



Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category 'A' by MHRD (now MoE)

Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC

Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test I – February 2025

Semester-II

Class : I PG

Branch : Mathematics

Time : 2 Hours

Max.Marks : 60

23MMAC07 –Advanced Algebra II

Course Outcomes:

CO1: Reduce the geometric problem to an algebraic problem.

CO2: Calculate the group of all automorphisms of a given field.

CO3: Find eigen values and eigen vectors of linear transformations.

CO4: Test orthogonality of given vectors.

CO5: Construct finite fields corresponding to a prime number.

Part A

6 x 1 = 6

Choose the Correct Answer

1. When two automorphisms σ and τ of K are said to be equal then
a. $\sigma(a) \neq \tau(a)$ b. $\sigma(a) = \tau(a)$ c. $\sigma(a) \neq 0$ d. $\sigma(a) = 0$ CO1K1
2. If k is a normal extension of a field F then k is a..... of some polynomial over F .
a. Splitting field b. fixed field c. finite extension d. normal extension CO1K1
3. If V is finite dimensional over F then $T \in A(V)$ is if and only if T maps V onto V .
a. regular b. singular c. left invertible d. Both (a) and (c) CO2K3
4. If $\dim V$ over a field F is greater than one then $A(V)$ is
a. commutative b. field c. group d. not commutative CO2K2
5. If $T, S \in A(V)$ and if S is regular then T and STS^{-1} have minimal polynomial.
a. same b. different c. n d. 1 CO3K2
6. The subspace W of V is invariant under $T \in A(V)$ if
a. $WT \subset W$ b. $WT = W$ c. $WT = V$ d. $WT \subset V$ CO3K1

Part B

Answer ALL questions

3 x 6 = 18

7. a. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F .
(or)
7. b. If $p(x) \in F[x]$ is solvable by radicals over F then prove that the Galois group of $p(x)$ is a solvable group. CO1K3
8. a. If V is finite dimensional over F then show that $T \in A(V)$ is singular if and only if there exists $v \neq 0$ in V such that $VT=0$.
(or)
8. b. If V is finite dimensional over F , then prove that $T \in A(V)$ is regular if and only if T maps V onto V . CO2K4

9. a. If V is n -dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis v_1, \dots, v_n and the matrix $m_2(T)$ in the basis w_1, w_2, \dots, w_n of V over F , then there is an element $C \in F_n$ such that $m_2(T) = Cm_1(T)C^{-1}$. CO2K4

(or)

9. b. If V is n dimensional over F and if $T \in A(V)$ has all its characteristic roots in F then prove that T satisfies a polynomial of degree n over F . CO3K4

Part C

Answer ALL questions

3 x 12 = 36

10. a. If K is a finite extension of F , then prove that $G(K, F)$ is a finite group and its order $o(G(K, F))$ satisfies the condition $o(G(K, F)) \leq [K: F]$ CO1K3

(or)

10. b. If K is a splitting field of some polynomial over F , then show that K is a normal extension of F . CO1K4

11. a. Let G' be the commutator subgroup of G then prove that

(i) G' is normal in G

(ii) G/G' is abelian

(iii) G' is the smallest subgroup of G such that G/G' is abelian. CO2K3

(or)

11. b. (i) If V is finite dimensional over F , then $T \in A(V)$ is invertible iff the constant term of the minimal polynomial for T is not 0.

(ii) If $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k$ are distinct characteristic roots of $T \in A(V)$ and if $v_1, v_2, v_3, \dots, v_k$ are characteristic vectors of T belonging to $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k$ respectively, then $v_1, v_2, v_3, \dots, v_k$ are linearly independent over F . CO2K4

12. a. (i) Define Characteristic vector. (ii) Prove that if V is an n -dimensional vector spaces over F , then $A(V)$ and F_n are isomorphic as algebras over F . CO2K3

(or)

12. b. If $T \in A(V)$ has all its characteristic roots in F , then prove that there is a basis of V in which the matrix of T is triangular. CO3K3

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