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GENERALIZED POSSIBILITY NEUTROSOPHIC SOFT SET AND IT'S APPLICATIONS

Thesis Submitted By

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GENERALIZED POSSIBILITY NEUTROSOPHIC SOFT SET AND IT'S APPLICATIONS

CHAPTER 1

1.INTRODUCTION

Lotfi A-Zadeh [21] introduced fuzzy set as an expansion of crisp set or non-fuzzy set in 1965. A fuzzy set is a class of objects with a continuum of grade of membership. Such a set is described by a membership ranging between zero and one. The intuitionistic fuzzy set are sets whose elements have degrees of membership and non-membership. Intuitionistic fuzzy sets was imported by Krassimir Atanassov [19] (1983) as an extension of Lotfi Zadeh's notation of fuzzy set, which itself develops the classical notation of a set.

The intuitionistic fuzzy sets can only operate the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. It does not operate the indeterminate and incompatible information which remain in belief system. Smarandache [12] introduced the approach of neutrosophic set which is a mathematical tool for handling problems contains imprecise, indeterminacy and incompatible data. The neutrosophic factors T, I, F which represents the membership, indeterminacy, and non-membership values respectively, where $]^{-}0,1^{+}[$ is the non-standard unit interval, and thus defines the neutrosophic set. Later, Salama and Alblow [1] defined generalized neutrosophic sets (GNSs), where the triplet functions satisfy the condition $T \wedge I \wedge F \leq 0.5$.

In 1999, Molodtsov [7] introduced the notion of soft set theory for dealing with complicated problems and different types of uncertainties and the approach has been applied diverse practical fields such as decision making [33, 36, 38, 39], data analysis [46], forecasting [47], optimization [8], etc. Several researchers have incorporated various mathematical hybrid structures such as fuzzy soft sets [29, 30, 34], intuitionistic fuzzy soft set theory [25, 26, 35], possibility fuzzy soft set [41], generalized fuzzy soft sets [13, 37], generalized intuitionistic fuzzy soft [20], possibility intuitionistic fuzzy soft set [23], vague soft set [45], possibility vague soft set [17], neutrosophic soft sets [38], weighted neutrosophic soft sets [36], etc by generalizing and expanding classical soft set theory of Molodtsov [7].

Maji [38] combined the notion of soft sets and neutrosophic set together by introducing a new notion called neutrosophic soft set and gave application of neutrosophic soft set in decision making problem. Currently, the properties and applications on the neutrosophic sets have studied increasingly [14, 15, 43, 44]. Currently, Broumi [44] studied generalized neutrosophic soft sets (GNSSs) and implemented some definitions and operations of the notion. He also implemented an application of GNSSs in decision making problems. Sahin, and Kucuk [40] discussed a method to find out similarity measures of two GNSSs and implemented an application of GNSSs in decision making problem. In [32] Maji et al. introduced several operators for soft set theory: equality of two soft sets, subsets and superset of soft sets, complement of soft set, null soft sets and absolute soft sets. But few of these definitions and their properties have few gaps, which have been pointed out by Ali et al. [24] and yang [3]. In 2010, Cagman and Enginoglo [27] made few modifications the operations of soft sets and filled in these gap. In 2014, Cagman [28] redefined soft sets using the single parameter set and compared definitions with those defined before.

Multi criteria decision-making (MCDM) is considered as a complex decision-making (DM) tool containing both qualitative and quantitaRRtive factors. In recent years, severs MCDM techniques and approaches have been recommended for choosing the best probable options. De et al. [9] studied the Sanchez’s approach for medical diagnosis and also they expanded this concept which is a generalization of fuzzy set theory with the concept of intuitionistic fuzzy set theory. TOPSIS (Technique for order preference by similarity to an ideal solution) method which is one of the most encourageble and effective MCDM methods to solve MCDM problems and most used classical MCDM methods has developed by Hwang and Yoon [4]. The fundamental idea of TOPSIS is that the chosen different should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. In classical MCDM methods, the aspects values and weights are determined precisely. Then the proposed set theories have implemented the different multi-criteria decision making methods. To handle with problems consisting of incomplete and vague information, in 2000 Chen [5] consult the fuzzy version of TOPSIS method for the first time. Chung and Chu [6] granted fuzzy TOPSIS method under group decision for facility location selection problems. The Boran [2] connected TOPSIS method with intuitionistic fuzzy set. They recommended a method to select best supplier in group decision making environment. Then the TOPSIS method for MCDM problems has expanded in interval valued intuitionistic fuzzy sets by Ye [16].

Liu and Wang [22] granted new methods in an intuitionistic fuzzy environment for clarifying multi-criteria decision-making problems. Firstly, they defined a calculation function for the decision-making problem and then introduced operators which will reduce the degree of ambiguously of the elements corresponding to an intuitionistic fuzzy set. Biswas et al. [31] expanded the cocept of TOPSIS method for MAGDM problems under single valued neutrosophic environment.

In this study, TOPSIS method merged with Prioritized Neutrosophic Soft set is used to select good college in group decision making environment. Finally, analytical example based on selection process is stated to illustrate the application of Prioritized Neutrosophic Soft set TOPSIS method.

REVIEW OF LITRATURE:

Zadeh (1965) introduced the concept of Fuzzy set. It gave an opportunity for the people to deal with non-statistical vague matters (concepts). It had also shown its importance and gained the interests of researchers in various fields such as medicine, engineering, political science, artificial intelligence, robotics, signal processing, network systems and attempted to quantify the same.

Atanassov (1986 and 1989) introduced the Intuitionistic fuzzy sets and its applications were found in various disciplines of research. Some degree of hesitation in an information had been captured and studied under Intuitionistic fuzzy sets which was the generalization of a fuzzy set.

The neutrosophic set (NS) was introduced by F. Smarandache who introduced the degree of indeterminacy (i) as indepedent component in his 1995 manuscris that was published in 1998. Neutrosophic set is a novel tool to deal with vagueness considering the truth-membership T, indeterminacy-membership I and falsity-membership F satisfying the condition $0 \leq T + I + F \leq 3$. It can be used to characterize the uncertain information more sufficiently and accurately than intuitionistic fuzzy set. Neutrosophic set has attracted great attention of many scholars that have been extended to new types and these extensions have been used in many areas such as aggregation operators, decision making, image processing, information measures, graph and algebraic structures. Because of such a growth, we present an overview on neutrosophic set with the aim of offering a clear perspective on the different concepts, tools and trends related to their extensions. A total of 137 neutrosophic set publication records from Web of Science are analyzed. The term "neutrosophic" because "neutrosophic" etymologically comes from "neutrosophy"[French neuter, Latin neuter, neutral, and Greek Sophia, skill/wisdom] which meansknowledge of neutral thought, and this third/neutral represents the main distinction between "fuzzy"/ "intuitionistic fuzzy" logic/setand „neutrosophic” logic/set, i.e. the included middle component (Lupasco-Nicolescu's logicin philosophy), (i.e.) the neutral/indeterminate/unknown part (besides the "truth"/"membership" and "falsehood"/"non-membership" components that both appear in fuzzy logic/set).

Molodtsov introduced the theory of soft sets, which can be seen as a new mathematical approach to vagueness. Maji et al. presented the concept of the fuzzy soft sets (fs-sets) by embedding the ideas of fuzzy sets. By using this definition of fs-sets many interesting applications of soft set theory have been expanded by some researchers. Roy and Maji gave some applications of fs-sets. Som defined soft relation and fuzzy soft relation on the theory of soft sets. Aktas and Cagman compared soft sets with the related concepts of fuzzy sets and rough sets. Yang et al. defined the operations on fuzzy soft sets which are based on three fuzzy logic operators: negation, triangular norm and triangular conorm. Zou and Xiao introduced the soft set and fuzzy soft set into the incomplete environment. Xiao et al. used forecasting accuracy as the criterion of fuzzy membership function, and purposed a combined forecasting approached based on fs-sets. Yang et al. presented the combination of interval-valued fuzzy set and soft set. Kong et al. defined the normal parameter reduction in the fs-sets, and showed that Roy and Maji's algorithm is not convenient in general cases. Naim Çağman, Filiz Çıtak and Serdar Enginoğlu we give definition of fuzzy parameterized fuzzy soft (fpfs) sets and their operations. We then define fpfs-aggregation operator to form fpfs-decision making method that allows constructing more efficient decision processes.

Maji et al. presented the concept of the intuitionistic fuzzy soft set theory by combining the intuitionistic fuzzy set with the soft set. Possibility intuitionistic fuzzy soft set and its operations are introduced by Maruah Bashir, Abdul Razak Salleh and Shawkat Alkhazaleh. An application of possibility intuitionistic fuzzy soft sets in decision making and a similarity measure of two possibility intuitionistic fuzzy soft sets and an application of this similarity measure in medical diagnosis had been shown by Maruah Bashir, Abdul Razak Salleh and Shawkat Alkhazaleh

Based soft set and neutrosophic sets a hybrid structure 'neutrosophic soft sets' had been initiated by PK Maji. Some neutrosophic soft set definitions, operations and some properties of this concept had been established by PK Maji . Maji introduced the concept of neutrosophic soft set. The parameters considered here are neutrosophic in nature.

Imposing the weights on the parameters (may be in a particular parameter also) aweighted neutrosophic soft sets had been introduced by Pabitra Kumar Maji.

PRELIMINARIES

DEFINITION 1.1

If X is a collection of objects denoted generically by x, then a fuzzy set Ã in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

$\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in Ã that maps X to the membership space M (When M contains only the two points 0 and 1, Ã is non fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a nonfuzzy set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

DEFINITION 1.2

An intuitionistic fuzzy set A in U is given by

$$A = \{\langle u, \mu_A(u), \vartheta_A(u)\rangle \mid u \in U\}$$

Where $\mu_A : U \to [0,1]\,, \vartheta_A : U \to [0,1]$

And $0 \le \mu_A(u) + \vartheta_A(u) \le 1 \; \forall u \in U$

For each u, the numbers $\mu_A(u)$ and $\vartheta_A(u)$ are the degree of membership and degree of non membership of u to A, respectively.

DEFINITION 1.3

A neutrosophic set A on the universe of discourse X is defined as A = $\{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$, where T, I, F: X → $]-0, 1+[$ and $-0 \le T_A(x) + I_A(x) + F_A(x) \le 3+$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]-0, 1+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]-0, 1+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0,1].

DEFINITION 1.4

A neutrosophic set A is contained in another neutrosophic set B i.e. A ⊆ B if $\forall x \in$ X , $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

EXAMPLE 1.5

Assume that the universe of discourse X = $\{x_1, x_2, x_3\}$, where x_1 characterises the capability, x_2 characterises the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a Neutrosophic Set (NS) of X, such that,

A = $\{< x_1, 0.4, 0.5, 0.3 >, < x_2, 0.7, 0.2, 0.4 >, < x_3, 0.8, 0.3, 0.4 >\}$,

where the degree of goodness of capability is 0.4, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.3 etc.

DEFINITION 1.6

Let U be an initial universe set and E be a set of parameters or attributes with respect to U. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

EXAMPLE 1.7

Assume that U = $\{h_1, h_2, h_3, h_4, h_5, h_6\}$ be a universal set consisting of a set of six houses under consideration, E = $\{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters with respect to U, where each parameter e_i, i = 1, 2, ⋯, 5 stands for 'expansive', 'beautiful', 'cheap', 'modern', 'wooden', respectively and A = $\{e_1, e_2, e_3\} \subset$ E. Suppose a soft set (F, A) describes the attractions of the houses, such that F(e_1) = $\{h_2, h_4\}$, F(e_2) = $\{h_1, h_3, h_5\}$ and F(e_3) = $\{h_3, h_4, h_5\}$. Then the soft set (F, A) is a parameterized family $\{F(e_i) : i = 1, 2, 3\}$ of subset of U defined as

(F, A) = $\{F(e_1), F(e_2), F(e_3)\}$, i.e., (F, A) = $\{\{h_2, h_4\}, \{h_1, h_3, h_5\}, \{h_3, h_4, h_5\}\}$.

