ψgα-Closed Sets in Topological Spaces

Naveena G.

(16PMA013)

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore-641 043

In Partial Fulfilment of the Requirements for the Degree of

Master of Science in Mathematics

April 2018

ψgα-Closed Sets in Topological Spaces

Naveena G.

(16PMA013)

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore-641 043

In Partial Fulfilment of the Requirements for the

Degree of Master of Science in Mathematics

April 2018

9.4.18

Signature of the Head of the Department

09-04-18

Signature of the Supervisor

ACKNOWLEDGMENT

First and foremost, I extremely thankful to the **LORD ALMIGHTY** for his graces and blessings showered on me.

I take immense pleasure in thanking **Dr.P.R.KRISHNAKUMAR,** Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore for providing the conducive infrastructure for the conduct of the research study.

I would like to thank **Dr. (Tmt.) PREMAVATHY VIJAYAN**, vice Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for the encouragement and for providing the opportunity to develop and establish my skills.

I extend my heartfelt thanks to **Dr. (Tmt.) S. KOWSALYA,** Registrar Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for the encouragement given by her during the investigation.

I would like to thank **Dr. (Tmt.) G.P.JEYANTHI**, Director (Research and Consultancy), Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for all the encouragement and for providing the opportunity to develop and establish my skills.

I express my heartfelt thanks to **Dr. (Tmt.) A. PARVATHI,** Professor, Dean, Faculty of Science, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her excellent support, unflinching encouragement and guidance during the course of the investigation.

I express my heartfelt thanks to **Dr. (Tmt.) K.UDAYA CHANDRIKA,** professor, Head of the Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her support and guidance during the course of the investigation.

I deeply indebted to my thesis supervisor (**Tmt.) N.BALAMANI,** Assistant professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her inspiring guidance, innovative ideas, critical suggestions and constant encouragement throughout the completion of this work.

I would like to express my sincere thanks to all the **STAFF MEMBERS OF THE DEPARTMENT OF MATHEMATICS** who were responsible for the good finish of this dissertation.

I owe my special thanks to my **BELOVED PARENTS, LOVING SISTERS, BROTHERS, FRIENDS AND ALSO THE GRACEFUL RELATIVES,** who helped me by providing full strength, support and encouragement to complete my project successfully.

CONTENT

CHAPTER	TITLE	PAGE No.
	Introduction	1
	Review of literature	**7**
1	**Preliminaries**	**8**
2	**ψgα-Closed Sets in Topological Spaces**	
	2.1 Introduction	**14**
	2.2 ψgα-closed sets	**14**
	2.3 ψgα-closure operator	**22**
	2.4 ψgα-open sets	**24**
	2.5 Applications of ψgα-closed sets	**29**
3	**ψgα-Continuous Functions in Topological Spaces**	
	3.1 Introduction	**34**
	3.2 ψgα-continuous functions	**34**
	Summary and conclusion	**48**
	Bibliography	

BIBLIOGRAPHY

[1] Abd EI-Monsef, M.E., Rose Mary, S. and Lellis Thivagar, M. (2007), **On αĝ-closed sets in topological spaces**, Assiut University Journal of Mathematics and Computer Science, 36, 43-51.

[2] Alli, K., Subramanian, A. and Pious Missier, S. (2013), **g#p#-closed set in topological spaces,** International Journal of Mathematics and Soft computing,3(3), 55-60**.**

[3] Andrijevic, D. (1986), **Semi-pre open sets**, Mat Vesnik, 38, 24-32.

[4] Arya., S.P. and Gupta, R. (1974), **On strongly continuous mappings,** Kyungpook Math. J., 14, 131-143.

[5] Arya , S. P. and Nour, T.(1990), **Characterizations of S-normal spaces,** Indian J. pure Appl. Math., 21, 717-719.

[6] Balachandran, K., Sundaram, P. and Maki H. (1991), **On generalized continuous maps in topological spaces,** Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 12, 5-13.

[7] Bhattacharyya,P. and Lahiri, B.K. (1987), **Semi-generalized closed sets in topolog**y, Indian J. Math., 29, 376-382.

[8] Crossely S. G. and Hildebrand, S.K. (1971), **On Semi-Closure**, Texas J. Sci., 22, 99-112.

[9] Devi, R. Balachandran , K. and Maki, H. (1997), **On generalized α-continuous maps and α-generalized continuous maps,** Far East J. Math. Sci. special vol,15.

[10] Dontchev , J. (1995), **On generalizing semi pre open sets,** Mem. Fac. Sci.Kochi.Ser. A, Math., 16, 35-48.

[11] Ganesan. s., Ravi. O. and Chandrasekar, S.(2011), **g̃-pre closed sets in topological spaces,** International Journal of Mathematical Archive, 2(2), 294-299.

[12] Gnanambal, Y. (1997), **On generalized pre regular closed sets in topological Spaces,** Indian J. pure. Appl. Math., 28, 351-360.

[13] Gnanambal, Y. and Balachandran, K. (1999),**On gpr-continuous functions in topological spaces,** Indian. J. Pure and Appl. Math., 30, 581-593.

[14] Jafari, S., Noiri, T., Rajesh, N. and LellisThivagar, M.(2008), **Another Generalization of closed sets**, Kochi. J. Math., 3, 25-38.

[15] Levine, N., (1963), **Semi-open sets and semi-continuity in topological spaces,** Amer. Math. Monthly, 70, 36-41.

[16] Levine, N., (1970), **Generalized closed sets in topological spaces,** Rend. Circ. Mat. Palermo, 19, 89-96.

[17] Levine, N., (1970), **Strongly continuity in topological spaces,** Amer. Math. Monthly, 67, 269-275.

[18] Maki, H., Devi, R. and Balachandran, K. (1993), **generalized α-closed sets in topology,** Bull. Fukuoka Univ, part III 42, 13-21.

[19] Maki, H., Devi, R. and Balachandran, K. (1994), **Associated topologies of Generalized α-closed sets and α-generalized closed sets,** Mem. Fac. Sci. Kochi. Univ. Ser.A.Math., 15, 51-63.

[20] Maki, H., Umehara, J. and Noiri, T. (1996), **Every topological spaces** is **pre-s**$\mathbf{T_{1/2}}$, Mem. Fac. Sci. Kochi Univ. Math., 17, 33-42.

[21] Mashhour, A.S., Abd EI-Monsef, M.E. and EI-Deeb, S.N. (1982), **On pre continuous and weak pre continuous mapping,** proc. Math. Phys. Soc. Egypt.,53, 47-53.

[22] Mashhour,A.s., Hasanein, I.A., and EI-Deeb, S.N.,(1983), **On α-continuous and α-open mappings,** Acta. Math. Hungarica, 41, 213-218.

[23] Njastad, O. (1965), **On some classes of nearly open sets**, Pacific J. Math., 15, 961-970.

[24] Pious Missier, S., Subramanian, A. and Alli, K. (2013), **On g#p#-continuous maps in topological spaces**, Journal of Global Research in Mathematical Archives, 1(5), 26-30.

[25] Rajamani, M. and Vishwanathan, K. (2004), **αgs-closed sets in topological spaces,** Acta Cienia Indica, XXXM,(3), 521-526.

[26] Rajamani M. and Viswnathan K. (2005) **On αgs-continuous Maps in topological spaces**, Acta Ciencia Indica, .XXXIM,(1), 293-303.

[27] Ramya, N. and Parvathi, A. (2011),**ψĝ-closed sets in topological spaces** Internationa l Journal of Mathematics Archive, 2(10), 1992-1996.

[28] Ramya, N. and Parvathi, A. (2012), **Strong forms of ψĝ-continuous function in topological spaces** J. Math. Comput. Sci., 1, 101-109.

[29] Rose Mary, S. and Lellis Thivagar, M. (2010), **Stronger form of αĝ-continuous functions,** Inter. J. of Eng. Comp. Sci. and Math., 1(1), 71-79.

[30] Stone,M., (1937), **Application of the theory of Boolean rings to general topology**, Trans. Amer. Math. Soc. 41, 374-481.

[31] Veera kumar, M.K.R.S.,(2000), **Between closed sets and g-closed sets,** Mem. Fac. Sci. Kochi. Univ. Ser. A, Math., 21, 1-19.

[32] Veera kumar, M.K.R.S., (2000), **Between semi-closed sets and semi-pre closed sets,** Rend. Istit.Mat. Univ. Trieste, (ITALY) XXXXII, 25-41.

[33] Veera Kumar, M.K.R.S. (2002), **g*-pre closed sets in topology,** Acta Ciencia India, 28(1), 51-60.

[34] Veera Kumar, M.K.R.S. (2003), **ĝ-closed sets in topological spaces,** Bull.Allah. Math. Soc., 18, 99-112.

[35] Veera Kumar, M.K.R.S. (2003), **g#-closed sets in topological spaces,** Mem. Fac Sci. Kochi Univ. Math., 24, 1-13.

[36] Veera Kumar, M.K.R.S. (2006), **Between g*-closed sets and g-closed sets,** Antarctica. J. Math., 3(1) 43-65.

INTRODUCTION

Topology is a branch of mathematics which is good at extracting global qualitative features from complicated geometric structures. General topology known as point set topology is the branch of topology dealing with the set theoretic definitions and constructions used in topology. It is the foundation of the most other branches of topology, including differential topology, geometric topology, algebraic topology. The fundamental concepts in point set topology are continuity, compactness and connectedness. The topological structures are modeled suitably in the fields of computer graphics, pattern recognition, artificial intelligence, data mining, information systems, rough set theory, quantum physics etc.

The notion of open sets is the powerful tool for defining a topological spaces. In the study of topological spaces many concepts of topology have been generalized by introducing the concept of semi open sets due to Levine (1963) instead of open sets. Njastad (1965) introduced α-open sets in topological spaces. Levine (1970) introduced the concept of generalized closed (briefly, g-closed) sets in topological spaces. Using this concept many researchers have introduced and studied various types of generalized closed sets.

Maki et al. (1993) introduced generalized α-closed set in topological spaces. Dontchev(1995) introduced generalized semi pre closed set in topological spaces. Maki et al. (1996) introduced generalized pre closed set in topological spaces. Veera kumar (2000) introduced ψ-closed sets in topological space.

Continuous maps are an important notions in the field of mathematics. Sevaral researchers working in the field of general topology have shown interest in studying the properties of generalizations of continuous maps. Levine (1963) introduced the semi-continuous maps in topological spaces. Levine (1970) introduced the concept of continuous maps in topological spaces. Mashhour et al. (1983) introduced and studied α-continuous maps in topological spaces. The generalized continuous (g-continuous) maps were introduced and studied by Balachandran et al. (1991).

The Present study focuses on the following concepts:

(i) $\psi g\alpha$-closed sets in topological spaces

(ii) $\psi g\alpha$-continuous maps in topological spaces

Chapter 1 deals with preliminary definitions in topological spaces that are needed for the present study.

