Introduction

INTRODUCTION

"Mathematics is of profound significance in the universe,

not because it exhibits principle, that we obey, but

because it exhibits principles that we impose"

- J.N.N.Sullivan

In a wide variety of problems one has to treat uncertain or incomplete information. Some kind of exact science is needed to describe and understand existing methods and to develop new attempts. Especially in applications of computer science, this is a fundamental problem. To handle such kind of information Zadeh [40] introduced the concept of fuzzy sets in 1965. Since then this concept has been used in the development of almost all branches of Mathematics, where the concepts of uncertainty information and complexity are explained by means of fuzzy sets. The notion of a fuzzy set has caused great interest among both 'pure' and applied Mathematicians. It has also raised enthusiasm among some Engineers, who use Mathematical ideas and methods in their research.

General topology was one of the first branches of pure Mathematics to which fuzzy sets have been applied systematically. In 1968, Chang [8] made the first "grafting" of the notion of a fuzzy set onto general topology and defined fuzzy topological space.

In 1985, Kubiak [25] and Sostak [39] have introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and fuzzy topology, in the sense that not only the objects are fuzzified, but also the axiomatics.

In 1992, Hazra, Samanta and Chattopadhyay [9] gave a new definition of fuzzy topology by introducing the concept of gradation of openness of fuzzy subsets. In 1993, Chattopadhyay and Samanta [10] have studied fuzzy closure operator and its characterizations.

In 1986, Atanassov [5] has introduced the concept of intuitionistic fuzzy set which is a generalization of fuzzy set in Zadeh's sense [40]. In 1997, Coker [12] has introduced the concept of intuitionistic fuzzy topological spaces by using the intuitionistic fuzzy sets, which is an extended concept of fuzzy topological spaces [8] in Chang's sense. In 2002, Mondal and Samanta [28] have introduced the concept of intuitionistic gradation of openness which is a generalization of the concept of gradation of openness defined by Chattopadhyay.

This thesis is devoted to the study of the following concepts:

(i) Gradation of openness

(ii) Intuitionistic gradation of openness

(iii) Some properties of (r, s)-T_0 and (r, s)-T_1 spaces.

The results discussed are contained in the following articles :

(i) "Gradation of openness : fuzzy topology" by K.C. Chattopadhyay, R.N.Hazra, S.K.Samanta [9].

(ii) "Fuzzy topology : fuzzy closure operator, fuzzy compactness and fuzzy connectedness" by K.C. Chattopadhyay, S.K.Samanta [10].

(iii) "Intuitionistic fuzzy sets" by Krassimir T. Atanassov [5].

(iv) "An introduction to intuitionistic fuzzy topological spaces" by Dogan Coker [12].

(v) "On intuitionistic gradation of openness" by T.K. Mondal, S.K. Samanta [28].

(vi) Some properties of (r, s)-T_0 and (r, s)-T_1 spaces by S.E.Abbas and Biljana Krsteska [2].

The first chapter deals with preliminary definitions and results on fuzzy sets, fuzzy topological spaces, gradation of openness and fuzzy closure operator.

The second chapter is devoted to the study of intuitionistic gradation of openness. In this chapter, preliminary definitions and results on intuitionistic fuzzy sets, intuitionistic fuzzy topological space and intuitionistic gradation of openness which are needed for our study are collected.

The third chapter is devoted to the study of some properties of (r, s)-T_0 and (r, s)-T_1 spaces. In section one, some preliminary definitions are discussed. In section two, properties of product intuitionistic fuzzy topological spaces are studied.