The soft set (F, A) can also be represented as a set of ordered pairs as follows:

(F, A) = $\{(e_1, F(e_1)), (e_2, F(e_2)), (e_3, F(e_3))\}$ i.e.,

(F, A) = $\{(e_1, \{h_2, h_4\}), (e_2, \{h_1, h_3, h_5\}), (e_3, \{h_3, h_4, h_5\})\}$

other notations for (F, A) are F_A or (F_A, E).

NEUTROSOPHIC SOFT SET

DEFINITION 1.8

Let U be an initial universe set and E be a set of parameters. Consider A ⊂ E. Let P(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F: A → P(U). For illustration we consider an example.

EXAMPLE 1.9

Let U be the universal set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider E = { beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive }. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by, U = $\{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters A = $\{e_1, e_2, e_3, e_4\}$, where e_1 stands for the parameter 'beautiful', e_2 stands for the parameter 'wooden', e_3 stands for the parameter 'costly'' and the parameter e_4 stands for 'moderate'. Suppose that,

F (beautiful) = $\{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$,

F (wooden) = $\{< h_1, 0.6, 0.3, 0.5 >, < h_2, 0.7, 0.4, 0.3 >, < h_3, 0.8, 0.1, 0.2 >, < h_4, 0.7, 0.1, 0.3 >, < h_5, 0.8, 0.3, 0.6 >\}$,

F (costly) = $\{< h_1, 0.7, 0.4, 0.3 >, < h_2, 0.6, 0.7, 0.2 >, < h_3, 0.7, 0.2, 0.5 >, < h_4, 0.5, 0.2, 0.6 >, < h_5, 0.7, 0.3, 0.4 >\}$,

F (moderate) = $\{< h_1, 0.8, 0.6, 0.4 >, < h_2, 0.7, 0.9, 0.6 >, < h_3, 0.7, 0.6, 0.4 >, < h_4, 0.7, 0.8, 0.6 >, < h_5, 0.9, 0.5, 0.7 >\}$.;

The neutrosophic soft set (NSS) (F, E) is a parametrized family $\{F(e_i),\ i = 1 \cdots 10\}$ of all neutrosophic sets of U and describes a collection of approximation of an object. The mapping F here is 'houses (.)', where dot (.) is to be filled up by a parameter e ∈ E. Therefore, F(e_1) means 'houses(beautiful)' whose functional-value is the neutrosophic set

$\{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$

Thus we can view the neutrosophic soft set (NSS) (F, A) as a collection of approximation as below:

(F, A) = beautiful houses = $\{< h_1, 0.5, 0.6, 0.3 \; > , < h_2, 0.4, 0.7, 0.6 >, h_3, 0.6, 0.2, 0.3 >$ $, < h_4, 0.7, \; 0.3, 0.2 > , < h_5, 0.8, 0.2, 0.3 >\}$,

wooden houses= $\{< h_1, 0.6, 0.3, 0.5 > , < \; h_2, 0.7, 0.4, 0.3 > , < h_3, 0.8, 0.1, 0.2 >, \; < h_4, 0.7,$ $0.1, 0.3 > , < h_5, 0.8, 0.3, 0.6 >\}$,

costly houses = $\{ \; < h_1, 0.8, 0.6, 0.4 \; > , < h_2, 0.6, \; 0.7, 0.2 \; > , < h_3, 0.7, 0.2, 0.5 \; > , < h_4, 0.5,$ $0.2, 0.6 > , \; < h_5, 0.7, 0.3, 0.4 > \; \}$,

moderate houses= $\{ \; < h_1, 0.7, 0.4, 0.3 \; >, < h_2, 0.7, 0.9, 0.6 \; > , < h_3, 0.7, 0.6, 0.4 > , <$ $h_4, 0.7, 0.8, 0.6 > , < h_5, 0.9, 0.5, 0.7 > \; \}$.

where each approximation has two parts:

(i) A predicate p

(ii) An approximate value-set v (or simply to be called value-set v).

FOR EXAMPLE,

For the approximation ‘beautiful houses = $\{ \; < h_1, 0.5, 0.6, 0.3 \; > , < h_2, 0.4, 0.7, 0.6 > ,$ $< h_3, 0.6, 0.2, 0.3 > , < h_4, 0.7, \; 0.3, 0.2 > , \; < h_5, 0.8, 0.2, 0.3 > \; \}$,

we have (i) the predicate name `beautiful houses', and (ii) the approximate value-set is $\{ \; < h_1, 0.5, 0.6, 0.3 \; > , \; < h_2, 0.4, 0.7, 0.6 > , \; < h_3, 0.6, 0.2, 0.3 >, \; < h_4, 0.7, \; 0.3, 0.2 > , \; <$ $h_5, 0.8, 0.2, 0.3 > \; \}$.

Thus, a neutrosophic soft set (F, E) can be viewed as a collection of approximation like (F, E) = $\{p_1 = \; v_1 \; , p_2 = v_2, \cdots \; p_{10} = \; v_{10}\}$. For the purpose of storing a neutrosophic soft set in a computer, we could represent it in the form of a table as shown below (corresponding to the neutrosophic soft set in the above Example).

In this table, the entries are c_{ij} corresponding to the house h_i and the parameter e_j, where c_{ij} = (true-membership value of h_i, indeterminacy-membership value of h_i, falsity-membership value of h_i) in F(e_j). The tabular representation of the neutrosophic soft set (F, A) is as follow:

U	beautiful	wooden	costly	moderate
$\boldsymbol{h_1}$	(0.5, 0.6, 0.3)	(0.6, 0.3, 0.5)	(0.7, 0.4, 0.3)	(0.8, 0.6, 0.4)
$\boldsymbol{h_2}$	(0.4, 0.7, 0.6)	(0.7, 0.4, 0.3)	(0.6, 0.7, 0.2)	(0.7, 0.9, 0.6)
$\boldsymbol{h_3}$	(0.6, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.5)	(0.7, 0.6, 0.4)
$\boldsymbol{h_4}$	(0.7, 0.3, 0.2)	(0.7, 0.1, 0.3)	(0.5, 0.2, 0.6)	(0.7, 0.8, 0.6)
$\boldsymbol{h_5}$	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.6)	(0.7, 0.3, 0.4)	(0.9, 0.5, 0.7)

Table 1: Tabular form of the NSS (F, A).

DEFINITION 1.10

The class of all value sets of a neutrosophic soft set (F, E) is called value-class of the neutrosophic soft set and is denoted by $C_{(F,E)}$. For the above Example, $C_{(F,E)} = \{ v_1 , v_2 , \cdots \ v_{10} \}$. Clearly, $C_{(F,E)} \subset P(U)$.

DEFINITION 1.11

Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if A ⊂ B and

$$T_{F(e)}(x) \le T_{G(e)}(x), I_{F(e)}(x) \le I_{G(e)}(x) \ , F_{F(e)}(x) \ge F_{G(e)}(x) \ , \forall \, e \in A \, , x \ \in U.$$

We denote it by (F, A) ⊆ (G, B). (F, A) is said to be neutrosophic soft super set of (G, B) if (G, B)is a neutrosophic soft subset of (F, A). We denote it by (F, A) ⊇ (G, B).

EXAMPLE 1.12

Consider the two NSSs (F, A) and (G, B) over the common universe U = $\{\sigma_1 , \sigma_2 , \sigma_3 , \sigma_4 , \sigma_5\}$ .The NSS (F, A) describes the sizes of the objects whereas the NSS (G, B) describes its surface textures. Consider the tabular representation of the NSS (F, A) is as follows:

U	small	large	moderate
$\boldsymbol{\sigma_1}$	(0.4, 0.3, 0.6)	(0.6, 0.1, 0.7)	(0.5, 0.7, 0.5)
$\boldsymbol{\sigma_2}$	(0.3, 0.1, 0.4)	(0.6, 0.7, 0.8)	(0.6, 0.3, 0.6)
$\boldsymbol{\sigma_3}$	(0.6, 0.2, 0.7)	(0.3, 0.1, 0.6)	(0.5, 0.3, 0.8)
$\boldsymbol{\sigma_4}$	(0.7, 0.1, 0.6)	(0.1, 0.5, 0.7)	(0.7, 0.5, 0.7)
$\boldsymbol{\sigma_5}$	(0.3, 0.2, 0.4)	(0.6, 0.1, 0.6)	(0.3, 0.2, 0.3)

Table 2: Tabular form of the NSS (F, A).

The tabular representation of the NSS (G, B) is as follows:

U	Small	large	moderate	very smooth
$\boldsymbol{\sigma_1}$	(0.6, 0.4, 0.3)	(0.7, 0.2, 0.5)	(0.6, 0.7, 0.4)	(0.6, 0.8, 0.7)
$\boldsymbol{\sigma_2}$	(0.7, 0.5, 0.2)	(0.6, 0.7, 0.6)	(0.7, 0.3, 0.5)	(0.5, 0.7, 0.6)
$\boldsymbol{\sigma_3}$	(0.6, 0.3, 0.5)	(0.7, 0.2, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.9, 0.4)
$\boldsymbol{\sigma_4}$	(0.7, 0.1, 0.6)	(0.3, 0.6, 0.4)	(0.7, 0.5, 0.6)	(0.7, 0.3, 0.5)
$\boldsymbol{\sigma_5}$	(0.5, 0.4, 0.2)	(0.6, 0.6, 0.5)	(0.6, 0.4, 0.3)	(0.5, 0.8, 0.3)

Table 3: Tabular form of the NSS (G, B).

Here (F, A) ⊂ (G, B).

DEFINITION 1.13

Equality of two neutrosophic soft sets. Two NSSs (F, A) and (G, B) over the common universe U are said to be equal if (F, A) is neutrosophic soft subset of (G, B) and (G, B) is neutrosophic soft subset of (F, A). We denote it by (F, A) = (G, B).

DEFINITION 1.14

NOT set of a set of parameters. Let E ={e_1 , e_2,⋯ e_n} be a set of parameters. The NOT set of E is denoted by ˥E is defined by ˥ E = { ˥e_1 , ˥e_2 ,⋯ ˥e_n } where ˥ e_i = not e_i, $\forall i$ (it may be noted that ˥ and ˥ are different operators).

EXAMPLE 1.15

Consider the Example 1.9. Here $\urcorner E$ = { not beautiful, not wooden, not costly, not moderate }.

DEFINITION 1.16

Complement of a neutrosophic soft set. The complement of a neutrosophic soft set (F, A) denoted by (F, A)^c and is defined as (F, A)^c = (F^c, $\urcorner$A); where F^c : $\urcorner$A → P(U) is a mapping given by F$^c(\alpha)$ = neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}$, $I_{F^c(x)} = I_{F(x)}$ and $F_{F^c(x)} = T_{F(x)}$.