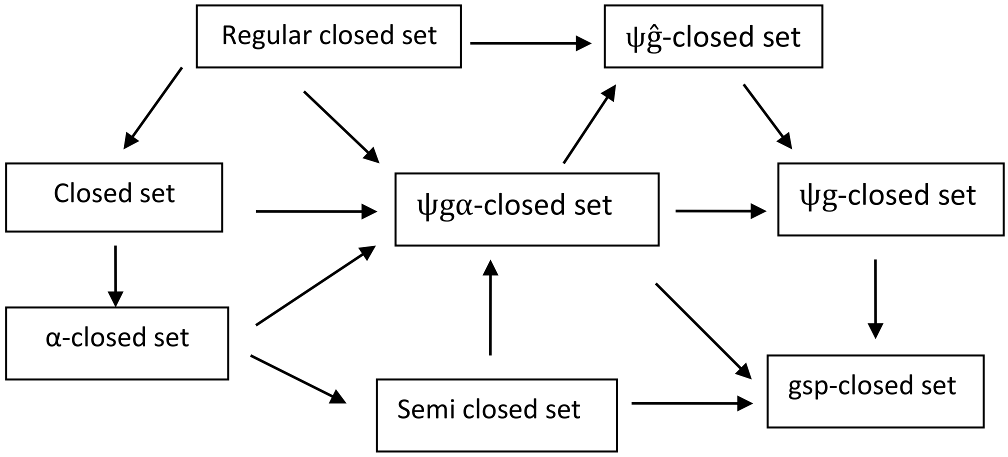
In Chapter 2, a new class of generalized closed sets called $\psi g\alpha$-closed sets is introduced in topological spaces. Properties and characterizations of $\psi g\alpha$-closed sets are derived. A comparative study between $\psi g\alpha$-closed sets and already existing various generalised closed sets is carried out. $\psi g\alpha$-closure operator is defined and its properties are analyzed. As an application of $\psi g\alpha$-closed sets five new spaces namely $\psi g\alpha T_c$-space, $\psi g\alpha T_\alpha$-space, $\psi g\alpha T_{sc}$-space, $\psi g T_{\psi g\alpha}$-space and $\psi \hat{g} T_{\psi g\alpha}$-space are introduced and their interrelations are obtained.

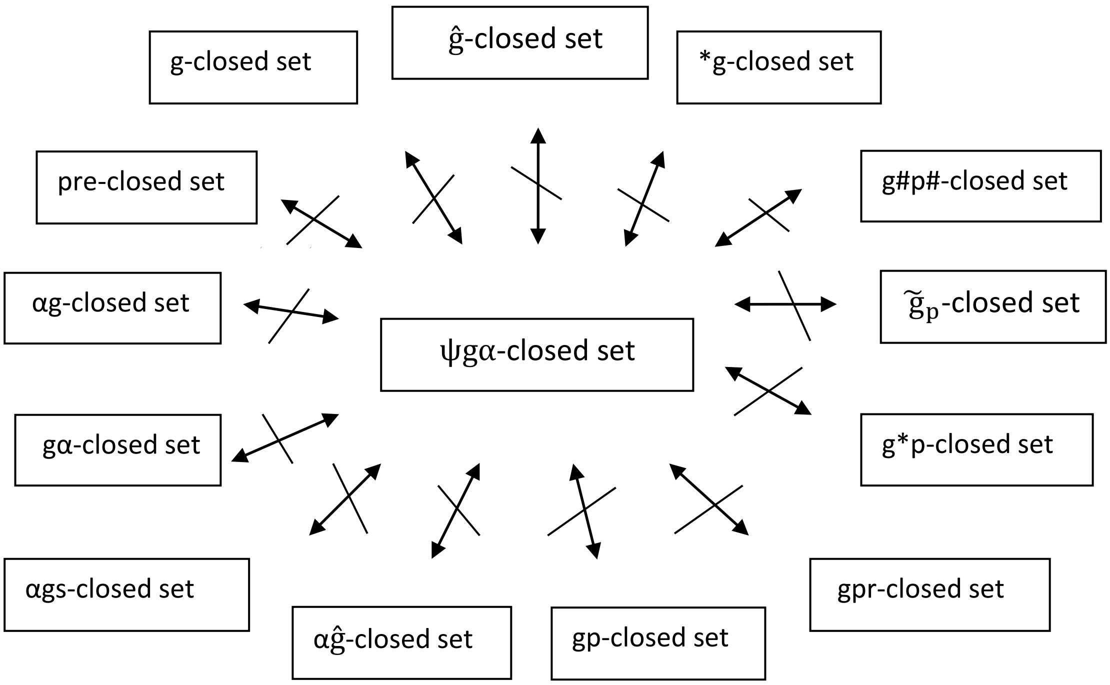
Important definitions and results:

A subset A of a topological space (X,τ) is said to be a **$\psi g\alpha$-closed set** if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g\alpha$-open in (X,τ).

The collection of all $\psi g\alpha$-closed sets of (X,τ) is denoted by $\psi g\alpha C(X,\tau)$.

The following diagrams exhibit the relations between ψgα-closed sets with other existing various generalized closed sets.





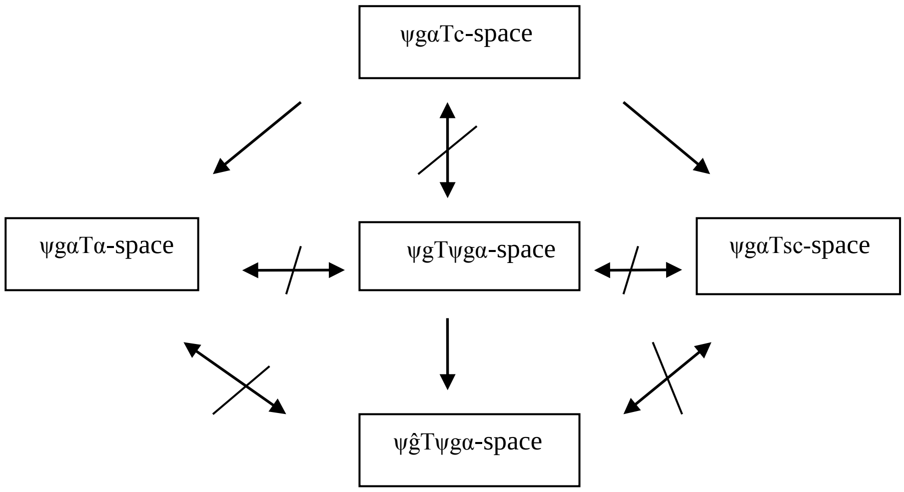
where A→B represents A implies B and A ↮ B represents A and B are independent.

- Let A and B be subsets of (X, τ) such that A⊆B⊆ ψcl(A). If A is a ψgα-closed set in (X, τ), then B is also a ψgα-closed set.
- Let A be a ψgα-closed set in (X,τ). Then ψcl(A)-A contains no non-empty closed sets.
- If a set A is ψgα-closed in (X, τ), then ψcl(A)-A contains no non-empty gα-closed set.
- Let A be a ψgα-closed set of (X, τ). Then A is ψ-closed if and only if ψcl(A)-A is gα-closed.

A topological space (X, τ) is said to be a

1) ψgαTc-space if every ψgα-closed subset of (X, τ) is closed in (X, τ).
2) ψgαTα-space if every ψgα-closed subset of (X, τ) is α-closed in (X, τ).
3) ψgαTsc-space if every ψgα-closed subset of (X, τ) is semi-closed in (X, τ).
4) ψgTψgα-space if every ψg-closed subset of (X, τ) is ψgα -closed in (X, τ).
5) ψĝTψgα-space if every ψĝ-closed subset of (X, τ) is ψgα -closed in (X, τ).

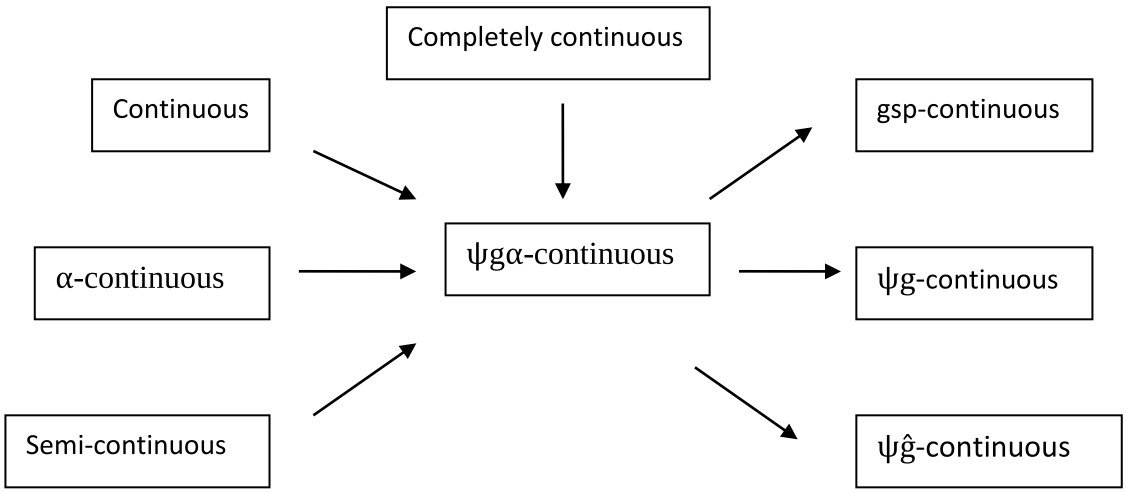
The following diagram exhibits the relations between the newly defined spaces.



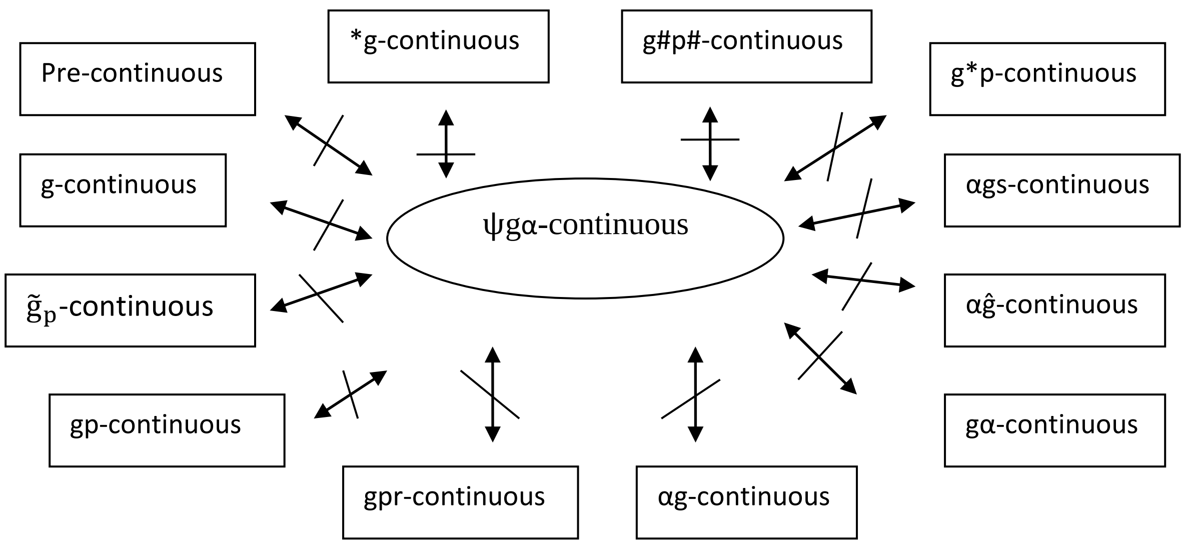
In Chapter 3, using ψgα-closed sets the concept of ψgα-continuity is introduced and some of their properties are analyzed in topological spaces. Also the relationship between ψgα-continuous maps with other existing continuous maps is obtained.

A function f: (X, τ)→(Y,σ) is called **ψgα-continuous** is $f^{-1}(V)$ is ψgα-closed in (X, τ) for every closed set V in (Y,σ).

