and $\hat{f}_2$
$\hat{\tau}_1 \hat{\times} \hat{\tau}_2$	- Second order fuzzy product topology
$\hat{A}_{bp}$	- Second order bipolar fuzzy set
$\hat{\tau}_{\mathfrak{B}}$	- Second order bipolar fuzzy topology
$\hat{x}_{(\alpha,\beta)}$	- Second order bipolar fuzzy point
$(\hat{A}_{bp})^c, (\hat{A}_{bp})_c$	- Two types of complements of $\hat{A}_{bp}$
$S_1(\hat{A}_{bp}), S_2(\hat{A}_{bp})$	- Two types of support of $\hat{f}$
$(\hat{\chi}_{bp})_M$	- Second order bipolar fuzzy characteristic function
$\hat{A}_{bp} \times \hat{B}_{bp}$	- Product of second order bipolar fuzzy sets $\hat{A}_{bp}$ and $\hat{B}_{bp}$

$\hat{\tau}_{\mathfrak{B}_1} \times \hat{\tau}_{\mathfrak{B}_2}$ - Second order bipolar fuzzy product topology

$\boldsymbol{\mathcal{G}}$ - Gradation of openness

$\mathcal{G}$ - Fuzzy Gradation of openness

$\tau(\mathcal{G})$ - Fuzzy topology induced by $\mathcal{G}$

$\hat{\mathcal{G}}$ - Second order fuzzy Gradation of openness

$\hat{\tau}(\hat{\mathcal{G}})$ - Second order fuzzy topology induced by $\mathcal{G}$

$\mathcal{G}_{bp}$ - Bipolar fuzzy gradation of openness

$\tau_{\mathfrak{B}}(\mathcal{G}_{bp})$ - Bipolar fuzzy topology induced by $\mathcal{G}_{bp}$

$\hat{\mathcal{G}}_{bp}$ - Second order bipolar fuzzy gradation of openness

$\hat{\tau}_{\mathfrak{B}}(\hat{\mathcal{G}}_{bp})$ - Second order bipolar fuzzy topology induced by $\hat{\mathcal{G}}_{bp}$

$[F_{ij}]_{u\times v}$ - Fuzzy matrix

$\left((\mathrm{A_{bp}})_{\mathrm{ij}}\right)_{\mathrm{p\times q}}$ - First order bipolar fuzzy matrix

$\left((\widehat{\mathrm{A}}_{\mathrm{bp}})_{\mathrm{ij}}\right)_{\mathrm{p\times q}}$ - Second order bipolar fuzzy matrix