EXAMPLE 1.17

Consider the Example 1.9. Then (F, A)^c describes the 'not attractiveness of the houses'. We have

F (not beautiful) = { $< h_1, 0.3, 0.6, 0.5 >, < h_2, 0.6, 0.7, 0.4 >,$
$< h_3, 0.3, 0.2, 0.6 >, < h_4, 0.2, 0.3, 0.7 >, h_5, 0.3, 0.2, 0.8 >$ }

F (not wooden) = { $< h_1, 0.5, 0.3, 0.6 >, < h_2, 0.3, 0.4, 0.7 >,$
$< h_3, 0.2, 0.1, 0.8 >, < h_4, 0.3, 0.1, 0.7 >, h_5, 0.6, 0.3, 0.8 >$ }

F (not costly) = { $< h_1, 0.3, 0.4, 0.7 >, < h_2, 0.2, 0.7, 0.6 >,$
$< h_3, 0.5, 0.2, 0.7 >, < h_4, 0.6, 0.2, 0.5 >, h_5, 0.4, 0.3, 0.7 >$ }

F (not moderate) = { $< h_1, 0.4, 0.6, 0.8 >, < h_2, 0.6, 0.9, 0.7 >,$
$< h_3, 0.4, 0.6, 0.7 >, < h_4, 0.6, 0.8, 0.7 >, h_5, 0.7, 0.5, 0.9 >$ }.

DEFINITION 1.18

Empty or Null neutrosophic soft set with respect to a parameter. A neutrosophic soft set (H, A) over the universe U is termed to be empty or null neutrosophic soft set with respect to the parameter A if $T_{H(e)}(m) = 0$, $F_{H(e)}(m) = 0$ and $I_{H(e)}(m) = 0$, $\forall m \in U$, $\forall e \in A$ In this case the null neutrosophic soft set (NNSS) is denoted by Φ_A.

EXAMPLE 1.19

Let U = $\{h_1, h_2, h_3, h_4, h_5\}$ the set of five houses be considered as the universal set and A = { beautiful, wooden, in the green surroundings } be the set of parameters that characterizes the houses. Consider the neutrosophic soft set (H, A) which describes the cost of the houses and

H(beautiful) = { $< h_1, 0, 0, 0 >, < h_2, 0, 0, 0 >, < h_3, 0, 0, 0 >, < h_4, 0, 0, 0 >$
$, < h_5, 0, 0, 0 >$ }

H(wooden) = { $< h_1, 0, 0, 0 >, < h_2, 0, 0, 0 >, < h_3, 0, 0, 0 >, < h_4, 0, 0, 0 >$
$, < h_5, 0, 0, 0 >$ }

H(in the green surroundings) = $\{ < h_1, 0, 0, 0 >, < h_2, 0, 0, 0 >, \; , < h_3, 0, 0, 0 >$
$< h_4, 0.5, 0, 0 >, < h_5, 0, 0, 0 > \}$

Here the NSS (H, A) is the null neutrosophic soft set.

DEFINITION 1.20

Union of two neutrosophic soft sets. Let (H, A) and (G, B) be two NSSs over the common universe U. Then the union of (H, A) and (G, B) is denoted by '(H, A) ∪ (G,B)',and is defined by (H, A) ∪ (G, B) = (K, C), where C = A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$T_{k(e)}(m) = T_{H(e)}(m)$, if $e \in A - B$

$= T_{G(e)}(m)$, if $e \in B - A$

$= max\left(T_{H(e)}(m), T_{G(e)}(m)\right)$, if $e \in A \cap B$.

$I_{k(e)}(m) = I_{H(e)}(m)$, if $e \in A - B$

$= I_{G(e)}(m)$, if $e \in B - A$

$= \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}$, if $e \in A \cap B$.

$F_{k(e)}(m) = F_{H(e)}(m)$, if $e \in A - B$

$= F_{G(e)}(m)$, if $e \in B - A$

$= min\left(F_{H(e)}(m), F_{G(e)}(m)\right)$, if $e \in A \cap B$.

EXAMPLE 1.21

Let (H, A) and (G, B) be two NSSs over the common universe U. Consider the tabular representation of the NSS (H, A) is as follow:

U	beautiful	wooden	moderate
$\boldsymbol{h_1}$	(0.6, 0.3, 0.7)	(0.7, 0.3, 0.5)	(0.6, 0.4, 0.5)
$\boldsymbol{h_2}$	(0.5, 0.4, 0.5)	(0.6, 0.7, 0.3)	(0.6, 0.5, 0.4)
$\boldsymbol{h_3}$	(0.7, 0.4, 0.3)	(0.7, 0.3, 0.5)	(0.7, 0.4, 0.5)
$\boldsymbol{h_4}$	(0.8, 0.4, 0.7)	(0.6, 0.3, 0.6)	(0.7, 0.5, 0.6)
$\boldsymbol{h_5}$	(0.6, 0.7, 0.2)	(0.7, 0.3, 0.4)	(0.8, 0.6, 0.5)

Table 4: Tabular form of the NSS (H, A).

The tabular representation of the NSS (G, B) is as follow:

U	costly	moderate
$\boldsymbol{h_1}$	(0.7, 0.6, 0.6)	(0.7, 0.8, 0.6)
$\boldsymbol{h_2}$	(0.8, 0.4, 0.5)	(0.8, 0.8, 0.3)
$\boldsymbol{h_3}$	(0.7, 0.4, 0.6)	(0.5, 0.6, 0.7)
$\boldsymbol{h_4}$	(0.6, 0.3, 0.5)	(0.8, 0.5, 0.6)
$\boldsymbol{h_5}$	(0.8, 0.5, 0.4)	(0.6, 0.3, 0.5)

Table 5: Tabular form of the NSS (G, B).

Then the union of (H, A) and (G, B) is (K, C) whose tabular representation is as:

U	beautiful	wooden	moderate	costly
$\boldsymbol{h_1}$	(0.6, 0.3, 0.7)	(0.7, 0.3, 0.5)	(0.7, 0.6, 0.5)	(0.7, 0.6, 0.6)
$\boldsymbol{h_2}$	(0.5, 0.4, 0.5)	(0.6, 0.7, 0.3)	(0.8, 0.65, 0.3)	(0.8, 0.4, 0.5)
$\boldsymbol{h_3}$	(0.7, 0.4, 0.3)	(0.7, 0.3, 0.5)	(0.7, 0.5, 0.5)	(0.7, 0.4, 0.6)
$\boldsymbol{h_4}$	(0.8, 0.4, 0.7)	(0.6, 0.3, 0.6)	(0.8, 0.5, 0.6)	(0.6, 0.3, 0.5)
$\boldsymbol{h_5}$	(0.6, 0.7, 0.2)	(0.7, 0.3, 0.4)	(0.8, 0.45, 0.5)	(0.8, 0.5, 0.4)

Table 6: Tabular form of the NSS (K, C).

DEFINITION 1.22

Intersection of two neutrosophic soft sets. Let (H, A) and (G, B) be two NSSs over the same universe U. Then the intersection of (H, A) and (G, B) is denoted by (H, A) ∩ (G,B) and is defined by (H, A) ∩ (G, B) = (K, C), where C = A ∩ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$T_{k(e)}(m) = min\ (T_{H(e)}(m), T_{G(e)}(m))$,

$I_{k(e)}(m) = \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}$ and

$F_{k(e)}(m) = max\ (F_{H(e)}(m), F_{G(e)}(m)), \forall e \in C$.

EXAMPLE 1.23

Consider the above Example 1.21. Then that tabular representation of (H, A) ∩ (G, B) is as follow:

U	moderate
h_1	(0.6, 0.6, 0.6)
h_2	(0.6, 0.65, 0.4)
h_3	(0.5, 0.5, 0.7)
h_4	(0.7, 0.5, 0.6)
h_5	(0.6, 0.45, 0.5)

Table 7: Tabular form of the NSS (K, C).

For any two NSSs (H, A) and (G, B) over the same universe U and on the basis of the operations defined above, we have the following propositions:

PROPOSITION 1.24

(1) (H, A) ∪ (H, A) = (H, A).

(2) (H, A) ∪ (G, B) = (G, B) ∪ (H, A).

(3) (H, A) ∩ (H, A) = (H, A).

(4) (H, A) ∩ (G, B) = (G, B) ∩ (H, A).

(5) (H, A) ∪ $\boldsymbol{\Phi}$ = (H, A).

(6) (H, A) ∩ $\boldsymbol{\Phi} = \boldsymbol{\Phi}$.

(7) $[(\mathrm{H}, \mathrm{A})^{c}]^{c}$ = (H, A).

Proof. The proof of the Proposition (1.24) 1 to 7 are obvious.

For any three NSSs (H, A), (G, B) and (K, C) over the same universe U, we have the following propositions:

PROPOSITION 1.25

(1) (H, A) ∪ [(G, B) ∪ (K, C)] = [(H, A) ∪ (G, B)] ∪ (K, C)

(2) (H, A) ∩ [(G, B) ∩ (K, C)] = [(H, A) ∩ (G, B)] ∩ (K, C)

(3) (H, A) ∪ [(G, B) ∩ (G, B)] = [(H, A) ∪ (G, B)] ∩ [(H, A) ∪ (K, C)]

(4) (H, A) ∩ [(G, B) ∪ (K, C)] = [(H, A) ∩ (G, B)] ∪ [(H, A) ∩ (K, C)]

Proof. Proofs are simple and thus omitted.

DEFINITION 1.26

AND operation on two neutrosophic soft sets. Let (H, A) and (G, B) be two NSSs over the same universe U. Then 'AND' operation on them is denoted by '(H, A) ∧ (G, B)' and is defined by (H, A) ∧ (G, B) = (K, A × B), where the truth-membership, indeterminacy-membership and falsity-membership of (K, A × B) are as follows:

$$T_{K(\alpha,\beta)}(m) = min\left(T_{H(\alpha)}(m), T_{G(\beta)}(m)\right), I_{K(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m)+I_{G(\beta)}(m)}{2},$$

$$F_{K(\alpha,\beta)}(m) = max\left(F_{H(\alpha)}(m), F_{G(\beta)}(m)\right), \forall\, \alpha \in \mathrm{A}, \forall\, \beta \in \mathrm{B}.$$

EXAMPLE 1.27

Consider the same Example 1.21 above. Then the tabular representation of (H, A) AND (G, B) is as follow:

U	(beautiful, costly)	(beautiful, moderate)	(wooden, costly)
h_1	(0.6, 0.45, 0.7)	(0.6, 0.55, 0.7)	(0.7, 0.45, 0.6)
h_2	(0.5, 0.4, 0.5)	(0.5, 0.6, 0.5)	(0.6, 0.55, 0.5)
h_3	(0.7, 0.4, 0.6)	(0.5, 0.5, 0.7)	(0.7, 0.35, 0.6)
h_4	(0.6, 0.35, 0.7)	(0.8, 0.45, 0.7)	(0.6, 0.3, 0.6)
h_5	(0.6, 0.6, 0.4)	(0.6, 0.5, 0.5)	(0.7, 0.4, 0.4)

U	(wooden, moderate)	(moderate, costly)	(moderate, moderate)
h_1	(0.7, 0.55, 0.6)	(0.6, 0.5, 0.6)	(0.6, 0.6, 0.6)
h_2	(0.6, 0.75, 0.3)	(0.6, 0.45, 0.5)	(0.6, 0.65, 0.4)
h_3	(0.5, 0.45, 0.7)	(0.7, 0.4, 0.6)	(0.5, 0.5, 0.7)
h_4	(0.6, 0.4, 0.6)	(0.6, 0.4, 0.6)	(0.7, 0.5, 0.6)
h_5	(0.6, 0.3, 0.5)	(0.8, 0.55, 0.5)	(0.6, 0.45, 0.5)

Table 8: Tabular representation of the NSS (K, $A \times B$).