The following diagrams are the output of the comparative study of ψgα-continuous maps with already existing continuous maps:



where A→B represents A implies B.



where A ↮ B represents A and B independent.

REVIEW OF LITERATURE

Topology is one of the widely studied areas of mathematics emerged through the works of the great mathematician Henri Poincare in the 19^{th} century. Topological structures on the collection of data are suitable mathematical models for mathematizing not only quantitativedata but also qualitative data.

Initially,open sets is the basic concept for the study of topological spaces. Later Stone (1937) introduced regular open sets in topological spaces.Levine (1963) introduced the notion of semi open sets in topological spaces. Njastad (1965) introduced α-open sets in topological spaces. The study of generalized closed sets was introduced by Levine (1970) in order to extend the topological properties of closed sets to a large family of sets. Mashhour et al. (1983) introduced α-closed sets. Using the concept of α-closed sets, Maki et al. (1993,1994) introduced the concept of generalized α-closed sets and α-generalized closed sets and investigated the properties between these two sets. Dontchev (1995) defined gsp-closed sets and studied their properties.

Veera Kumar (2000) investigated a new class of closed sets called ψ-closed sets in topological spaces. Veera Kumar introduced g*-closed sets (2000), g*p-closed sets (2002), ĝ-closed sets (2003) and *g-closed sets (2006) in topological spaces. Abd El.Monsef et al. (2007) defined αĝ-closed sets. Ramya and Parvathi (2011) introduced ψg-closed sets and ψĝ-closed sets in topological spaces.

Continuous maps are important notions in the field of mathematics. Several researchers working in the field of general topology have shown interest in studying the properties of generalizations of continuous maps. Mashhour et al. (1983) defined and studied α-continuous maps. Devi et al. (1997) introduced and studied αg-continuous maps and gα-continuous maps. Veera Kumar introduced ψ-continuous (2000), g*-continuous (2000), ĝ-continuous (2003), *g-continuous (2006) maps in topological spaces.

CHAPTER-1

PRELIMINARIES

Definition 1.1

Let (X,τ) be a topological space. The intersection of all closed sets containing A is called **closure of A** and is denoted by **cl(A).** The union of all open sets contained in A is called **interior of A** and is denoted by **int(A).**

Definition 1.2[30]

A subset A of a topological space (X,τ) is called **regular-open** if A=int(cl(A)) and **regular-closed** if cl(int(A))=A.

Definition1.3[23]

A subset A of a topological space (X,τ) is called **α-open** if A⊆int(cl(int(A))) and **α-closed** if cl(int(cl(A))) ⊆A.

Definition 1.4[15]

A subset A of a topological space (X,τ) is called **semi-open** if A⊆cl(int(A)) and **semi-closed** if int(cl(A)) ⊆A.

Definition 1.5[21]

A subset A of a topological space (X,τ) is called **pre-open** if A⊆int(cl(A)) and **pre-closed** if cl(int(A)) ⊆A.

Definition 1.6[3]

A subset A of a topological space (X,τ) is called **semi pre-open** if A⊆cl(int(cl(A))) and **semi pre-closed** if int(cl(int(A)))⊆A.

Definition 1.7[16]

A subset A of a topological space (X,τ) is called **generalized closed** (briefly g-closed) if cl(A) ⊆U whenever A⊆U and U is open in (X,τ).

Definition 1.8[7]

A subset A of a topological space (X,τ) is called **semi-generalized closed** (briefly sg-closed) if scl(A) ⊆U whenever A⊆U and U is semi-open in (X,τ).

Definition 1.9[5]

A subset A of a topological space (X,τ) is called **generalized semi closed** (briefly gs-closed) if scl(A) ⊆U whenever A⊆U and U is open in (X,τ).

Definition 1.10[18]

A subset A of a topological space (X,τ) is called **generalized α-closed** (briefly gα-closed) if αcl(A)⊆U whenever A⊆U and U is α-open in (X,τ).

Definition 1.11[19]

A subset A of a topological space (X,τ) is called **α-generalized closed** (briefly αg-closed) if αcl(A) ⊆U whenever A⊆U and U is open in (X,τ).

Definition 1.12[10]

A subset A of a topological space (X,τ) is called **generalized semi pre closed** (briefly gsp-closed) if spcl(A) ⊆U whenever A⊆U and U is open in (X,τ).

Definition 1.13[20]

A subset A of a topological space (X,τ) is called **generalized pre closed** (briefly gp-closed) if pcl(A) ⊆U whenever A⊆U and U is open in (X,τ).

Definition 1.14[12]

A subset A of a topological space (X,τ) is called **generalized pre regular closed** (briefly gpr-closed) if pcl(A) ⊆U whenever A⊆U and U is regular open in (X,τ).

Definition 1.15[32]

A subset A of a topological space (X,τ) is called **ψ-closed** if scl(A) ⊆U whenever A⊆U and U is sg-open in (X,τ).

Definition 1.16[31]

A subset A of a topological space (X,τ) is called **g*-closed** if cl(A) ⊆U whenever A⊆U and U is g-open in (X,τ).

Definition 1.17[35]

A subset A of a topological space (X,τ) is called **g#-closed** if cl(A) ⊆U wheneverA⊆U and U is αg-open in (X,τ).

Definition 1.18[34]

A subset A of a topological space (X,τ) is called **ĝ-closed** if cl(A) ⊆U whenever A⊆U and U is semi-open in (X,τ).

Definition 1.19[2]

A subset A of a topological space (X,τ) is called **g#p#-closed** if cl(A) ⊆U whenever A⊆U and U is g#-open in (X,τ).

Definition 1.20[11]

A subset A of a topological space (X,τ) is called **$\tilde{g}_p$-closed** if pcl(A) ⊆U whenever A⊆U and U is #gs-open in (X,τ).

Definition 1.21[14]

A subset A of a topological space (X,τ) is called **g̃-closed** if cl(A) ⊆U whenever A⊆U and U is #gs-open in (X,τ).

Definition 1.22[27]

A subset A of a topological space (X,τ) is called **ψĝ-closed** if ψcl(A) ⊆U whenever A⊆U and U is ĝ-open in (X,τ).

Definition 1.23[25]

A subset A of a topological space (X,τ) is called **αgs-closed** if αcl(A) ⊆U whenever A⊆U and U is semi-open in (X,τ).

Definition 1.24[27]

A subset A of a topological space (X,τ) is called **ψg-closed** if ψcl(A) ⊆U whenever A⊆U and U is open in (X,τ).

Definition 1.25[36]

A subset A of a topological space (X,τ) is called ***g-closed** if cl(A) ⊆U whenever A⊆U and U is ĝ-open in (X,τ).

Definition 1.26[33]

A subset A of a topological space (X,τ) is called **g*p-closed** if pcl(A) ⊆U whenever A⊆U and U is g-open in (X,τ).

Definition 1.27[1]

A subset A of a topological space (X,τ) is called **αĝ-closed** if αcl(A) ⊆U whenever A⊆U and U is ĝ-open in (X,τ).

Definition 1.28[17]

A function f:(X, τ)→(Y,σ) is said to be **continuous** if $f^{-1}(V)$ is closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.29[4]

A function f:(X, τ)→(Y,σ) is said to be **completely continuous** if $f^{-1}(V)$ is regular-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.30[22]

A function f:(X, τ)→(Y,σ) is said to be **α-continuous** if $f^{-1}(V)$ is α-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.31[15]

A function f:(X, τ)→(Y,σ) is said to be **semi-continuous** if $f^{-1}(V)$ is semi-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.32[6]

A function f:(X, τ)→(Y,σ) is said to be **generalized continuous** (briefly g-continuous) if f^{-1}(V) is g-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.33[9]

A function f:(X, τ)→(Y,σ) is said to be **α-generalized continuous** (briefly αg-continuous) if f^{-1}(V) is αg-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.34[26]

A function f:(X, τ)→(Y,σ) is said to be **αgs-continuous** if f^{-1}(V) is αgs-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.35[29]

A function f:(X, τ)→(Y,σ) is said to be **αĝ-continuous** if f^{-1}(V) is αĝ-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.36[8]

A function f:(X, τ)→(Y,σ) is said to be **gsp-continuous** if f^{-1}(V) is gsp-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.37[24]

A function f:(X, τ)→(Y,σ) is said to be **g#p#-continuous** if f^{-1}(V) is g#p#-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.38[31]

A function f:(X, τ)→(Y,σ) is said to be **g*-continuous** if f^{-1}(V) is g*-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.39[28]

A function f:(X, τ)→(Y,σ) is said to be **ψg-continuous** if f^{-1}(V) is ψg-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.40[28]

A function f:(X, τ)→(Y,σ) is said to be **ψĝ-continuous** if $f^{-1}(V)$ is ψĝ-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.41[9]

A function f:(X, τ)→(Y,σ) is said to be **gα-continuous** if $f^{-1}(V)$ is gα-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.42[6]

A function f:(X, τ)→(Y,σ) is said to be **gp-continuous** if $f^{-1}(V)$ is gp-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.43[33]

A function f:(X, τ)→(Y,σ) is said to be **g*p-continuous** if $f^{-1}(V)$ is g*p -closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.44[11]

A function f:(X, τ)→(Y,σ) is said to be $\tilde{g}_p$**-continuous** if $f^{-1}(V)$ is $\tilde{g}_p$-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.45[30]

A function f:(X, τ)→(Y,σ) is said to be **pre-continuous** if $f^{-1}(V)$ is pre-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.46[36]

A function f:(X, τ)→(Y,σ) is said to be ***g-continuous** if $f^{-1}(V)$ is $*$g-closed in (X,τ) for every closed set V of (Y, σ).

Definition 1.47[13]

A function f:(X, τ)→(Y,σ) is said to be **gpr-continuous** if $f^{-1}(V)$ is gpr-closed in (X,τ) for every closed set V of (Y, σ).

CHAPTER-2

ψgα-Closed Sets in Topological Spaces

2.1 Introduction

Levine (1970) introduced generalized closed sets in topological spaces. Maki et al. (1993) introduced gα-closed sets in topological spaces. Veera Kumar (2000) defined the concept of ψ-closed sets in topological spaces.

In this chapter a new class of generalized closed sets called ψgα-closed sets in topological spaces is introduced. The relationship between ψgα-closed sets with various existing generalized closed sets are obtained. As an application of ψgα-closed sets five new spaces are defined and their interrelations are derived.

2.2 ψgα-Closed sets

In this section a new class of generalized closed sets called ψgα-closed sets in topological spaces is introduced and some of their properties and characterizations are analyzed.

Definition 2.2.1

A subset A of a topological space (X,τ) is said to be a **ψgα-closed set** if ψcl(A) ⊆ U whenever A⊆U and U is gα-open in (X,τ).

The collection of all ψgα-closed sets of (X,τ) is denoted by ψgαC(X,τ).

Example 2.2.2

Let X=$\{a, b, c\}$,$\tau = \{\phi, \{a\}, \{b, c\}, X\}$.Then $\phi, \{b, c\}, X$ are ψgα-closed sets.

Proposition 2.2.3

Every closed set in (X,τ) is ψgα-closed but not conversely.

Proof

Let A be a closed set and U be any gα-open containing A in X. Since A is closed, cl(A)=A. For every subset A of X, ψcl(A) ⊆ cl(A)=A⊆U. Therefore A is ψgα-closed.