Some of the results discussed here are:

1) Let $(X, \tau_\beta, \tau^*_{\beta^*})$ be a product space of a family $\{(X_i, \tau_i, \tau^*_i) / i\in\Gamma\}$ of IFTS's. Then the following statements are equivalent:

   i) a projection map $\pi_j : (X, \tau_\beta, \tau^*_{\beta^*}) \to (X_j, \tau_j, \tau^*_j)$ is IF open;

   ii) for every $\mu = \bigwedge_{i\in\Gamma} \pi_i^{-1}(\lambda_i)$ such that $\bigvee_{x\in X_i} \lambda_i(x) = \alpha_i$ for each $\alpha_i \in I$ and $i\in\Gamma_0$ such that a finite index subset Γ_0 of Γ-{j} and $\tau_i(\lambda_i) > 0$, then $\bigwedge_{i\in\Gamma_0} \tau_i(\lambda_i) \le \tau_j(\underline{\alpha})$ and $\bigwedge_{i\in\Gamma_0} \tau^*_i(\lambda_i) \ge \tau^*_j(\underline{\alpha})$, where $\alpha = \bigwedge_{i\in\Gamma_0} \alpha_i$.

2) Let (X, τ, τ^*) be a product space of a family $\{(X_i, \tau_i, \tau^*_i) / i\in\Gamma\}$ of IFTSs. Then, the following properties hold:

   1) $C_{\tau,\tau^*}(\prod_{i\in\Gamma} \lambda_i, r, s) \le \prod_{i\in\Gamma} C_{\tau,\tau^*}(\lambda_i, r, s)$, $\forall \lambda_i \in I^{X_i}$, $r\in I_0$, $s\in I_1$,

   2) if $C_{\tau_i,\tau_i^*}(\lambda_i, r, s) = \lambda_i$, $\forall \lambda_i \in I^{X_i}$, $r\in I_0$, $s\in I_1$, then $C_{\tau,\tau^*}(\prod_{i\in\Gamma} \lambda_i, r, s) = \prod_{i\in\Gamma} \lambda_i$.

In section three, some properties of (r, s)-T_0 and (r, s)-T_1 spaces are discussed.

An IFTS (X, τ, τ^*) is said to be:

1) **(r, s)-quasi-T_0 space** if for each x_t, $x_m \in P_t(X)$ and $t<m$, there exists $\lambda \in Q(x_m, r, s)$ such that $x_t \bar{q} \lambda$.

2) **(r, s)-sub-T_0 space** if for each $x \neq y \in X$, there exists $t \in I_0$ such that there exists $\lambda \in Q(x_t, r, s)$ such that $y_t \bar{q} \lambda$, or there exists $\mu \in Q(y_t, r, s)$ such that $x_t \bar{q} \mu$.

3) **(r, s)-T_0 space** if for each x_t, $y_m \in P_t(X)$, there exists $\lambda \in Q(x_t, r, s)$ such that $y_m \bar{q} \lambda$, or there exists $\mu \in Q(y_m, r, s)$ such that $x_t \bar{q} \mu$.

4) **(r, s)-T_1 space** if for each x_t, $y_m \in P_t(X)$, such that $x_t \not\leq y_m$, there exists $\lambda \in Q(x_t, r, s)$ such that $y_m \bar{q} \lambda$.

Some interesting properties of these spaces and relationships between them investigated are :

1. Let (X, τ, τ^*) be an IFTS. Then the following statements are equivalent.

   1) (X, τ, τ^*) is (r, s)-quasi-T_0 space ;

   2) for each x_t, $x_m \in P_t(X)$, $Q(x_t, r, s) \neq Q(x_m, r, s)$;

   3) for each x_t, $x_m \in P_t(X)$, then $x_t \notin C_{\tau,\tau^*}(x_m, r, s)$ or $x_m \notin C_{\tau,\tau^*}(x_t\, r, s)$.

2. 1) Every (r, s)-T_0 space is both (r, s)-quasi-T_0 and (r, s)-sub T_0.

   2) Every (r, s)-T_1 space is (r, s)-T_0 space.

3. Every subspace of (r, s)-quasi-T_0 (resp. (r, s)-sub-T_0, (r, s)-T_0, and (r, s)-T_1) space is (r, s)-quasi-T_0 (resp, (r, s)-sub-T_0, (r, s)-T_0 and (r, s)-T_1) space.