DEFINITION 1.28

If (F, A) and (G, B) be two NSSs over the common universe U then '(F, A) OR (G, B)' denoted by (F, A) ∨ (G, B) is defined by (F, A) ∨ (G, B) = (O, A × B), where, the truth-membership, indeterminacy-membership and falsity-membership of O(α, β) are given as follows:

$$T_{O(\alpha,\beta)}(m) = max\left(T_{H(\alpha)}(m), T_{G(\beta)}(m)\right),$$

$$I_{O(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m) + I_{G(\beta)}(m)}{2},$$

$$F_{O(\alpha,\beta)}(m) = min\left(F_{H(\alpha)}(m), F_{G(\beta)}(m)\right), \forall\ \alpha \in \text{A}, \forall\ \beta \in \text{B}.$$

EXAMPLE 1.29

Consider the same Example 1.21 above. Then the tabular representation of (H, A) OR (G, B) is as follow:

U	(beautiful, costly)	(beautiful, moderate)	(wooden, costly)
h_1	(0.7, 0.45, 0.6)	(0.7, 0.55, 0.6)	(0.7, 0.45, 0.5)
h_2	(0.8, 0.4, 0.5)	(0.8, 0.6, 0.3)	(0.8, 0.55, 0.3)
h_3	(0.7, 0.4, 0.3)	(0.7, 0.5, 0.3)	(0.7, 0.35, 0.5)
h_4	(0.8, 0.35, 0.5)	(0.8, 0.45, 0.6)	(0.6, 0.3, 0.5)
h_5	(0.8, 0.6, 0.2)	(0.8, 0.5, 0.2)	(0.8, 0.4, 0.4)

U	(wooden, moderate)	(moderate, costly)	(moderate, moderate)
h_1	(0.7, 0.55, 0.5)	(0.7, 0.5, 0.5)	(0.7, 0.6, 0.5)
h_2	(0.8, 0.75, 0.3)	(0.8, 0.45, 0.4)	(0.8, 0.65, 0.3)
h_3	(0.7, 0.45, 0.5)	(0.7, 0.4, 0.5)	(0.7, 0.5, 0.5)
h_4	(0.8, 0.4, 0.6)	(0.7, 0.4, 0.5)	(0.8, 0.5, 0.6)
h_5	(0.7, 0.3, 0.4)	(0.8, 0.55, 0.4)	(0.8, 0.45, 0.5)

Table 9: Tabular representation of the NSS (O, $A \times B$).

For any two NSSs (H, A) and (G, B) over the common universe U, the De Morgan's types of results are true.

CHAPTER-II

GENERALIZED POSSIBILITY NEUTROSOPHIC SOFT SETS

In this section, we first introduce a new type of set called priority weighted neutrosophic soft set (PWNSS), and further construct a priority weighted neutrosophic soft decision making method.

DEFINITION 2.1

Let U be an initial universe, E be a parameter set, N(U) be the collection of all neutrosophic sets of U and F(U) is collection of all fuzzy subset of U. A priority weighted neutrosophic soft set (PWNS-set) $P_{[\alpha,\beta]}$ over U is a set of triple defined by

$$P_{[\alpha,\beta]} = \left\{\left(e_{k,} \left\{\left(\frac{u_j}{P(e_k)(u_j)}, \alpha(e_k)(u_j)\right)\right\}, \beta(P(e_k))\right) : e_k \in E\right\}$$

or a mapping defined by $P_{[\alpha,\beta]} : E \rightarrow$ N(U) × F(U) × N(U), where, i ∈ $\wedge_1$ and k ∈ $\wedge_2$, P is a mapping given by P : E → N(U) and $\alpha\ (e_k)$ is a fuzzy set such that α : E → F(U).

For each parameter

$$e_k \in \text{E}, \text{P}(e_k) = \{\langle u_j, T_{P(e_k)}(u_j), I_{P(e_k)}(u_j), F_{P(e_k)}(u_j)\rangle : u_j \in U\}$$

Indicates neutrosophic value set of parameter e_k and where T, I, F are the truth, indeterminacy and falsity values respectively of the element $u_i \in$ U. For each $u_i \in$ U and $e_k \in$ E, $0 \leq T_{P(e_k)}(u_j) + I_{P(e_k)}(u_j) + F_{P(e_k)}(u_j) \leq 3$. Also α (e_k), degrees of priority given for the belongingness of elements of U in P(e_k) and $\beta(e_k)$ the weightage given to the parameters by the experts. So we can write

$$P_{[\alpha,\beta]}(e_k) = \left\{\left(\frac{u_1}{P(e_k)(u_1)}, \alpha(e_k)(u_1)\right)\left(\frac{u_2}{P(e_k)(u_2)}, \alpha(e_k)(u_2)\right), \ldots, \left(\frac{u_n}{P(e_k)(u_n)}, \alpha(e_k)(u_n)\right), \beta(e_k)\right\}$$

EXAMPLE 2.2

Let U = $\{u_{1,}u_{2,}u_3\}$ be a set of three restaurants. Let E =$\{e_{1,}e_{2,}e_3\}$ be a set of qualities where e_1=Taste, e_2 = Variety,e_3 =Service and let $\alpha : \mathrm{E} \rightarrow \mathrm{F(U)}$ and $\beta : \mathrm{E} \rightarrow \mathrm{N(U)}$. We can define a function $P_{[\alpha,\beta]} : E \rightarrow \mathrm{N(U)} \times \mathrm{F(U)} \times \mathrm{N(U)}$ as follow:

$$P_{[\alpha,\beta]}=$$

$$\left\{\begin{array}{l} P_{[\alpha,\beta]}(e_1) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.4}\,,\mathbf{0.1}\,,\mathbf{0.5})}\,,0.7\right),\left(\frac{u_2}{(\mathbf{0.6}\,,\mathbf{0.2}\,,\mathbf{0.4})}\,,0.3\right),\left(\frac{u_3}{(\mathbf{0.3}\,,\mathbf{0.4}\,,\mathbf{0.7})}\,,0.6\right)\right],(0.3,0.5,0.7)\right\} \\ P_{[\alpha,\beta]}(e_2) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.7}\,,\mathbf{0.3}\,,\mathbf{0.4})}\,,0.5\right),\left(\frac{u_2}{(\mathbf{0.4}\,,\mathbf{0.6}\,,\mathbf{0.1})}\,,0.7\right),\left(\frac{u_3}{(\mathbf{0.6}\,,\mathbf{0.2}\,,\mathbf{0.8})}\,,0.3\right)\right],(0.6,0.8,0.2)\right\} \\ P_{[\alpha,\beta]}(e_3) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.5}\,,\mathbf{0.6}\,,\mathbf{0.7})}\,,0.2\right),\left(\frac{u_2}{(\mathbf{0.4}\,,\mathbf{0.2}\,,\mathbf{0.6})}\,,0.5\right),\left(\frac{u_3}{(\mathbf{0.5}\,,\mathbf{0.4}\,,\mathbf{0.3})}\,,0.4\right)\right],(0.5,0.3,0.8)\right\} \end{array}\right\}$$

For the purpose of storing a PWNS in a computer, we can use matrix notation of priority weighted neutrosophic soft set $P_{[\alpha,\beta]}$. For Example, matrix notation of PWNS $P_{[\alpha,\beta]}$ can be written as follows: for m, n $\in \wedge$,

$$P_{[\alpha,\beta]} = \begin{pmatrix} (\langle 0.4\,,0.1\,,0.5\rangle\,,0.7)(\langle 0.6\,,0.2\,,0.4\rangle\,,0.3)(\langle 0.3\,,0.4\,,0.7\rangle\,,0.6)(0.3,0.5,0.7) \\ (\langle 0.7\,,0.3\,,0.4\rangle\,,0.5)(\langle 0.4\,,0.6\,,0.1\rangle\,,0.7)(\langle 0.6\,,0.2\,,0.8\rangle\,,0.3)(0.6,0.8,0.2) \\ (\langle 0.5\,,0.5\,,0.4\rangle\,,0.1)(\langle 0.4\,,0.2\,,0.6\rangle\,,0.5)(\langle 0.5\,,0.4\,,0.3\rangle\,,0.4)(0.5,0.3,0.8) \end{pmatrix}$$

where the m-th row vector shows P(e_m) and n-th column vector shows u_n

DEFINITION 2.3

Let $P_{[\alpha,\beta]}, Q_{[\gamma,\delta]} \in$ PWN (U, E). Then, $P_{[\alpha,\beta]}$ is said to be a priority weighted neutrosophic soft subset (PWNS-subset) of $Q_{[\gamma,\delta]}$, and denoted by $P_{[\alpha,\beta]} \sqsubseteq Q_{[\gamma,\delta]}$ if

1. α(e) and β(e) are a fuzzy subset of γ(e) and δ(e), for all e $\in$ E
2. P is a neutrosophic subset of.

EXAMPLE 2.4

Let U = $\{u_{1,}u_{2,}u_3\}$be a set of three broadband service, and let E =$\{e_{1,}e_{2,}e_3\}$ be a set of qualities where e_1= Stability, e_2= Security, e_3 = price. Let $P_{[\alpha,\beta]}$ be a PWNS-set define as follows:

$$P_{[\alpha,\beta]} = \begin{cases} P_{[\alpha,\beta]}(e_1) = \left\{\left[\left(\frac{u_1}{(0.4,0.7,0.6)},0.8\right),\left(\frac{u_2}{(\mathbf{0.8,0.6,0.8})},0.3\right),\left(\frac{u_3}{(\mathbf{0.4,0.5,0.8})},0.7\right)\right],(0.3,0.5,0.7)\right\} \\ P_{[\alpha,\beta]}(e_2) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.7,0.4,0.6})},0.6\right),\left(\frac{u_2}{(\mathbf{0.5,0.7,0.5})},0.7\right),\left(\frac{u_3}{(\mathbf{0.7,0.4,0.8})},0.5\right)\right],(0.6,0.8,0.2)\right\} \\ P_{[\alpha,\beta]}(e_3) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.5,0.6,0.7})},0.2\right),\left(\frac{u_2}{(\mathbf{0.6,0.5,0.8})},0.5\right),\left(\frac{u_3}{(\mathbf{0.6,0.5,0.7})},0.4\right)\right],(0.5,0.3,0.8)\right\} \end{cases}$$

Also we can define a function $Q_{[\gamma,\delta]}$: E ⟶ N(U) × F(U)× N(U) as follows:

$$Q_{[\gamma,\delta]} \begin{cases} Q_{[\gamma,\delta]}(e_1) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.4,0.7,0.6})},0.8\right),\left(\frac{u_2}{(\mathbf{0.8,0.6,0.8})},0.3\right),\left(\frac{u_3}{(\mathbf{0.4,0.5,0.8})},0.7\right)\right],(0.3,0.5,0.7)\right\} \\ Q_{[\gamma,\delta]}(e_2) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.8,0.6,0.5})},0.7\right),\left(\frac{u_2}{(\mathbf{0.6,0.8,0.3})},0.8\right),\left(\frac{u_3}{(\mathbf{0.8,0.5,0.6})},0.5\right)\right],(0.6,0.8,0.2)\right\} \\ Q_{[\gamma,\delta]}(e_3) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.6,0.7,0.5})},0.4\right),\left(\frac{u_2}{(\mathbf{0.7,0.6,0.7})},0.7\right),\left(\frac{u_3}{(\mathbf{0.8,0.6,0.4})},0.6\right)\right],(0.5,0.3,0.8)\right\} \end{cases}$$

It is clear that $P_{[\alpha,\beta]}$ is PWNS –subset of $Q_{[\gamma,\delta]}$.