Example 2.2.4

Let X={a, b, c}, $\tau = \{\phi, \{a\}, X\}$. Then the subset{c} is ψgα-closed but not closed in (X,τ). `

Proposition 2.2.5

Every regular closed set in (X,τ) is ψgα-closed but not conversely.

Proof

Follows from the fact that every regular closed set is closed and by proposition 2.2.3.

Example 2.2.6

Let X={a, b, c}, $\tau = \{\phi, \{a\}, X\}$.Then the subset {b} is ψgα-closed but not regular closed in (X,τ).

Proposition 2.2.7

Every α-closed set in (X,τ) is ψgα-closed but not conversely.

Proof

Let A be an α-closed set and U be any gα-open set containing A in X. Since A is α-closed, αcl(A)=A .For every subset A of X, ψcl(A) ⊆ αcl(A)=A⊆U. Therefore A is ψgα-closed.

Example 2.2.8

Let X={a, b, c}, $\tau = \{\phi, \{a, b\}, X\}$.Then the subset {b, c} is ψgα-closed but not α-closed in (X,τ).

Proposition 2.2.9

Every semi-closed set in (X,τ) is $\psi g\alpha$-closed but not conversely.

Proof

Let A be a semi-closed set and U be any $g\alpha$-open set containing A in X. Since A is semi-closed, scl(A)=A . For every subset A of X, $\psi cl(A) \subseteq scl(A)=A\subseteq U$.Therefore A is $\psi g\alpha$-closed.

Example 2.2.10

Let $X=\{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$.Then the subset $\{a, c\}$ is $\psi g\alpha$-closed but not semi-closed in (X,τ).

Proposition 2.2.11

Every $\psi g\alpha$-closed set in (X,τ) is ψg-closed but not conversely.

Proof

Let A be a $\psi g\alpha$-closed set and U be any open set containing A in X. Since every open set is $g\alpha$-open and A is $\psi g\alpha$-closed, $\psi cl(A) \subseteq U$. Hence A is ψg-closed.

Example 2.2.12

Let $X=\{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$.Then the subset $\{a, b\}$ is ψg-closed but not $\psi g\alpha$-closed in (X,τ).

Proposition 2.2.13

Every $\psi g\alpha$-closed set in (X,τ) is gsp-closed but not conversely.

Proof

Let A be a $\psi g\alpha$-closed set and U be any open set containing A in X. Since every open set is $g\alpha$-open and A is $\psi g\alpha$-closed, $\psi cl(A) \subseteq U$. For every subset A of X, $spcl(A) \subseteq \psi cl(A)$ and so $spcl(A) \subseteq U$.Therefore A is gsp-closed.

Example 2.2.14

Let X={a, b, c}, $\tau = \{\phi, \{a\}, X\}$.Then the subset {a, c} is gsp-closed but not ψgα-closed in (X,τ).

Proposition 2.2.15

Every ψgα-closed set in (X,τ) is ψĝ-closed but not conversely.

Proof

Let A be a ψgα-closed set and U be any ĝ-open set containing A in X. Since every ĝ-open set is gα-open and A is ψgα-closed, ψcl(A) ⊆U. Hence A is ψĝ-closed.

Example 2.2.16

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Then the subset {a, c} is ψĝ-closed but not ψgα-closed in (X,τ).

Remark 2.2.17

The following examples show that ψgα-closedness is independent from pre-closedness, g*p-closedness and $\widetilde{g_p}$-closedness.

Example 2.2.18

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.Then the subset {a} is ψgα-closed but not pre-closed,g*p-closed and $\widetilde{g_p}$-closed.

Example 2.2.19

Let X={a, b, c}, $\tau = \{\phi, \{a, b\}, X\}$.Then the subset {b} is pre-closed, g*p-closed, $\widetilde{g_p}$-closed but not ψgα-closed.

Remark 2.2.20

The following examples show that ψgα-closedness is independent from g-closedness, αg-closedness, αĝ-closedness and gp-closedness.

Example 2.2.21

Let X={a, b, c}, $\tau = \{\phi, \{a\}, X\}$.Then the subset {a, c} is g-closed,αg-closed,αĝ-closed and gp-closed but not ψgα-closed.

Example 2.2.22

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.Then the subset {a} is ψgα-closed but not g-closed,αg-closed,αĝ-closed and gp-closed.

Remark 2.2.23

The following examples show that ψgα-closedness is independent from gα-closedness and αgs-closedness.

Example 2.2.24

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.Then the subset {a} is ψgα-closed but not gα-closed and αgs-closed.

Example 2.2.25

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$.Then the subset {c} is gα-closed and αgs-closed but not ψgα-closed.

Remark 2.2.26

The following examples show that ψgα-closedness is independent from ĝ-closedness.

Example 2.2.27

Let X={a, b, c}, $\tau = \{\phi, \{a\}, X\}$.Then the subset {c} is ψgα-closed but not ĝ-closed.

Example 2.2.28

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$.Then the subset {b} is ĝ-closed but not ψgα-closed.

Remark 2.2.29

The following example show that ψgα-closedness is independent from *g-closedness and g#p# closedness.

Example 2.2.30

Let X={a, b, c}, τ = {ϕ, {a}, {a, b}, X}.Then the subset {b} is ψgα-closed but not *g-closed and g#p# closed. The subset {a,c} is *g-closed and g#p# closed but not ψgα-closed.

Remark 2.2.31

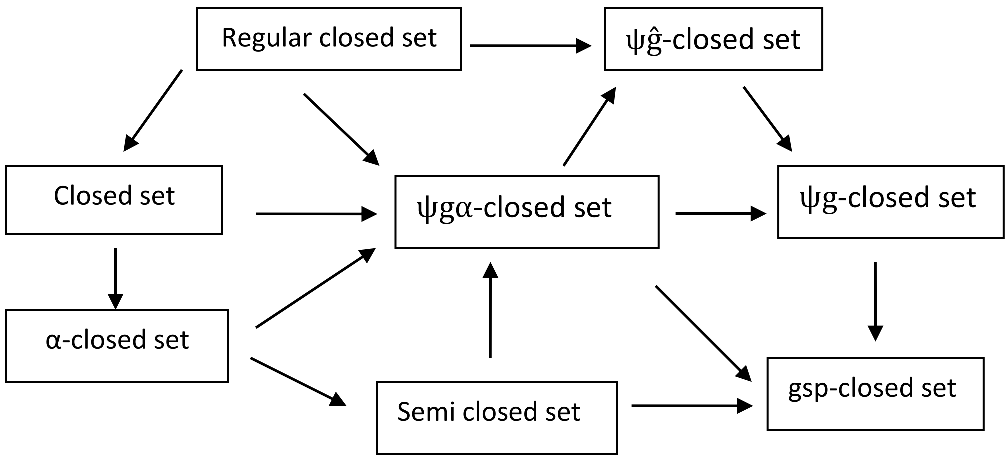
The following example show that ψgα-closedness is independent from gpr-closedness.

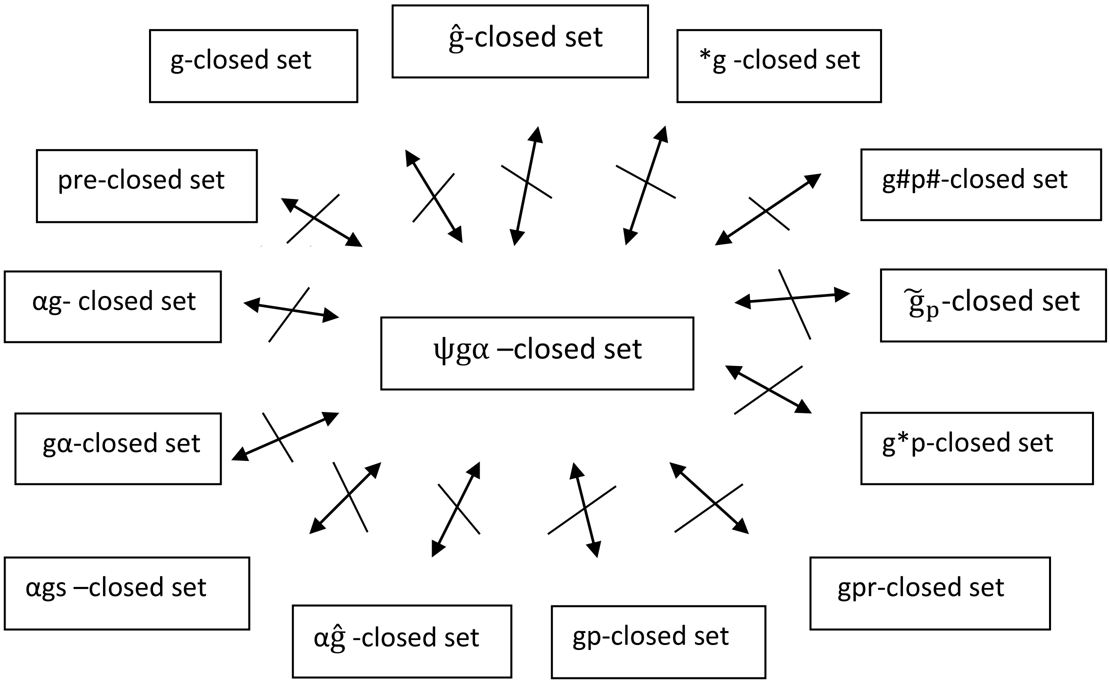
Example 2.2.32

Let X={a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, X}.Then the subset {a} is ψgα-closed but not gpr-closed. The subset {a, b} is gpr-closed but not ψgα -closed.

Remark 2.2.33

The following diagrams exhibit the relation between ψgα-closed sets with already existing various generalized closed sets.





Proposition 2.2.34

If A is both gα-open and ψgα-closed set of (X, τ) then A is ψ-closed in (X, τ).

Proof

Let A be gα-open and ψgα-closed. Then by definition, ψcl(A) ⊆ A, Always A⊆ ψcl(A). Therefore ψcl(A)=A, Hence A is ψ-closed.

Proposition 2.2.35

Let A and B be subsets of (X, τ) such that A⊆B⊆ ψcl(A). If A is a ψgα-closed set in (X, τ), then B is also a ψgα-closed set.

Proof

Let A and B be subsets such that A⊆B⊆ ψcl(A). Suppose that A is a ψgα-closed set. Let U be a gα-open set of (X, τ) such that B⊆U. Then A⊆U. Since A is ψgα-closed, ψcl(A) ⊆U. Also since B⊆ ψcl(A), ψcl(B) ⊆ ψcl(ψcl(A))= ψcl(A). Hence ψcl(B) ⊆U. Hence B is also a ψgα-closed set in (X, τ).

Proposition 2.2.36

Let A be a ψgα-closed set in (X, τ). Then ψcl(A)-A contains no non-empty closed set.

Proof

Suppose that A is ψgα-closed.Let F be a closed subset of ψcl(A)-A. Then F^c is open and hence gα-open such that A⊆ F^c.Since A is a ψgα-closed set, ψcl(A) ⊆ F^c. Thus F⊆ (ψcl(A))^C. Also F ⊆ ψcl(A)-A implies F⊆ ψcl(A). Hence F ⊆ ψcl(A) ∩ (ψcl(A))^C= ϕ.Therefore F= ϕ.

The converse of the above proposition is not true as seen from the following example.

Example 2.2.37

` Let X={a, b, c}, τ = {ϕ, {a}, X}.If A={a,b} then ψcl(A)-A=X-{a,b}={c} does not contain non-empty closed set. However A is not ψgα-closed.

Proposition 2.2.38

If a set A is ψgα-closed in (X,τ) then ψcl(A)-A contains no non-empty gα closed set.

Proof

Suppose that A is a ψgα-closed set. Let F be a gα-closed set contained in ψcl(A)- A. F^c is a g α-open in X such that A⊆ F^c.Since A is a ψgα-closed set of X, ψcl(A) ⊆ F^c. Thus F ⊆ (ψcl(A))^C.Also F⊆ ψcl(A)-A implies F⊆ ψcl(A).Therefore F⊆ ψcl(A)∩(ψcl(A))^C= ϕ. Therefore F= ϕ.

Proposition 2.2.39

Let A be a ψgα-closed set in (X, τ). Then A is ψ-closed if and only if ψcl(A)-A is gα-closed.

Proof

(Neccessity): Suppose that A is $\psi g\alpha$-closed. Let A be an ψ-closed subset of (X, τ). Then $\psi cl(A)=A$. Hence $\psi cl(A)-A= \phi$ is $g\alpha$-closed in (X, τ).

(Sufficiency): Let $\psi cl(A)-A$ be a $g\alpha$-closed set. Since A is $\psi g\alpha$-closed, by Proposition 2.2.38, $\psi cl(A)-A$ contains no non-empty $g\alpha$-closed set which implies $\psi cl(A)A= \phi$.That is $\psi cl(A)=A$. Hence A is ψ-closed.

Remark 2.2.40

Union of two $\psi g\alpha$-closed sets need not be $\psi g\alpha$-closed as seen from the following example.

Example 2.2.41

Let $X=\{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then $A=\{a\}$, $B=\{b\}$ are $\psi g\alpha$-closed but $A \cup B = \{a, b\}$ is not $\psi g\alpha$-closed in (X, τ).

2.3 $\psi g\alpha$-closure operator

In this section $\psi g\alpha$-closure operator is introduced and studied its basic properties.

Definition 2.3.1

The $\psi g\alpha$-closure of a subset A of a topological space (X, τ) denoted by $\psi g\alpha cl(A)$ is defined as follows.

$$\psi g\alpha cl(A) = \cap \{F \subseteq X : A \subseteq F \text{ and } F \text{ is } \psi g\alpha\text{-closed in } (X, \tau)\}$$

Remark 2.3.2

For a subset A of a topological space (X, τ), $A \subseteq \psi g\alpha cl(A) \subseteq cl(A)$

Proof

Follows from proposition 2.2.3 and definition 2.3.1

Remark 2.3.3

Both inclusion relation in Remark 2.3.2 may be proper as seen from the following example.

Example 2.3.4

Let $X=\{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $A=\{b\}$.Then $\psi g\alpha cl(A)=\{b, c\}$ and $cl(A)=\{X\}$and so $A \subset \psi g\alpha cl(A) \subset cl(A)$.

Proposition 2.3.5

Let A be any subset of (X, τ). If A is ψgα-closed in (X, τ), then $\psi g\alpha cl(A)=A$.

Proof

Follows from the definition 2.3.1.

Proposition 2.3.

Let A and B be any two subsets of (X, τ). Then the following statements are true

(a) $\psi g\alpha cl(\phi)=\phi$ and $\psi g\alpha cl(X)=X$.

(b) If $A\subseteq B$, then $\psi g\alpha cl(A) \subseteq \psi g\alpha cl(B)$

(c) $\psi g\alpha cl(A)\cup \psi g\alpha cl(B) \subseteq \psi g\alpha cl(A \cup B)$

(d) $\psi g\alpha cl(A \cap B) \subseteq \psi g\alpha cl(A)\cap \psi g\alpha cl(B)$

(e) $\psi g\alpha cl(\psi g\alpha cl(A))= \psi g\alpha cl(A)$

Proof

(a) and **(b)** follow from the **Definition 2.3.1**

(c) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, by (b) $\psi g\alpha cl(A) \subseteq \psi g\alpha cl(A \cup B)$ and $\psi g\alpha cl(B) \subseteq \psi g\alpha cl(A \cup B)$. Hence $\psi g\alpha cl(A) \cup \psi g\alpha cl(B) \subseteq \psi g\alpha cl(A \cup B)$.

(d) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$,by (b) $\psi g\alpha cl(A \cap B) \subseteq \psi g\alpha cl(A)$ and $\psi g\alpha cl(A \cap B) \subseteq \psi g\alpha cl(B)$. Therefore $\psi g\alpha cl(A \cap B) \subseteq \psi g\alpha cl(A) \cap \psi g\alpha cl(B)$.

(e) Follows from the **Definition 2.3.1**

Remark 2.3.7

The Reverse inclusion of (c) in proposition 2.3.6 is not true in general as seen from the following example.

Example 2.3.8

Let X={a, b, c} , $\tau = \{\phi, \{a\}, \{b\}\{a, b\}, X\}$. If A={a},B={b} then ψgαcl(A)= {a} and ψgαcl(B)={b} and ψgαcl(A ∪ B)=X, but ψgαcl(A) ∪ ψgαcl(B) = {a, b}.

Remark 2.3.9

The reverse inclusion of (d) in proposition 2.3.6 is not true in general as seen from the following example.

Example 2.3.10

Let X={a, b, c}, $\tau = \{\phi, \{a, b\}, X\}$. If A={a} and B={b} the ψgαcl(A)=X and ψgαcl(B)= {b} and ψgαcl(A) ∩ ψgαcl(B)= {b} but ψgαcl(A ∩ B) = ϕ.

2.4 ψgα-open sets

In this section the concept of ψgα-open sets is introduced and their properties are studied in topological spaces.

Definition 2.4.1

A subset A of a topological space (X, τ) is called ψgα-open if its complement A^c is ψgα-closed in (X, τ).

The collection of all ψgα-open sets of (X, τ) is denoted by ψgαO(X, τ).

Example 2.4.2

Let X={a, b, c}, $\tau = \{\phi, \{a\}, X\}$.Then ϕ, {a}, {a, b}, {a, c}, X are ψgα-open sets.

Proposition 2.4.3

Every open set in (X, τ) is a ψgα-open set but not conversely.

Proof

Let A be an open set in (X, τ).Then A^c is closed in (X, τ).Since every closed set is ψgα-closed, A^c is ψgα-closed in (X, τ). Therefore A is ψgα-open in (X, τ).

Example 2.4.4

Let X=$\{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$.Then the subset $\{a\}$ is ψgα-open but not open in (X, τ).

Proposition 2.4.5

Every α-open set in (X, τ) is a ψgα-open set but not conversely.

Proof

Let A be an α-open set in (X, τ). Then A^c is α-closed in (X, τ).Since every α-closed set in (X, τ) is ψgα -closed, A^c is ψgα-closed in (X, τ).Hence A is ψgα-open in (X, τ).

Example 2.4.6

Let X=$\{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.Then the subset $\{a, c\}$ is ψgα-open but not α-open in (X, τ).

Proposition 2.4.7

Every regular open set in (X, τ) is ψgα-open but not conversely.

Proof

Let A be a regular open set in (X, τ). Then A^c is regular closed in (X, τ).Since every regular closed set in (X, τ) is ψgα-closed, A^c is ψgα-closed in (X, τ).Therefore A is ψgα-open in (X, τ).

Example 2.4.8

Let X=$\{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$.Then the subset $\{a\}$ is ψgα-open but not regular-open in (X, τ).

Proposition 2.4.9

Every semi-open set in (X, τ) is ψgα-open but not conversely.

Proof

Let A be a semi-open set in (X, τ). Then A^c is semi-closed in (X, τ).Since every semi-closed set in (X, τ) is ψgα-closed, A^c is ψgα-closed in (X, τ).Therefore A is ψgα-open in (X, τ).

Example 2.4.10

Let X={a, b, c}, $\tau = \{\phi, \{a, b\}, X\}$.Then the subset {a} is ψgα-open but not semi-open in (X, τ).

Proposition 2.4.11

Every ψgα-open set in (X, τ) is ψg-open but not conversely

Proof

Let A be a ψgα-open set in (X, τ). Then A^c is ψgα-closed in (X, τ).Since every ψgα-closed set in (X, τ) is ψg-closed, A^c is ψg-closed in (X, τ).Therefore A is ψg-open in (X, τ).

Example 2.4.12

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$.Then the subset {b} is ψg-open but not ψgα-open in (X, τ).

Proposition 2.4.13

Every ψgα-open set in (X, τ) is gsp-open but not conversely.

Proof

Let A be a ψgα-open set in (X, τ). Then A^c is ψgα-closed in (X, τ).Since every ψgα –closed set in (X, τ) is gsp-closed, A^c is gsp-closed in (X, τ).Therefore A is gsp-open in (X, τ).

Example 2.4.14

Let X=$\{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$.Then the subset $\{a, c\}$ is gsp-open but not ψgα-open in (X, τ).

Proposition 2.4.15

Every ψgα-open set in (X, τ) is ψĝ-open but not conversely.

Proof

Let A be a ψgα-open set in (X, τ). Then A^c is ψgα-closed in (X, τ).Since every ψgα-closed set in (X, τ) is ψĝ-closed, A^c is ψĝ-closed in (X, τ).Therefore A is ψĝ-open in (X, τ).

Example 2.4.16

Let X=$\{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$.Then the subset $\{b\}$ is ψĝ-open but not ψgα-open in (X, τ).

Remark 2.4.17

For a subset A of (X, τ), $\psi cl(X - A) = X - \psi int(A)$.

Proposition 2.4.18

A subset A of a topological space (X, τ) is ψgα-open if and only if $U \subseteq \psi int(A)$ whenever $U \subseteq A$ and U is gα-closed.

Proof

Assume that A is ψgα-open in (X, τ).Then A^c is ψgα-closed. Let U be a gα-closed set in (X, τ) contained in A. Then U^c is a gα-open set in (X, τ) containing A^c. Since A^c is a ψgα-closed,$\psi cl(A^c) \subseteq U^c$ equivalently $U \subseteq \psi int(A)$.

Conversely assume that U is contained in $\psi int(A)$ whenever U is contained in A and U is gα-closed in (X, τ). Let A^c be contained in U, where U is gα-open. Then U^c is contained in A. By criteria, $U^c \subseteq \psi int(A)$.This implies $(\psi int(A))^c \subseteq U$ that is $\psi cl(A^c) \subseteq U$. Therefore A^c is ψgα-closed.Therefore A is ψgα-open in (X, τ).

Proposition 2.4.19

If ψint(A) $\subseteq$ B $\subseteq$ A and A is ψgα-open in (X, τ) then B is ψgα-open in (X, τ).

Proof

Follows from Remark 2.4.17 and proposition 2.2.35.

Proposition 2.4.20

If a subset A is ψgα-closed in (X, τ) then ψcl(A)-A is ψgα-open.

Proof

Suppose that A is ψgα-closed in (X, τ). Let F $\subseteq$ ψcl(A)-A and F be gα-closed. Since A is ψgα-closed, ψcl(A)-A does not contain nonempty gα-closed set (by proposition 2.2.38) Hence F= ϕ. Thus F $\subseteq$ ψint [ψcl(A)-A].Therefore ψcl(A)-A is ψgα-open.

Proposition 2.4.21

Let A $\subseteq$ B $\subseteq$ ψcl(A) and Let A be a ψgα-closed set .Then ψcl(B)-B is ψgα-open.