DEFINITION 2.5

Let $P_{[\alpha,\beta]}, Q_{[\gamma,\delta]} \in$ PWN (U, E). Then, $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ are called priority weighted neutrosophic soft equal set and denote by $P_{[\alpha,\beta]} = Q_{[\gamma,\delta]}$ if $P_{[\alpha,\beta]} \subseteq Q_{[\gamma,\delta]}$ and $P_{[\alpha,\beta]} \supseteq Q_{[\gamma,\delta]}$.

DEFINITION 2.6

Let $P_{[\alpha,\beta]} \in$ PWN (U, E). Then, $P_{[\alpha,\beta]}$ is said to be priority weighted neutrosophic soft null set, denoted by $\phi_{[\alpha,\beta]}$, if $\forall e \in$, $\phi_{[\alpha,\beta]} : E \to$ N(U) × F(U) × N(U) such that $\phi_{[\alpha,\beta]}(e) = \left\{\left[\frac{u}{\phi(e)(u)}, \alpha(e)(u) : u \in U\right] \beta(P(e))\right\}$ where ϕ(e) $=\{\ \langle u, 0, 0, 1\rangle : u \in U\ \}$ and $\alpha(e) = \{(u, 0) : u \in U\}$ and $\beta(e) = \{(u, 0,0,1) : u \in U\}$.

DEFINITION 2.7

Let $P_{[\alpha,\beta]} \in$ PWN(U,E) .Then ,$P_{[\alpha,\beta]}$ is said to be priority weighted neutrosophic soft universal set, denoted by $U_{[\alpha,\beta]}$, if $\forall e \in E$, $U_{[\alpha,\beta]} : E \to$ N(U) × F(U) × N(U) such that $U_{[\alpha,\beta]}(e) = \left\{\left[\frac{u}{U(e)(u)}, \alpha(e)(u) : u \in U\right] \beta(e)\right\}$ where U(e) = $\{\langle u, 1, 1, 0\rangle : u \in U\}$ and $\alpha(e) = \{(u, 1) : u \in U\}$ and $\beta(e) = \{(u, 1,1,0) : u \in U\}$

PROPOSITION 2.8

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]}$ and $H_{[\theta,\phi]}$ ∈ PWN (U, E). Then,

1. $\phi_{[\alpha,\beta]} \subseteq P_{[\alpha,\beta]}$
2. $P_{[\alpha,\beta]} \subseteq U_{[\alpha,\beta]}$
3. $P_{[\alpha,\beta]} \subseteq \phi_{[\alpha,\beta]}$ and $\phi_{[\alpha,\beta]} \subseteq H_{[\theta,\phi]} \rightarrow P_{[\alpha,\beta]} \subseteq H_{[\theta,\phi]}$

***Proof* .** The proof follows from the Definitions (2.5) - (2.7)

DEFINITION 2.9

Let $P_{[\alpha,\beta]}$ ∈ PWN (U,E), where

$P_{[\alpha,\beta]}(e_k) = \{([P(e_k)(u_i), \alpha(e_k)(u_i)], \beta(\mathrm{P}(e_k))): e_k \in E, u_i \in U\}$

$P(e_k) = \{\langle u, T_{P(e_k)}(u_i), I_{P(e_k)}(u_i), F_{P(e_k)}(u_i)\rangle\} \ \forall e_k \in E, u \in U.$

Then for $e_k \in E$ and $u_i \in U$, and

1. $P^T_{[\alpha,\beta]}$ is said to be truth- membership part of $P_{[\alpha,\beta]}$

$P^T_{[\alpha,\beta]} = \{(P^T_{kj}(e_k), \alpha_{kj}(e_k))\}$

and $P^T_{kj}(e_k) = \{(u_j, T_{P(e_k)}(u_j))\}$, $\alpha_{kj}(e_k) = \{(u_j, \alpha(e_k)(u_j))\}$

2. $P^I_{[\alpha,\beta]}$ is said to be indeterminacy - membership part of $P_{[\alpha,\beta]}$

$P^I_{[\alpha,\beta]} = \{(P^I_{kj}(e_k), \alpha_{kj}(e_k))\}$

and $P^I_{kj}(e_k) = \{(u_j, I_{P(e_k)}(u_j))\}$, $\alpha_{kj}(e_k) = \{(u_j, \alpha(e_k)(u_j))\}$

3. $P^F_{[\alpha,\beta]}$ is said to be falsity- membership part of $P_{[\alpha,\beta]}$

$P^F_{[\alpha,\beta]} = \{(P^F_{kj}(e_k), \alpha_{kj}(e_k))\}$

and $P^F_{kj}(e_k) = \{(u_j, F_{P(e_k)}(u_j))\}$, $\alpha_{kj}(e_k) = \{(u_j, \alpha(e_k)(u_j))\}$

We can write a PWNS in form $P_{[\alpha,\beta]} = \left(P^T_{[\alpha,\beta]}, P^I_{[\alpha,\beta]}, P^F_{[\alpha,\beta]}\right)$.

A PWNS can be expressed in matrix form.

Let us consider priority weighted neutrosophic soft set $P_{[\alpha,\beta]}$ given in Example 2.4. Then priority weighted neutrosophic soft set $P_{[\alpha,\beta]}$ can be expressed in

matrix form as follows :

$$P^T_{[\alpha,\beta]} = \begin{pmatrix} (0.4, 0.8)(0.8, 0.3)(0.4, 0.7)(0.3) \\ (0.7, 0.6)(0.5, 0.7)(0.7, 0.5)(0.6) \\ (0.5, 0.2)(0.6, 0.5)(0.6, 0.4)(0.5) \end{pmatrix}$$

$$P^I_{[\alpha,\beta]} = \begin{pmatrix} (0.7, 0.8)(0.6, 0.3)(0.5, 0.7)(0.5) \\ (0.4, 0.6)(0.7, 0.7)(0.4, 0.5)(0.8) \\ (0.6, 0.2)(0.5, 0.5)(0.5, 0.4)(0.3) \end{pmatrix}$$

$$P^F_{[\alpha,\beta]} = \begin{pmatrix} (0.6, 0.8)(0.8, 0.3)(0.8, 0.7)(0.7) \\ (0.6, 0.6)(0.5, 0.7)(0.8, 0.5)(0.2) \\ (0.7, 0.2)(0.8, 0.5)(0.7, 0.4)(0.8) \end{pmatrix}$$

DEFINITION 2.10

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]} \in$ PWN (U, E). The union of two PWNSs $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ over U, denote by $P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}$ is defined by

$$P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]} = \left\{\left(e_k, \left\{\left(\rho, \alpha_{kj}(e_k) \oplus \gamma_{kj}(e_k)\right)\right\}, \beta_{kj}(P(e_k)) \oplus \delta_{kj}(e_k)\right) : e_k \in E\right\}$$

Where

$$\rho = \frac{u_j}{\left(P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k), P^I_{kj}(e_k) \oplus Q^I_{kj}(e_k), P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k)\right)}$$

and $\oplus$ represents n-conorm and $\otimes$ represents n-norm functions respectively.

DEFINITION 2.11

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]} \in$ PWN (U, E). The intersection of two PWNSs

$P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ over U , denote by $P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}$ is defined by

$$P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]} = \left\{\left(e_k, \left\{\left(\theta, \alpha_{kj}(e_k) \otimes \gamma_{kj}(e_k)\right)\right\}, \beta_{kj}(P(e_k)) \otimes \delta_{kj}(e_k)\right) : e_k \in E\right\}$$

Where

$$\theta = \frac{u_j}{\left(P_{kj}^T(e_k) \otimes Q_{kj}^T(e_k),\ P_{kj}^I(e_k) \otimes Q_{kj}^I(e_k),\ P_{kj}^F(e_k) \oplus Q_{kj}^F(e_k)\right)}$$

and $\oplus$ represents n-conorm and $\otimes$ represents n-norm functions respectively .

EXAMPLE 2.12

Let us consider the PWNSs $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ Defined as in 2.4. Let us suppose that n-norm is defined by a $\oplus$ b $= min\{a,b\}$ and the n-conorm is defined by a $\otimes$ b $= max\{a,b\}$ f(0.3,0.5,0.7)or a, b ∈ [0, 1]

$P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]} =$

$$\left\{\begin{array}{l}
\left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)(e_1) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.4,0.7,0.6})}, 0.8\right), \left(\frac{u_2}{(\mathbf{0.8,0.6,0.8})}, 0.3\right), \left(\frac{u_3}{(\mathbf{0.4,0.5,0.8})}, 0.7\right)\right], (0.3,0.5,0.7)\right\} \\
\left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)(e_2) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.7,0.4,0.5})}, 0.6\right), \left(\frac{u_2}{(\mathbf{0.5,0.7,0.3})}, 0.7\right), \left(\frac{u_3}{(\mathbf{0.8,0.5,0.6})}, 0.5\right)\right], (0.6,0.8,0.2)\right\} \\
\left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)(e_3) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.6,0.7,0.5})}, 0.4\right), \left(\frac{u_2}{(\mathbf{0.7,0.6,0.7})}, 0.7\right), \left(\frac{u_3}{(\mathbf{0.8,0.6,0.4})}, 0.6\right)\right], (0.5,0.3,0.8)\right\}
\end{array}\right\}$$

And

$P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]} =$

$$\left\{\begin{array}{l}
\left(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}\right)(e_1) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.4,0.7,0.6})}, 0.8\right), \left(\frac{u_2}{(\mathbf{0.8,0.6,0.8})}, 0.3\right), \left(\frac{u_3}{(\mathbf{0.4,0.5,0.8})}, 0.7\right)\right], (0.3,0.5,0.7)\right\} \\
\left(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}\right)(e_2) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.8,0.6,0.5})}, 0.7\right), \left(\frac{u_2}{(\mathbf{0.6,0.8,0.3})}, 0.8\right), \left(\frac{u_3}{(\mathbf{0.8,0.5,0.6})}, 0.5\right)\right], (0.6,0.8,0.2)\right\} \\
\left(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}\right)(e_3) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.6,0.7,0.5})}, 0.4\right), \left(\frac{u_2}{(\mathbf{0.7,0.6,0.7})}, 0.7\right), \left(\frac{u_3}{(\mathbf{0.8,0.6,0.4})}, 0.6\right)\right], (0.5,0.3,0.8)\right\}
\end{array}\right\}$$

PROPOSITION 2.13

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]}$ and $H_{[\theta,\phi]} \in$ PWN (U, E). Then,

1. $P_{[\alpha,\beta]} \cap \phi_\alpha = \phi_\alpha$ and $P_{[\alpha,\beta]} \cap U_\alpha = P_{[\alpha,\beta]}$
2. $P_{[\alpha,\beta]} \cup \phi = P_{[\alpha,\beta]}$ and $P_{[\alpha,\beta]} \cup U_\alpha = U_\alpha$
3. $P_{[\alpha,\beta]} \cap (Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}$ and $P_{[\alpha,\beta]} \cup (Q_{[\gamma,\delta]} \cup H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}) \cap H_{[\theta,\phi]}$
4. $P_{[\alpha,\beta]} \cap (Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}) \cup (P_{[\alpha,\beta]} \cap H_{[\theta,\phi]})$ and $P_{[\alpha,\beta]} \cup (Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}) \cap (P_{[\alpha,\beta]} \cup H_{[\theta,\phi]})$