Proof

Let A $\subseteq$ B $\subseteq$ ψcl(A) and A be ψgα-closed .By proposition 2.2.35, B is ψgα-closed set. Then by proposition 2.4.20 ψcl(B)-B is ψgα-open.

Proposition 2.4.22

Let A be a subset of X and x $\in$ X,then x $\in$ ψgαcl(A) if and only if U$\cap$A $\neq$ ϕ for every ψgα-open set U containing x.

Proof

Let x $\in$ ψgαcl(A) and there exist a ψgα-open set U containing X such that U$\cap$A= ϕ . Then A $\subseteq$ X-U and ψgαcl(A) $\subseteq$ X-U. Hence x $\notin$ U, which is a contradiction. Therefore U $\cap$ A $\neq$ ϕ.

Conversely, assume that $U \cap A \neq \phi$ for every ψgα-open set U containing x. Suppose that $x \notin$ ψgαcl(A).Then there exists a ψgα-closed set F containing A such that $x \notin F$. Then $x \in X - F$ and $X - F$ is ψgα-open. By assumption, $(X\text{-}F) \cap A \neq \phi$. Since $A \subseteq F$, $(X\text{-}F) \cap A = \phi$, which is a contradiction. Hence $x \in$ ψgαcl(A).

2.5 Applications of ψgα-closed set

As an application of ψgα-closed sets five new spaces namely ψgαTc-space, ψgαTα-space, ψgαTsc-space, ψgTψgα-space and ψĝTψgα-space are introduced and their properties and interrelations are obtained.

Definition 2.5.1

A topological space (X, τ) is said to be a

1) ψgαTc-space if every ψgα-closed subset of (X, τ) is closed in (X, τ).
2) ψgαTα-space if every ψgα-closed subset of (X, τ) is α-closed in (X, τ).
3) ψgαTsc-space if every ψgα-closed subset of (X, τ) is semi-closed in (X, τ).
4) ψgTψgα-space if every ψg-closed subset of (X, τ) is ψgα-closed in (X, τ).
5) ψĝTψgα-space if every ψĝ-closed subset of (X, τ) is ψgα-closed in (X, τ).

Proposition 2.5.2

Every ψgαTc-space is a ψgαTα-space but not conversely.

Proof

Let A be a ψgα-closed set in (X, τ). Since (X, τ) is a ψgαTc-space, A is closed in (X, τ). Since every closed set is α-closed, A is α-closed in (X, τ). Hence (X, τ) is ψgαTα-space.

Example 2.5.3

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$.Then (X, τ) is a ψgαTα-space but not ψgαTc-space,since the subset {b} is ψgα-closed but not closed in (X, τ).

Proposition 2.5.4

Every ψgαTc-space is a ψgαTsc-space but not conversely.

Proof

Let A be a ψgα-closed set in (X, τ). Since (X, τ) is a ψgαTc-space, A is closed in (X, τ). Since every closed set is semi-closed, A is semi-closed in (X, τ).Therefore (X, τ) is a ψgαTsc-space.

Example 2.5.5

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}X\}$.Then (X, τ) is a ψgαTsc-space but not ψgαTc-space, since the subset {a} is ψgα-closed but not closed in (X, τ).

Proposition 2.5.6

Every ψgαTα-space is a ψgαTsc-space but not conversely.

Proof

Let A be a ψgα-closed set in (X, τ). Since (X, τ) is a ψgαTα-space, A is α-closed in (X,τ). Since every α-closed set is semi-closed, A is semi-closed in (X, τ). Therefore (X,τ) is a ψgαTsc-space.

Example 2.5.7

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}X\}$. Then (X, τ) is a ψgαTsc-space but not ψgαTα-space,since the subset {b} is ψgα-closed but not α-closed in (X, τ).

Proposition 2.5.8

Every ψgTψgα-space is a ψĝTψgα-space but not conversely.

Proof

Let A be a $\psi\hat{g}$-closed set in (X, τ). Since every $\psi\hat{g}$-closed set is ψg-closed in (X, τ) and (X, τ) is a $\psi gT_{\psi g\alpha}$-space, A is ψgα-closed.Therefore (X, τ) is a $\psi\hat{g}T_{\psi g\alpha}$-space.

Example 2.5.9

Let X={a, b, c}, τ = {ϕ, {a}, {b, c}, X}.Then (X, τ) is a $\psi\hat{g}T_{\psi g\alpha}$-space but not $\psi gT_{\psi g\alpha}$-space, since the subset {b} is ψg-closed but not ψgα-closed in (X, τ).

Remark 2.5.10

The following examples show that $\psi g\alpha T_c$-space and $\psi gT_{\psi g\alpha}$-space are independent.

Example 2.5.11

Let X={a, b, c}, τ = {ϕ, {a}, {b, c}, X}.Then (X, τ) is a $\psi g\alpha T_c$-space but not $\psi gT_{\psi g\alpha}$-space, since the subset {c} is ψg-closed but not ψgα closed in (X, τ).

Example 2.5.12

Let X={a, b, c}, τ = {ϕ, {a, b}, X}.Then (X, τ) is a $\psi gT_{\psi g\alpha}$-space but not a $\psi g\alpha T_c$-space, since the subset {a, c} is ψgα-closed but not closed in (X, τ).

Remark 2.5.13

The following examples show that $\psi g\alpha T_\alpha$-space and $\psi gT_{\psi g\alpha}$-space are independent.

Example 2.5.14

Let X={a, b, c}, τ = {ϕ, {a}, {b, c}, X}.Then (X, τ) is a $\psi g\alpha T_\alpha$-space but not $\psi gT_{\psi g\alpha}$-space, since the subset {b} is ψg-closed but not ψgα-closed in (X, τ).

Example 2.5.15

Let X={a, b, c}, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.Then (X, τ) is a ψgTψgα-space but not a ψgαTα-space, since the subset {a} is ψgα-closed but not α-closed in (X, τ).

Remark 2.5.16

The following examples show that ψgαTα-space and ψĝTψgα-space are independent.

Example 2.5.17

Let X={a, b, c}, $\tau = \{\phi, \{a\}, X\}$.Then (X, τ) is a ψgαTα-space but not a ψĝTψgα-space, since the subset {a, b} is ψĝ-closed but not ψgα-closed in (X, τ).

Example 2.5.18

Let X={a, b, c}, $\tau = \{\phi, \{a, b\}, X\}$.Then (X, τ) is a ψĝTψgα-space but not a ψgαTα-space, since the subset {a, c} is ψgα-closed but not α-closed in (X, τ).

Remark 2.5.19

The following examples show that ψgαTsc-space and ψgTψgα-space are independent.

Example 2.5.20

Let X={a, b, c}, $\tau = \{\phi, \{a\}, X\}$.Then (X, τ) is a ψgαTsc-space but not a ψgTψgα-space, since the subset {a, c} is ψg-closed but not ψgα-closed in (X, τ).

Example 2.5.21

Let X={a, b, c}, $\tau = \{\phi, \{a, b\}, X\}$.Then (X, τ) is a ψgTψgα-space but not ψgαTsc-space,since the subset {b, c} is ψgα-closed but not semi-closed in (X, τ).

Remark 2.5.22

The following examples show that ψgαTsc-space and ψĝTψgα-space are independent.

Example 2.5.23

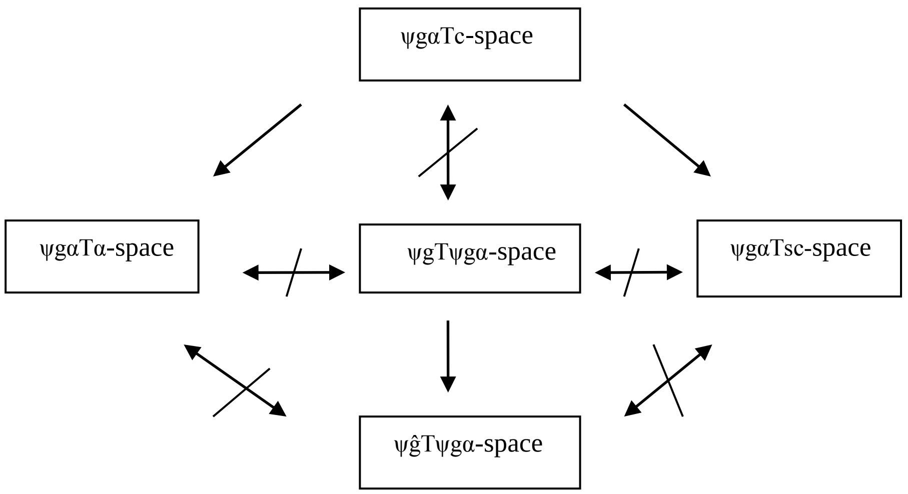
Let X={a, b, c}, $\tau = \{\phi, \{a\}, X\}$.Then (X, τ) is a ψgαTsc-space but not ψĝTψgα-space, since the subset {a, c} is ψĝ-closed but not ψgα-closed in (X, τ).

Example 2.5.24

Let X={a, b, c}, $\tau = \{\phi, \{a, b\}, X\}$.Then (X, τ) is a ψĝTψgα-space but not ψgαTsc-space,since the subset {b, c} is ψgα-closed but not semi-closed in (X, τ).

Remark 2.5.25

The following diagram exhibits the interrelations between the newly defined spaces.



CHAPTER-3

ψgα-Continuous Maps in Topological spaces

3.1 Introduction

Continuity is an important concept in mathematics and various types of continuous maps have been introduced and analyzed over the years. In this chapter ψgα-continuous maps using ψgα-closed sets is introduced and obtained some of their properties.

3.2 ψgα-Continuous Maps

In this section ψgα-continuous maps in topological spaces are introduced and discussed some of their basic properties.

Definition 3.2.1

A map f : (X, τ)→(Y,σ) is called ψgα-continuous if $f^{-1}(V)$ is ψgα-closed in (X, τ) for every closed set V in (Y,σ).

Example 3.2.2

Let X = Y = $\{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let f : (X, τ)→(Y,σ) be a map defined by f(a)=b, f(b)=a, f(c)=c. Then f is ψgα-continuous.

Proposition 3.2.3

Every continuous map f : (X, τ)→(Y,σ) is a ψgα-continuous map.

Proof

Let V be any closed set in (Y,σ). Since f is continuous, $f^{-1}(V)$ is closed in (X, τ). Since every closed set is ψgα-closed,$f^{-1}(V)$ is ψgα-closed in (X, τ). Therefore f is ψgα-continuous.

The converse of the above proposition need not be true as seen from the following example.

Example 3.2.4

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$ and σ =$\{\phi, \{a, b\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not continuous, since for the closed set {c} in (Y,σ), $f^{-1}\{c\}$ ={c} is not closed in (X, τ).

Proposition 3.2.5

Every completely-continuous map f:(X, τ)→(Y,σ) is a ψgα-continuous map.

Proof

Let V be any closed set in (Y,σ). Since f is completely-continuous,$f^{-1}(V)$ is regular-closed in (X, τ). Since every regular-closed set is ψgα-closed,$f^{-1}(V)$ is ψgα-closed in (X, τ). Therefore f is ψgα-continuous.

The converse of the above proposition need not be true as seen from the following example.

Example 3.2.6

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$ and σ =$\{\phi, \{a\}, \{a, b\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not completely continuous,since for the closed set $\{b, c\}$ in (Y,σ),$f^{-1}(\{b, c\}) = \{b, c\}$ is not regular closed in (X, τ).

Proposition 3.2.7

Every α-continuous map f: (X, τ)→(Y,σ) is ψgα-continuous.

Proof

Let V be any closed set in (Y,σ). Since f is α-continuous, $f^{-1}(V)$ is α-closed in (X, τ). Since every α-closed set is ψgα-closed,$f^{-1}(V)$ is ψgα-closed in (X, τ).Therefore f is ψgα-continuous.

The converse of the above proposition need not be true as seen from the following example.

Example 3.2.8

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not α-continuous,since for the closed set $\{a, c\}$ in (Y,σ),$f^{-1}(\{a, c\}) = \{a, c\}$ is not α-closed in (X, τ).

Proposition 3.2.9

Every semi-continuous map f:(X, τ)→(Y,σ) is ψgα-continuous.

Proof

Let V be any closed set in (Y,σ). Since f is semi-continuous, $f^{-1}(V)$ is semi-closed in (X, τ). Since every semi-closed set is ψgα-closed,$f^{-1}(V)$ is ψgα-closed in (X, τ). Hence f is ψgα-continuous.

The converse of the above proposition need not be true as seen from the following example.

Example 3.2.10

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not semi-continuous,since for the closed set $\{a, c\}$ in (Y,σ),$f^{-1}(\{a, c\}) = \{a, c\}$ is not semi-closed in (X, τ).

Proposition 3.2.11

Every ψgα-continuous map f:(X, τ)→(Y,σ) is ψg-continuous.

Proof

Let V be any closed set in (Y,σ). Since f is ψgα-continuous, $f^{-1}(V)$ is ψgα-closed in (X, τ). Since every ψgα-closed set is ψg-closed, $f^{-1}(V)$ is ψg-closed in (X, τ). Therefore f is ψg-continuous.

The converse of the above proposition need not be true as seen from the following example.

Example 3.2.12

Let X=Y={a, b, c},with $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let f be the identity map. Then f is ψg-continuous but not ψgα-continuous,since for the closed set {a, c} in (Y,σ),$f^{-1}(\{a, c\}) = \{a, c\}$ is not ψgα-closed in (X, τ).

Proposition 3.2.13

Every ψgα-continuous map f : (X, τ)→(Y,σ) is gsp-continuous.

Proof

Let V be any closed set in (Y,σ). Since f is ψgα-continuous, $f^{-1}(V)$ is ψgα-closed in (X, τ). Since every ψgα-closed set is gsp-closed,$f^{-1}(V)$ is gsp-closed in (X, τ). Therefore f is gsp-continuous.

The converse of the above proposition need not be true as seen from the following example.

Example 3.2.14

Let X=Y={a, b, c} with $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let f be the identity map. Then f is gsp-continuous but not ψgα-continuous,since for the closed set {a, c} in (Y,σ),$f^{-1}(\{a, c\}) = \{a, c\}$ is not ψgα-closed in (X, τ).

Proposition 3.2.15

Every ψgα-continuous map f : (X, τ)→(Y,σ) is ψĝ-continuous.

Proof

Let V be any closed set in (Y,σ). Since f is ψgα-continuous, $f^{-1}(V)$ is ψgα-closed in (X, τ). Since every ψgα-closed set is ψĝ-closed,$f^{-1}(V)$ is ψĝ-closed in (X, τ).Therefore f is ψĝ-continuous.

The converse of the above proposition need not be true as seen from the following example.

Example 3.2.16

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$ and σ =$\{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let f be the identity map. Then f is ψĝ-continuous but not ψgα-continuous,since for the closed set $\{a, c\}$ in (Y,σ),$f^{-1}(\{a, c\})$ = $\{a, c\}$ is not ψgα-closed in (X, τ).

Remark 3.2.17

The following examples show that ψgα-continuity is independent from pre-continuity and g*p-continuity.

Example 3.2.18

Let X = Y = $\{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not pre-continuous and g*p-continuous, since for the closed set $\{b\}$ in (Y,σ), $f^{-1}(\{b\})$= $\{b\}$ is ψgα-closed but not pre-closed and g*p-closed in (X, τ).

Example 3.2.19

Let X = Y = $\{a, b, c\}$, with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let f be the identity map. Then f is pre-continuous and g*p-continuous but not ψgα-continuous, since for the closed set $\{c\}$ in (Y,σ),$f^{-1}(\{c\})$ = $\{c\}$ is pre-closed and g*p-closed but not ψgα-closed in (X, τ).

Remark 3.2.20

The following examples show that ψgα-continuity is independent from g-continuity, αg-continuity, αĝ-continuity,*g-continuity and gp-continuity.

Example 3.2.21

Let X = Y = $\{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and σ=$\{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not g-continuous, αg-continuous,αĝ-continuous, *g-continuous and gp-continuous,since for the closed $\{b\}$ in (Y,σ),$f^{-1}(\{b\})$= $\{b\}$ is ψgα-closed but not g-closed, αg-closed, αĝ-closed, *g-closed and gp-closed in (X, τ).

Example 3.2.22

Let X=Y={a, b, c} with $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let f be the identity map. Then f is g-continuous, αg-continuous, αĝ-continuous, *g-continuous and gp-continuous but not ψgα-continuous, since for the closed set {a, c} in (Y,σ), $f^{-1}(\{a, c\}) = \{a, c\}$ is g-closed, αg-closed, αĝ-closed , *g-closed and gp-closed but not ψgα-closed in (X, τ).

Remark 3.2.23

The following examples show that ψgα-continuity is independent from gα-continuity and αgs-continuity.

Example 3.2.24

Let X=Y={a, b, c} with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not gα-continuous and αgs-continuous, since for the closed {a} in (Y,σ), $f^{-1}(\{a\}) = \{a\}$ is ψgα-closed but not gα-closed and αgs-closed in (X, τ).

Example 3.2.25

Let X=Y={a, b, c} with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Let f be the identity map. Then f is gα-continuous and αgs-continuous but not ψgα-continuous since for the closed {b} in (Y,σ), $f^{-1}(\{b\}) = \{b\}$ is gα-closed, αgs-closed, but not ψgα-closed in (X, τ).

Remark 3.2.26

The following examples show that ψgα-continuity is independent from ĝ-continuity.

Example 3.2.27

Let X=Y={a, b, c} with $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not ĝ-continuous, since for the closed {c} in (Y,σ), $f^{-1}(\{c\}) = \{c\}$ is ψgα-closed but not ĝ-closed in (X, τ)

Example 3.2.28

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let f be the identity map. Then f is ĝ-continuous but not ψgα-continuous,since for the closed set {c} in (Y,σ), $f^{-1}(\{c\})$ = {c} is ĝ-closed but not ψgα-closed in (X, τ).

Remark 3.2.29

The following examples show that ψgα-continuity is independent from $\tilde{g}_p$-continuity.

Example 3.2.30

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not $\tilde{g}_p$-continuous, since for the closed {a} in (Y,σ), $f^{-1}(\{a\})$= {a} is ψgα-closed but not $\tilde{g}_p$-closed in (X, τ).

Example 3.2.31

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$ and σ =$\{\phi, \{a\}, \{b, c\}, Y\}$. Let f be the identity map. Then f is $\tilde{g}_p$-continuous but not ψgα-continuous, since for the closed sets {c} in (Y,σ), $f^{-1}(\{c\})$= {c} is $\tilde{g}_p$-closed but not ψgα- closed in (X, τ).

Remark 3.2.32

The following examples show that ψgα-continuity is independent from gpr-continuity.

Example 3.2.33

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and σ =$\{\phi, \{a, b\}, Y\}$. Let f be the identity map. Then f is gpr-continuous but not ψgα-continuous,since for the closed {c} in (Y,σ), $f^{-1}(\{c\})$= {c} is gpr-closed but not ψgα-closed in (X, τ).

Example 3.2.34

Let X=Y=$\{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and σ =$\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not gpr-continuous, since for the closed sets {b} in (Y,σ),$f^{-1}(\{b\})$= {b} is ψgα-closed but not gpr-closed in (X, τ).

Remark 3.2.35

The following examples show that ψgα-continuity is independent from g#p#-continuity.

Example 3.2.36

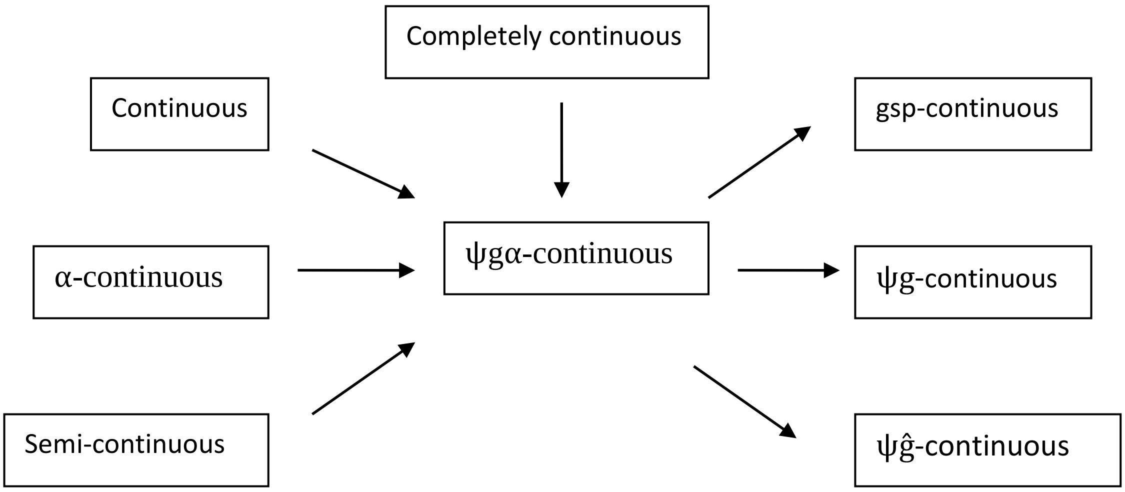
Let X=Y={a, b, c} with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Let f be the identity map. Then f is g#p#-continuous but not ψgα -continuous,since for the closed {c} in (Y,σ), $f^{-1}(\{c\})$= {c} is g#p#-closed but not ψgα-closed in (X, τ).

Example 3.2.37

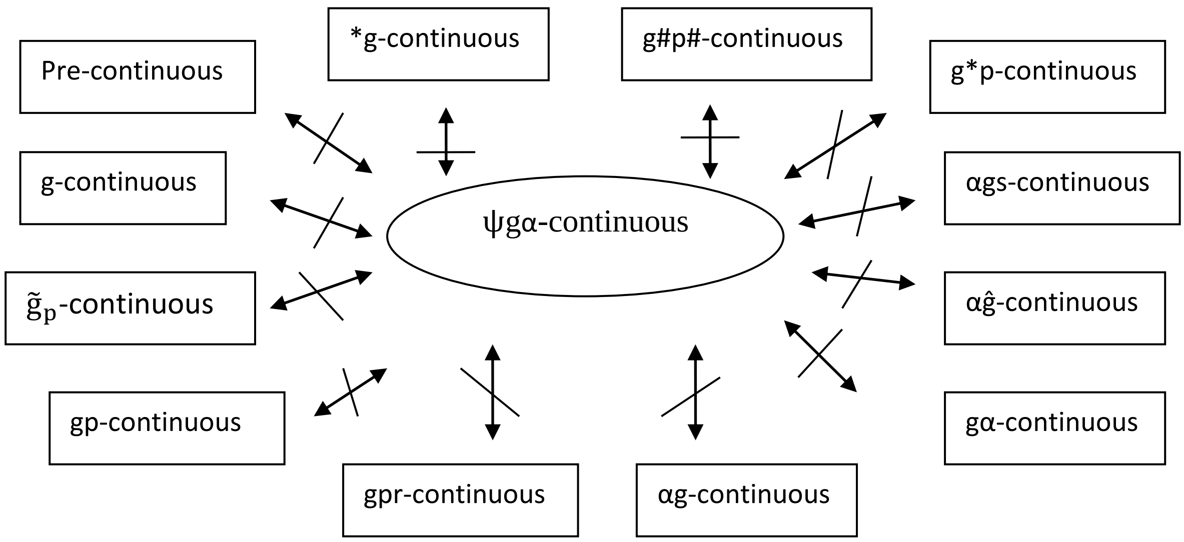
Let X=Y={a, b, c} with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$. Let f be the identity map. Then f is ψgα-continuous but not g#p#-continuous,since for the closed {b} in (Y,σ), $f^{-1}(\{b\})$= {b} is ψgα-closed but not g#p#-closed in (X, τ).