$proof$. The proof can be obtained from Definitions 2.10 and 2.11

DEFINITION 2.14

Let $\mathrm{P}_{[\alpha,\beta]} \epsilon$ PWN (U, E). Complement of PWNS $\mathrm{P}_{[\alpha,\beta]}$, denoted by $P^{c}_{[\alpha,\beta]}$, is defined by

$$P^{c}_{[\alpha,\beta]} = \left\{\left(e, \left\{\left(\frac{u_j}{n(P(e_k))}, \mathrm{N}\left(\alpha_{kj}(e_k)(u_j)\right)\right), \mathrm{N}(\beta_{kj}(P(e_k))(u_j)\right\}\right), e\epsilon \mathrm{E}\right\}$$

Where,

$$\left(\sim(\mathrm{P}_{kj})(e_k)\right) = \left(\sim (P^{T}_{kj}(e_k)), \sim(P^{I}_{kj}(e_k)), \sim(P^{F}_{kj}(e_k))\right), \forall\ k\epsilon\wedge_1, j\epsilon\wedge_2.$$

EXAMPLE 2.15

Let us consider the PWNS $\mathrm{P}_{[\alpha,\beta]}$ define in Example 2.4. Suppose that the negation is defined by $\left(\sim P^{T}_{kj}(e_k)\right) = P^{F}_{kj}(e_k)$, $\sim(\mathrm{P}^{\mathrm{F}}_{\mathrm{kj}}(e_k)) = P^{T}_{kj}(e_k)$, $\sim (P^{I}_{kj}(e_k)) = 1- P^{I}_{kj}(e_k)$ and $\sim\left(\alpha_{ij}(e_k)\right) = 1- \alpha_{kj}(e_k)$ and $\sim(\beta^{T}_{ij}(e_k)) = \beta^{F}_{ij}(e_k)$ $\sim(\beta^{F}_{ij}(e_k)) = \beta^{T}_{ij}(e_k)$, $\sim(\beta^{I}_{ij}(e_k)) = 1-\beta^{I}_{ij}(e_k)$ respectively.

Then, $\sim\mathrm{P}_{[\alpha,\beta]}$ is defined as follow:

$\sim\mathrm{P}_{[\alpha,\beta]}$

$$= \begin{cases} \sim\mathrm{P}_{[\alpha,\beta]}(e_1) = \left\{\left[\left(\frac{u_1}{(0.6,0.3,0.4)}, 0.2\right), \left(\frac{u_2}{(0.8,0.6,0.8)}, 0.7\right), \left(\frac{u_3}{(0.8,0.5,0.4)}, 0.2\right)\right], (0.7,0.5,0.3)\right\} \\ \sim\mathrm{P}_{[\alpha,\beta]}(e_2) = \left\{\left[\left(\frac{u_1}{(0.6,0.6,0.7)}, 0.4\right), \left(\frac{u_2}{(0.5,0.3,0.5)}, 0.3\right), \left(\frac{u_3}{(0.8,0.6,0.7)}, 0.3\right)\right], (0.2,0.2,0.6)\right\} \\ \sim\mathrm{P}_{[\alpha,\beta]}(e_3) = \left\{\left[\left(\frac{u_1}{(0.7,0.4,0.5)}, 0.8\right), \left(\frac{u_2}{(0.8,0.6,0.6)}, 0.5\right), \left(\frac{u_3}{(0.7,0.5,0.6)}, 0.4\right)\right], (0.8,0.7,0.5)\right\} \end{cases}$$

PROPOSITION 2.16

Let $\mathrm{P}_{[\alpha,\beta]} \in \mathrm{PWN}$ (U, E). Then

1. $\sim\phi_{[\alpha,\beta]} = U_{[\alpha,\beta]}$
2. $\sim U_{[\alpha,\beta]} = \phi_{[\alpha,\beta]}$
3. $\sim(\sim\mathrm{P}_{[\alpha,\beta]} = \mathrm{P}_{[\alpha,\beta]}$.

Proof. It is clear from Definition 2.14.

PROPOSITION 2.17

Let $\mathrm{P}_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]} \in$ PW N(U,E). Then De Morgans law is valid.

1. $\sim(\mathrm{P}_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}) = \sim\mathrm{P}_{[\alpha,\beta]} \cap \sim Q_{[\gamma,\delta]}$.
2. $\sim(\mathrm{P}_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}) = \sim\mathrm{P}_{[\alpha,\beta]} \cup \sim Q_{[\gamma,\delta]}$.

Proof:

1. Let i, j ϵ A, $\sim\left(\mathrm{P}_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)$

$$= \sim\left\{\left(e_k, \left\{\left(\left(\frac{u_j}{\left(P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k),\ P^I_{kj}(e_k) \oplus Q^I_{kj}(e_k),\ P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k)\right)}{}', \alpha_{kj}(e_k) \oplus \gamma_{kj}(e_k)\right), \beta_{kj}(e_k) \oplus \delta_{kj}(e_k)\right\}\right) : u_j \in \mathrm{U}, e_k \in \mathrm{E}\right\}$$

$$= \Bigg\{\Bigg(e_k, \Bigg\{\Bigg(\frac{u_j}{\Big(P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k),\ \sim\big(\ P^I_{kj}(e_k) \oplus Q^I_{kj}(e_k)\big),\ P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k)\Big)}{}' \sim(\alpha_{kj}(e_k)$$

$$\oplus\, \gamma_{kj}(e_k))\ \Bigg), \sim\Big((\beta_{kj}(e_k) \oplus\ \delta_{kj}(e_k)\Big)\Bigg\}\Bigg): u_j \in \mathrm{U}, e_k \in \mathrm{E}\ \Bigg\}$$

$$= \Bigg\{\Bigg(e_k, \Bigg\{\Bigg(\frac{u_j}{\Big(P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k),\ \sim\big(P^I_{kj}(e_k)\big) \oplus \sim\big(Q^I_{kj}(e_k)\big), P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k)\Big)}{}' \sim(\alpha_{kj}(e_k))$$

$$\otimes\ \sim(\gamma_{kj}(e_k))\Bigg), \sim\Big((\beta_{kj}(e_k)\Big) \otimes \sim\Big(\ \delta_{kj}(e_k)\Big)\Bigg\}\Bigg): u_j \in \mathrm{U}, e_k \in \mathrm{E}\ \Bigg\}$$

$$= \Bigg\{\Bigg(e_k, \Bigg\{\Bigg(\frac{u_j}{\Big(P^F_{kj}(e_k),\ \sim\big(P^I_{kj}(e_k)\big),\ P^T_{kj}(e_k)\Big)}{}' \sim(\alpha_{kj}(e_k))\Bigg), \sim\Big(\beta_{kj}(e_k)\Big)\Bigg\}\Bigg): u_j \in \mathrm{U}, e_k \in \mathrm{E}\ \Bigg\}$$

$$= \Bigg\{\Bigg(e_k, \Bigg\{\Bigg(\frac{u_j}{\Big(P^T_{kj}(e_k), \big(P^I_{kj}(e_k)\big), P^F_{kj}(e_k)\Big)}{}'\ (\alpha_{kj}(e_k))\Bigg), \Big(\beta_{kj}(e_k)\Big)\Bigg\}\Bigg): u_j \in \mathrm{U}, e_k \in \mathrm{E}\Bigg\}^c$$

$$\cap \Bigg\{\Bigg(e_k, \Bigg\{\Bigg(\frac{u_j}{Q^T_{kj}(e_k),\ Q^I_{kj}(e_k),\ Q^F_{kj}(e_k)}{}'\ \gamma_{kj}(e_k)\ \Bigg), \Big(\delta_{kj}(e_k)\Big)\Bigg\}\Bigg): u_j \in \mathrm{U}, e_k \in \mathrm{E}\Bigg\}^c$$

$$= \sim P^c_{[\alpha,\beta]} \cap \sim Q^c_{[\gamma,\delta]}$$

2. The techniques used to prove (2) are similar to those used for (1), therefore we skip this proof.

DEFINITION 2.18

Let $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]} \in$ PW N(U, E). Then 'AND' product of PWNS set $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ denoted by $P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta]}$ id defined as follows:

$$P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta\ ]} =$$

$$\left\{\left((e_k, e_1), P_{kj}^{T}(e_k) \wedge g_{lj}^{T}(e_1), P_{kj}^{I}(e_k) \wedge g_{lj}^{I}(e_1), P_{kj}^{F}(e_k) \vee g_{lj}^{F}(e_1)), \alpha_{kj}(e_k) \wedge \gamma_{lj}(e_1)), \beta_{kj}(e_k) \wedge \delta_{lj}(e_1)\right) : (e_k, e_1) \in \mathrm{E} \times \mathrm{E}, \mathrm{j}, \mathrm{k}, \mathrm{l} \in \wedge\right\}$$

DEFINITION 2.19

Let $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]} \in$ PWN (U, E). Then 'OR' product of PWNS set $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ denoted by $P_{[\alpha,\beta]} \vee Q_{[\gamma,\delta]}$ id defined as follows:

$$P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta]} =$$

$$\left\{\left((e_k, e_1), P_{kj}^{T}(e_k) \vee Q_{lj}^{T}(e_1), P_{kj}^{I}(e_k) \vee Q_{lj}^{I}(e_1), P_{kj}^{F}(e_k) \wedge Q_{lj}^{F}(e_1)), \alpha_{kj}(e_k) \vee \gamma_{lj}(e_1)), \beta_{kj}(e_k) \vee \delta_{lj}(e_1)\right) : (e_k, e_1) \in \mathrm{E} \times \mathrm{E}, \mathrm{j}, \mathrm{k}, \mathrm{l} \in \wedge\right\}$$

CHAPTER 3

APPLICATION OF GENERALIZED POSSIBILITY NEUTROSOPHIC SOFT SET IN DECISION MAKING

DEFINITION 3.1

Let $Q_{[\gamma,\delta]}$,$L_{[\theta,\phi]} \epsilon$ PWN (U, E), $P_{[\alpha,\beta]} = Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}$ and $P^{T}_{[\alpha,\beta]}$, $P^{I}_{[\alpha,\beta]}$ and $P^{F}_{[\alpha,\beta]}$ be the truth ,indeterminacy and falsity matrices of $\wedge$ −product matrix, respectively. Then, weighted matrices of $\left(P_{[\alpha,\beta]}\right)^{T}$, $\left(P_{[\alpha,\beta]}\right)^{I}$ and $\left(P_{[\alpha,\beta]}\right)^{F}$ denoted $\wedge^{\mathrm{T}}$, $\wedge^{\mathrm{I}}$ and $\wedge^{\mathrm{F}}$ are defined as follows:

- $\wedge^{\mathrm{T}}(e_{kj}, u_r) = T_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}(u_r) + (\gamma_{kr}\ (e_k) \wedge \theta_{jr}(e_j)) - T_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}\ (u_r) \times (\alpha_{\mathrm{kr}}\ (e_k) \wedge (\gamma_{jr}(e_j)), \beta(P(e_k))$
- $\wedge^{\mathrm{I}}\ (e_{kj}, u_r) = I_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}(u_r) + (\gamma_{kr}\ (e_k) \wedge \theta_{jr}(e_j)) - I_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}(u_r) \times (\alpha_{\mathrm{kr}}\ (e_k) \wedge (\gamma_{jr}(e_j)), \beta(P(e_k))$
- $\wedge^{\mathrm{F}}\ (e_{kj}, u_r) = P_{Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}(e_{kj})}\ (u_r) \times (\gamma_{kr}(e_k) \wedge \theta_{jr}(e_j)), \beta(\mathrm{P}(e_k))$

DEFINITION 3.2

Let $Q_{[\gamma,\delta]}$, $L_{[\theta,\phi]} \epsilon$ PWN(U, E), $P_{[\alpha,\beta]} = Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}$ and let $\wedge^{\mathrm{T}}$, $\wedge^{\mathrm{I}}$ and $\wedge^{\mathrm{F}}$ be the weighted matrices of $P_{[\alpha,\beta]})^{T}$, $P_{[\alpha,\beta]})^{I}$ and $P_{[\alpha,\beta]})^{F}$ respectively. Then, in the weighted matrices $\wedge^{\mathrm{T}}$, $\wedge^{\mathrm{I}}$ and $\wedge^{\mathrm{F}}$ scores of $u_n \epsilon$ U denoted by $s^{T}(u_n)$, $s^{I}(u_n)$ and $s^{F}(u_n)$ are defined as follows:

$$s^{T}(u_n) = \textstyle\sum_{\mathrm{k,j}\, \epsilon \wedge}(\pi^{T}_{kj}\ (u_n) \times \beta(\mathrm{P}(e_k)))$$

$$s^{I}(u_n) = \textstyle\sum_{\mathrm{k,j}\, \epsilon \wedge}(\pi^{I}_{kj}\ (u_n) \times \beta(P(e_k)))$$

$$s^{F}(u_n) = \textstyle\sum_{\mathrm{k,j}\, \epsilon \wedge}(\pi^{F}_{kj}\ (u_n) \times \beta(P(e_k)))$$

Where $\pi^{T}_{kj}(u_m) = \begin{cases} \wedge^{\mathrm{T}}\ (e_{kJ}, u_n),\ \wedge^{\mathrm{T}}\ (e_{kJ}, u_n)\ =\ \max\{\ \wedge^{\mathrm{T}}\ (e_{kJ}, u_m) : u_m \epsilon\ \mathrm{U}\ \} \\ \qquad 0, \quad otherwise \end{cases}$

$$\pi^{I}_{kj}(u_m) = \begin{cases} \wedge^{I}(e_{kJ}, u_n), & \wedge^{I}(e_{kJ}, u_n) = \max\{\wedge^{I}(e_{kJ}, u_m) : u_m \epsilon \text{ U}\} \\ 0, & otherwise \end{cases}$$

$$\pi^{F}_{kj}(u_m) = \begin{cases} \vee^{\boldsymbol{F}}(e_{kJ}, u_n), & \vee^{\boldsymbol{F}}(e_{kJ}, u_n) = \max\{\vee^{F}(e_{kJ}, u_m) : u_m \epsilon \text{ U}\} \\ 0, & otherwise \end{cases}$$

DEFINITION 3.3

Let $s^T(u_n)$, $s^I(u_n)$ and $s^F(u_n)$be scores of $u_n \epsilon$ U in the weighted matrices $\wedge^{\mathrm{T},}$ $\wedge^{\mathrm{I}}$ and $\wedge^{\mathrm{F}}$.Then, decision score of $u_n \epsilon$ U, denoted by ds(u_n), is defined by ds(u_n) = $s^T(u_n) + s^I(u_n) - s^F(u_n)$. Now, we construct a PWNS –decision making method by the following algorithm:

ALGORITHM

Step 1: Input the PWNSs,

Step 2: Construct the ∧-product matrix,

Step3: Construct the truth, indeterminacy and falsity matrices of the ∧-product matrix,

Step4: Construct the weighted matrices $\wedge^T$, $\wedge^I$ and$\wedge^F$.

Step5: Compute score of $u_t \in U$,for each of the weighted matrices,

Step6: Compute decision score, for all $u_t \in U$,

Step7: The optimal decision is to select $u_t = maxds(u_i)$

Example

Assume that U = $\{C_{1,}C_{2,}C_3\}$ is a set of three colleges and E = $\{e_{1,}e_{2,}e_3\}$ = {Academics, Sports, Fine Arts} is a set of parameters which is best college among the three, α-represents priority given to the parameters by the college and β-represents the weightage given to the parameters by the experts.

Suppose that a committee wants to award a particular college.

Step 1: Based on the choice parameters, PWNSs $Q_{[\gamma,\delta]}$ and $L_{[\theta,\phi]}$, constructed by two experts are as follows:

$$Q_{[\gamma,\delta]} = \begin{cases} Q_{[\gamma,\delta]}(e_1) = \left\{\left(\frac{u_1}{(\mathbf{0.6,0.5,0.6})},0.7\right),\left(\frac{u_2}{(\mathbf{0.7,0.3,0.5})},0.3\right),\left(\frac{u_3}{(\mathbf{0.8,0.7,0.6})},0.5\right),(0.3,0.5,0.4)\right\} \\ Q_{[\gamma,\delta]}(e_2) = \left\{\left(\frac{u_1}{(\mathbf{0.4,0.3,0.6})},0.5\right),\left(\frac{u_2}{(\mathbf{0.8,0.9,0.4})},0.6\right),\left(\frac{u_3}{(\mathbf{0.3,0.5,0.5})},0.7\right),(0.7,0.3,0.6)\right\} \\ Q_{[\gamma,\delta]}(e_3) = \left\{\left(\frac{u_1}{(\mathbf{0.8,0.7,0.6})},0.5\right),\left(\frac{u_2}{(\mathbf{0.5,0.6,0.3})},0.4\right),\left(\frac{u_3}{(\mathbf{0.6,0.4,0.6})},0.3\right),(0.9,0.2,0.3)\right\} \end{cases}$$

also we can define a function $Q_{[\gamma,\delta]}$: E ⟶ N(U) × F(U) as follows:

$$L_{[\theta,\phi]} = \begin{cases} L_{[\theta,\phi]}(e_1) = \left\{\left(\frac{u_1}{(\mathbf{0.4,0.5,0.6})},0.7\right),\left(\frac{u_2}{(\mathbf{0.8,0.4,0.5})},0.3\right),\left(\frac{u_3}{(\mathbf{0.5,0.6,0.3})},0.5\right),(0.3,0.6,0.2)\right\} \\ L_{[\theta,\phi]}(e_2) = \left\{\left(\frac{u_1}{(\mathbf{0.5,0.7,0.6})},0.5\right),\left(\frac{u_2}{(\mathbf{0.3,0.6,0.4})},0.8\right),\left(\frac{u_3}{(\mathbf{0.5,0.7,0.3})},0.9\right),(o.8,0.1,0.5)\right\} \\ L_{[\theta,\phi]}(e_3) = \left\{\left(\frac{u_1}{(\mathbf{0.3,0.2,0.7})},0.8\right),\left(\frac{u_2}{(\mathbf{0.9,0.5,0.6})},0.4\right),\left(\frac{u_3}{(\mathbf{0.7,0.5,0.4})},0.5\right),(0.4,0.3,0.9)\right\} \end{cases}$$

Step 2: Let us consider PWNS ∧-product $P_{[\alpha,\beta]} = Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}$ which is the mapping ∧ **:** E × E ⟶ N(U) × F(U) × N(U) given as follows:

$$\left(\begin{array}{c|cccc} \wedge & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta \\ \hline e_{11} & (\langle 0.4,0.5,0.6\rangle,0.7) & (\langle 0.7,0.3,0.5\rangle,0.3) & (\langle 0.5,0.6,0.6\rangle,0.5) & (0.3,0.5,0.4) \\ e_{12} & (\langle 0.5,0.5,0.6\rangle,0.5) & (\langle 0.3,0.3,0.5\rangle,0.3) & (\langle 0.5,0.7,0.6\rangle,0.5) & (0.3,0.1,0.5) \\ e_{13} & (\langle 0.3,0.2,0.7\rangle,0.6) & (\langle 0.7,0.3,0.6\rangle,0.2) & (\langle 0.7,0.5,0.6\rangle,0.6) & (0.3,0.3,0.9) \\ e_{21} & (\langle 0.4,0.3,0.6\rangle,0.5) & (\langle 0.8,0.4,0.5\rangle,0.3) & (\langle 0.3,0.5,0.5\rangle,0.5) & (0.3,0.3,0.6) \\ e_{22} & (\langle 0.4,0.3,0.6\rangle,0.5) & (\langle 0.3,0.6,0.4\rangle,0.6) & (\langle 0.3,0.5,0.5\rangle,0.7) & (0.7,0.1,0.6) \\ e_{23} & (\langle 0.3,0.2,0.7\rangle,0.5) & (\langle 0.8,0.5,0.6\rangle,0.5) & (\langle 0.3,0.5,0.5\rangle,0.5) & (0.4,0.3,0.9) \\ e_{31} & (\langle 0.4,0.3,0.6\rangle,0.5) & (\langle 0.5,0.4,0.5\rangle,0.3) & (\langle 0.5,0.4,0.6\rangle,0.3) & (0.3,0.2,0.5) \\ e_{32} & (\langle 0.5,0.3,0.6\rangle,0.5) & (\langle 0.3,0.6,0.4\rangle,0.4) & (\langle 0.5,0.4,0.6\rangle,0.3) & (0.8,0.1,0.5) \\ e_{33} & (\langle 0.3,0.2,0.6\rangle,0.5) & (\langle 0.5,0.5,0.6\rangle,0.4) & (\langle 0.6,0.4,0.6\rangle,0.3) & (0.4,0.3,0.9) \end{array}\right)$$

Step 3: We construct matrices $P^T_{[\alpha,\beta]}$, $P^I_{[\alpha,\beta]}$ and $P^F_{[\alpha,\beta]}$ as follows:

$$\left(\begin{array}{c|cccc} \wedge & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta^T \\ \hline e_{11} & (0.4,0.7) & (0.7,0.3) & (0.5,0.5) & (0.3) \\ e_{12} & (0.5,0.5) & (0.3,0.3) & (0.5,0.5) & (0.3) \\ e_{13} & (0.3,0.7) & (0.7,0.3) & (0.7,0.6) & (0.3) \\ e_{21} & (0.4,0.5) & (0.8,0.3) & (0.3,0.5) & (0.3) \\ e_{22} & (0.4,0.5) & (0.3,0.6) & (0.3,0.7) & (0.7) \\ e_{23} & (0.3,0.5) & (0.8,0.5) & (0.3,0.5) & (0.4) \\ e_{31} & (0.4,0.5) & (0.5,0.3) & (0.5,0.3) & (0.3) \\ e_{32} & (0.5,0.5) & (0.4,0.4) & (0.5,0.3) & (0.8) \\ e_{33} & (0.3,0.5) & (0.5,0.4) & (0.6,0.3) & (0.4) \end{array}\right)$$

$$\left(\begin{array}{c|cccc}
\wedge & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta^I \\
\hline
e_{11} & (0.4,0.7) & (0.3,0.3) & (0.6,0.5) & (0.5) \\
e_{12} & (0.4,0.5) & (0.3,0.3) & (0.7,0.5) & (0.1) \\
e_{13} & (0.2,0.7) & (0.3,0.3) & (0.5,0.6) & (0.3) \\
e_{21} & (0.3,0.5) & (0.4,0.3) & (0.5,0.5) & (0.3) \\
e_{22} & (0.3,0.5) & (0.6,0.6) & (0.5,0.7) & (0.1) \\
e_{23} & (0.2,0.5) & (0.5,0.5) & (0.5,0.5) & (0.3) \\
e_{31} & (0.3,0.5) & (0.4,0.3) & (0.4,0.3) & (0.2) \\
e_{32} & (0.3,0.5) & (0.6,0.4) & (0.4,0.3) & (0.1) \\
e_{33} & (0.2,0.5) & (0.5,0.4) & (0.4,0.3) & (0.3)
\end{array}\right)$$

$$\left(\begin{array}{c|cccc}
\wedge & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta^F \\
\hline
e_{11} & (0.6,0.7) & (0.5,0.3) & (0.6,0.5) & (0.4) \\
e_{12} & (0.6,0.5) & (0.5,0.3) & (0.6,0.5) & (0.5) \\
e_{13} & (0.7,0.7) & (0.6,0.3) & (0.6,0.6) & (0.9) \\
e_{21} & (0.6,0.5) & (0.5,0.3) & (0.5,0.5) & (0.6) \\
e_{22} & (0.6,0.5) & (0.4,0.6) & (0.5,0.7) & (0.6) \\
e_{23} & (0.7,0.5) & (0.6,0.5) & (0.5,0.5) & (0.9) \\
e_{31} & (0.6,0.5) & (0.5,0.3) & (0.6,0.3) & (0.5) \\
e_{32} & (0.6,0.5) & (0.4,0.4) & (0.6,0.3) & (0.5) \\
e_{33} & (0.7,0.5) & (0.6,0.4) & (0.6,0.3) & (0.9)
\end{array}\right)$$