Remark 3.2.38

The following diagrams show the relationship between ψgα-continuous map with the existing continuous map.



where A→B represents A implies B.



where A ↮ B represents A and B independent.

Theorem 3.2.39

A map f : (X, τ)→(Y,σ) is ψgα-continuous if and only if $f^{-1}(V)$ is ψgα-open for each open set V in (Y,σ).

Proof (Neccessity):

Let V be an open set in (Y,σ). Then Y-V is closed in (Y,σ) .Since f is ψgα-continuous, $f^{-1}(Y-V)$ = X - $f^{-1}(V)$ is ψgα-closed in (X, τ). Hence $f^{-1}(V)$ is ψgα-open in (X, τ).

(Sufficiency):

Assume that $f^{-1}(V)$ is ψgα-open in (X, τ) for each open set V in (Y,σ). Let G be any closed set in (Y,σ). Then Y-G is open in (Y,σ). By assumption,$f^{-1}(Y-G)$=X-$f^{-1}(G)$ is ψgα-open in (X, τ) which implies that $f^{-1}(G)$ is ψgα-closed in (X, τ). Therefore f is ψgα-continuous.

Theorem 3.2.40

If f : (X, τ)→(Y,σ) is ψgα-continuous then f(ψgαcl(U)) ⊆ cl(f(U)) for every subset U of (X, τ).

Proof

Let U be any subset of (X, τ).Then cl(f(U)) is closed in (Y,σ). Since f is ψgα-continuous, f^{-1}(cl(f(U))) is ψgα-closed in (X, τ), since f(U) ⊆ cl(f(U)),U ⊆ f^{-1}(f(U)) ⊆ f^{-1}(cl(f(U))) and hence f^{-1}(cl(f(U))) is a ψgα-closed set containing U. By definition of ψgα-closure, we have ψgαcl(U)) ⊆ f^{-1}(cl(f(U))) which implies that f(ψgαcl(U)) ⊆ cl(f(U)) .

Corollary 3.2.41

Let f : (X, τ)→(Y,σ) be a continuous map. Then for every subset U of (X, τ), f(ψgαcl(U)) ⊆ cl(f(U)).

Proof

As every continuous map f : (X, τ)→(Y,σ) is ψgα-continuous and by Theorem 3.2.40 the result follows.

Proposition 3.2.42

If f : (X, τ)→(Y,σ) is a ψgα-continuous map and g : (Y, σ) → (Z, μ) is a continuous map, then g ∘ f : (Y, σ) → (Z, μ) is a ψgα-continuous map.

Proof

Let V be a closed set in (Z, μ). Since g is continuous, g^{-1}(V) is closed in (Y, σ). Since f is ψgα-continuous, $(g \circ f)^{-1}$(V)= f^{-1}(g^{-1}(V)) is ψgα-closed in (X, τ). Therefore g ∘ f is a ψgα-continuous map.

Proposition 3.2.43

If f : (X, τ)→(Y,σ) is a continuous(resp.α-continuous) map and g : (Y, σ) → (Z, μ) is a continuous map, then g ∘ f : (X, τ) → (Z, μ) is a ψgα-continuous map.

Proof

Let V be a closed set in (Z, μ). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is continuous (resp.α-continuous),$(g \circ f)^{-1}(V)$= $f^{-1}(g^{-1}(V))$ is closed (resp.α-closed). Since every closed(resp. α-closed) set is ψgα-closed, $(g \circ f)^{-1}(V)$ is ψgα-closed . Therefore $g \circ f$ is a ψgα-continuous map.

Proposition 3.2.44

If f : (X, τ)→(Y,σ) be a semi-continuous map and $g : (Y, \sigma) \to (Z, \mu)$ be a continuous map. Then $g \circ f : (X, \tau) \to (Z, \mu)$ is a ψgα-continuous map.

Proof

Let V be a closed set in (Z, μ). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is semi-continuous, $(g \circ f)^{-1}(V)$=$f^{-1}(g^{-1}(V))$ is semi-closed in (X, τ). Since every semi-closed set is ψgα-closed, $(g \circ f)^{-1}(V)$ is ψgα-closed. Therefore $g \circ f$ is a ψgα-continuous map.

Proposition 3.2.45

Let f : (X, τ) → (Y,σ) be completely-continuous map and $g : (Y, \sigma) \to (Z, \mu)$ be continuous map. Then $g \circ f : (X, \tau) \to (Z, \mu)$ is a ψgα-continuous map.

Proof

Let V be a closed set in (Z, μ). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is completely-continuous, $(g \circ f)^{-1}(V)$ =$f^{-1}(g^{-1}(V))$ is regular-closed in (X, τ). Since every regular-closed set is ψgα-closed, $(g \circ f)^{-1}(V)$ is ψgα closed. Therefore $g \circ f$ is a ψgα-continuous map.

Proposition 3.2.46

Let f : (X, τ) → (Y,σ) be a ψgα -continuous map and $g : (Y, \sigma) \to (Z, \mu)$ be a continuous map. Then $g \circ f : (X, \tau) \to (Z, \mu)$ is a gsp-continuous map.

Proof

Let V be a closed set in (Z, μ). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is ψgα-continuous. $(g \circ f)^{-1}(V)$=$f^{-1}(g^{-1}(V))$ is ψgα-closed in

(X,τ). Since every ψgα-closed set is gsp-closed, $(g \circ f)^{-1}(V)$ is gsp-closed, Therefore $g \circ f$ is a gsp-continuous map.

Proposition 3.2.47

Let $f : (X, \tau) \rightarrow (Y,\sigma)$ be a ψgα -continuous map and $g : (Y, \sigma) \rightarrow (Z, \mu)$ be a continuous map. Then $g \circ f : (X, \tau) \rightarrow (Z, \mu)$ is a ψg-continuous map.

Proof

Let V be a closed set in (Z, μ). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is ψgα-continuous. $(g \circ f)^{-1}(V)= f^{-1}(g^{-1}(V))$ is ψgα-closed in (X, τ). Since every ψgα-closed set is ψg-closed, $(g \circ f)^{-1}(V)$ is ψg-closed, Therefore $g \circ f$ is a ψg-continuous map.

Proposition 3.2.48

Let $f : (X, \tau) \rightarrow (Y,\sigma)$ be a ψgα-continuous map and $g : (Y, \sigma) \rightarrow (Z, \mu)$ be a continuous map. Then $g \circ f : (X, \tau) \rightarrow (Z, \mu)$ is a ψĝ-continuous map.

Proof

Let V be a closed set in (Z, μ). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is ψgα-continuous., $(g \circ f)^{-1}(V)=f^{-1}(g^{-1}(V))$ is ψgα-closed in (X, τ). Since every ψgα-closed set is ψĝ-closed, $(g \circ f)^{-1}(V)$ is ψĝ-closed, Therefore $g \circ f$ is a ψĝ-continuous map.

Remark 3.2.49

The composition of two ψgα-continuous map need not be a ψgα-continuous map as seen from the following example.

Example 3.2.50

Let X=Y=Z=$\{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$, $\sigma =\{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$ and $\mu = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Z\}$. Let $f : (X, \tau) \rightarrow (Y,\sigma)$ be a map defined by f(a)=a, f(b)=b, f(c)=c and $g : (Y, \sigma) \rightarrow (Z, \mu)$ be a map defined by g(a)=a, g(b)=b, g(c)=c. Then the map f and g are ψgα-continuous but their composition $g \circ f : (X, \tau) \rightarrow (Z, \mu)$ is not a

$\psi g\alpha$-continuous map, since for the closed set $\{a, c\}$ in (Z, μ), $(g \circ f)^{-1}(\{a, c\}) = \{a, c\}$ is not $\psi g\alpha$-closed in (X, τ).

Proposition 3.2.51

Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \mu)$ be $\psi g\alpha$-continuous maps. Then $g \circ f : (X, \tau) \to (Z, \mu)$ is also a $\psi g\alpha$-continuous map, if (Y, σ) is a $\psi g\alpha T_c$-space.

Proof

Let V be a closed set in (Z, μ). Since g is $\psi g\alpha$-continuous, $g^{-1}(V)$ is $\psi g\alpha$-closed in (Y, σ). Since (Y, σ) is a $\psi g\alpha T_c$-space, $g^{-1}(V)$ is closed in (Y, σ). Since f is $\psi g\alpha$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi g\alpha$-closed in (X, τ). Hence $g \circ f$ is a $\psi g\alpha$-continuous map.

Proposition 3.2.52

Let $f : (X, \tau) \to (Y, \sigma)$ be a ψg-continuous map and $g : (Y, \sigma) \to (Z, \mu)$ be a continuous map. Then $g \circ f : (X, \tau) \to (Z, \mu)$ is a $\psi g\alpha$-continuous map, if (X, τ) is a $\psi g T_{\psi g\alpha}$-space.

Proof

Let V be a closed set in (Z, μ). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is ψg-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψg-closed in (X, τ). Since (X, τ) is a $\psi g T_{\psi g\alpha}$-space, $(g \circ f)^{-1}(V)$ is $\psi g\alpha$-closed in (X, τ).Hence $g \circ f$ is a $\psi g\alpha$-continuous map.

Proposition 3.2.53

Let $f : (X, \tau) \to (Y, \sigma)$ be $\psi \hat{g}$-continuous map and $g : (Y, \sigma) \to (Z, \mu)$ be a continuous map. Then $g \circ f : (X, \tau) \to (Z, \mu)$ is a $\psi g\alpha$-continuous map, if (X, τ) is a $\psi \hat{g} T_{\psi g\alpha}$-space.

Proof

Let V be a closed set in (Z, μ). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ).Since f is $\psi \hat{g}$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi \hat{g}$-closed in (X, τ).

Since (X, τ) is a $\psi\hat{g}T_{\psi g\alpha}$-space, $(g \circ f)^{-1}(V)$ is $\psi g\alpha$-closed in (X, τ).Hence $g \circ f$ is a $\psi g\alpha$-continuous map.

Summary and Conclusion

Preliminary definition are presented in **Chapter 1**.

In **Chapter 2**, ψgα-closed sets, ψgα-closure and ψgα-open sets are introduced in topological spaces. Properties and relations between ψgα-closed sets with other existing various generalized closed sets are obtained.

In **Chapter 3**, ψgα-continuous maps in topological spaces are introduced. Properties and Characterizations are derived.