Matrix $P^F_{[\alpha,\beta]}$ of $\wedge$ -product

Step 4: We obtain weighted matrices $\wedge^T$, $\wedge^I$ and $\wedge^F$ by using Definition 2.21 as follws:

$$\left(\begin{array}{c|cccc}
\wedge^T & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta^T \\
\hline
e_{11} & \underline{0.82} & 0.79 & 0.75 & 0.3 \\
e_{12} & \underline{0.75} & 0.51 & 0.70 & 0.3 \\
e_{13} & 0.79 & 0.79 & \underline{0.88} & 0.3 \\
e_{21} & 0.70 & \underline{0.86} & 0.65 & 0.3 \\
e_{22} & 0.70 & 0.72 & \underline{0.79} & 0.7 \\
e_{23} & 0.65 & \underline{0.90} & 0.65 & 0.4 \\
e_{31} & \underline{0.70} & 0.65 & 0.65 & 0.3 \\
e_{32} & \underline{0.75} & 0.58 & 0.65 & 0.8 \\
e_{33} & 0.65 & 0.7 & \underline{0.72} & 0.4
\end{array}\right),$$

$$\left(\begin{array}{c|cccc} \wedge^I & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta^I \\ \hline e_{11} & \underline{0.85} & 0.51 & 0.80 & 0.5 \\ e_{12} & 0.70 & 0.51 & \underline{0.85} & 0.1 \\ e_{13} & 0.76 & 0.51 & \underline{0.80} & 0.3 \\ e_{21} & 0.65 & 0.58 & \underline{0.75} & 0.3 \\ e_{22} & 0.65 & 0.84 & \underline{0.85} & 0.1 \\ e_{23} & 0.60 & 0.75 & \underline{0.75} & 0.3 \\ e_{31} & \underline{0.65} & 0.58 & 058 & 0.2 \\ e_{32} & 0.65 & \underline{0.76} & 0.58 & 0.1 \\ e_{33} & 0.60 & \underline{0.70} & 0.58 & 0.3 \end{array}\right),$$

$$\left(\begin{array}{c|cccc} \wedge^F & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta^F \\ \hline e_{11} & \underline{0.42} & 0.15 & 0.15 & 0.4 \\ e_{12} & \underline{0.15} & 0.12 & \underline{0.15} & 0.5 \\ e_{13} & \underline{0.49} & 0.18 & 0.24 & 0.9 \\ e_{21} & 0.30 & 0.12 & \underline{0.15} & 0.6 \\ e_{22} & 0.15 & \underline{0.24} & 0.21 & 0.6 \\ e_{23} & 0.35 & 0.20 & 0.20 & 0.9 \\ e_{31} & 0.30 & 0.09 & 0.09 & 0.5 \\ e_{32} & \underline{0.15} & 0.12 & 0.09 & 0.5 \\ e_{33} & \underline{0.30} & 0.12 & 0.12 & 0.9 \end{array}\right)$$

Matrix $P^T_{[\alpha,\beta]}$, $P^I_{[\alpha,\beta]}$ and $P^F_{[\alpha,\beta]}$ from left to right, respectively

Step 5: For all $u \in U$, we find a scores by using Definition 2.22 as follows:

$s^T(u_1) = 1.281$, $s^T(u_2) = 0.618$, $s^T(u_3) = 1.105$

$s^I(u_1) = 0.555$, $s^I(u_2) = 0.286$, $s^I(u_3) = 0.86$

$s^F(u_1) = 1.599$, $s^F(u_3) = 0.144$, $s^F(u_3) = 0.075$

Step 6: For all $u \in U$, we find a scores by using Definition 2.22 as follows:

$ds(u_1) = 1.281 + 0.555 - 1.599 = 0.237$

$ds(u_2) = 0.618 + 0.286 - 0.44 = 0.464$

$ds(u_3) = 1.105 + 0.86 - 0.075 = 1.89$

Step 7: Then the optimal selection of the committee is the college

A NUMERICAL EXAMPLE

Assume that U = $\{C_{1,}C_{2,}C_3\}$ is a set of three colleges and E = $\{e_{1\,,}e_{2\,,}e_3\}$ = {Academics, Sports, Fine Arts} is a set of parameters which is best college among the three, α-represents priority given to the parameters by the college and β-represents the weightage given to the parameters by the experts.

Suppose that a committee wants to award a particular college.

Step 1: The PWNSs $Q_{[\gamma\,,\delta]}$ and $L_{[\theta,\phi]}$, constructed by the experts are:

$$Q_{[\gamma\,,\delta]}= \left\{\begin{matrix} Q_{[\gamma\,,\delta]}(e_1) = \left\{\left(\frac{u_1}{\mathbf{(0.6,0.5,0.6)}}\,,0.7\right),\left(\frac{u_2}{\mathbf{(0.7,0.3,0.5)}}\,,0.3\right),\left(\frac{u_3}{\mathbf{(0.8,0.7,0.6)}}\,,0.5\right)\,,0.9\right\} \\ Q_{[\gamma\,,\delta]}(e_2) = \left\{\left(\frac{u_1}{\mathbf{(0.4,0.3,0.6)}}\,,0.5\right),\left(\frac{u_2}{\mathbf{(0.8,0.9,0.4)}}\,,0.6\right),\left(\frac{u_3}{\mathbf{(0.3,0.5,0.5)}}\,,0.7\right)\,,0.8\right\} \\ Q_{[\gamma\,,\delta]}(e_3) = \left\{\left(\frac{u_1}{\mathbf{(0.8,0.7,0.6)}}\,,0.5\right),\left(\frac{u_2}{\mathbf{(0.5,0.6,0.3)}}\,,0.4\right),\left(\frac{u_3}{\mathbf{(0.6,0.4,0.6)}}\,,0.3\right)\,,0.7\right\} \end{matrix}\right\}$$

$$L_{[\theta,\phi]}= \left\{\begin{matrix} L_{[\theta,\phi]}(e_1) = \left\{\left(\frac{u_1}{\mathbf{(0.4,0.5,0.6)}}\,,0.7\right),\left(\frac{u_2}{\mathbf{(0.8,0.4,0.5)}}\,,0.3\right),\left(\frac{u_3}{\mathbf{(0.5,0.6,0.3)}}\,,0.5\right)\,,0.7\right\} \\ L_{[\theta,\phi]}(e_2) = \left\{\left(\frac{u_1}{\mathbf{(0.5,0.7,0.6)}}\,,0.5\right),\left(\frac{u_2}{\mathbf{(0.3,0.6,0.4)}}\,,0.8\right),\left(\frac{u_3}{\mathbf{(0.5,0.7,0.3)}}\,,0.9\right)\,,0.8\right\} \\ L_{[\theta,\phi]}(e_3) = \left\{\left(\frac{u_1}{\mathbf{(0.3,0.2,0.7)}}\,,0.8\right),\left(\frac{u_2}{\mathbf{(0.9,0.5,0.6)}}\,,0.4\right),\left(\frac{u_3}{\mathbf{(0.7,0.5,0.4)}}\,,0.5\right)\,,0.6\right\} \end{matrix}\right\}$$

Step 2: Let us consider PWNS ∧-product $P_{[\alpha,\beta]} = Q_{[\gamma\,,\delta]} \wedge L_{[\theta,\phi]}$ defined by ∧**:** E × E → N(U) × F(U) × F(U) given by:

$P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta\ ]} = D_G =$

$$\left(\begin{array}{c|cccc}
\wedge & u_1,\alpha & u_2,\alpha & u_3,\alpha & \beta \\
\hline
d_1 & (\langle 0.4,0.5,0.6\rangle, 0.7) & (\langle 0.7,0.3,0.5\rangle, 0.3) & (\langle 0.5,0.6,0.6\rangle, 0.5) & (0.7) \\
d_2 & (\langle 0.5,0.5,0.6\rangle, 0.5) & (\langle 0.3,0.3,0.5\rangle, 0.3) & (\langle 0.5,0.7,0.6\rangle, 0.5) & (0.8) \\
d_3 & (\langle 0.3,0.2,0.7\rangle, 0.6) & (\langle 0.7,0.3,0.6\rangle, 0.2) & (\langle 0.7,0.5,0.6\rangle, 0.6) & (0.6) \\
d_4 & (\langle 0.4,0.3,0.6\rangle, 0.5) & (\langle 0.8,0.4,0.5\rangle, 0.3) & (\langle 0.3,0.5,0.5\rangle, 0.5) & (0.7) \\
d_5 & (\langle 0.4,0.3,0.6\rangle, 0.5) & (\langle 0.3,0.6,0.4\rangle, 0.6) & (\langle 0.3,0.5,0.5\rangle, 0.7) & (0.8) \\
d_6 & (\langle 0.3,0.2,0.7\rangle, 0.5) & (\langle 0.8,0.5,0.6\rangle, 0.5) & (\langle 0.3,0.5,0.5\rangle, 0.5) & (0.6) \\
d_7 & (\langle 0.4,0.3,0.6\rangle, 0.5) & (\langle 0.5,0.4,0.5\rangle, 0.3) & (\langle 0.5,0.4,0.6\rangle, 0.3) & (0.7) \\
d_8 & (\langle 0.5,0.3,0.6\rangle, 0.5) & (\langle 0.3,0.6,0.4\rangle, 0.4) & (\langle 0.5,0.4,0.6\rangle, 0.3) & (0.7) \\
d_9 & (\langle 0.3,0.2,0.6\rangle, 0.5) & (\langle 0.5,0.5,0.6\rangle, 0.4) & (\langle 0.6,0.4,0.6\rangle, 0.3) & (0.6)
\end{array}\right)$$

Step 3: Calculation of Entropy value of the parametric weights H_j (j = 1, 2, 3, 4,⋯,9) with respect to the j-th choice can be obtained by:

$H_1 = 0.9407$, $H_2 = 0.9157$, $H_3 = 0.9072$, $H_4 = 0.9351$, $H_5 = 0.9242$, $H_6 = 0.94$,
$H_7 = 0.9239$, $H_8 = 0.9202$, $H_9 = 0.9316$.

The normalized entropy weights are determined as :

$w_1 = 0.0896$, $w_2 = 0.1274$, $w_3 = 0.1403$, $w_4 = 0.0981$, $w_5 = 0.1145$, $w_6 = 0.0907$,
$w_7 = 0.1150$, $w_8 = 0.1206$, $w_9 = 0.1034$,
where $\sum_{j-1}^{9} w_j = 1$.

Step 4: Formulating the weighted decision

D_G^w	$\mathbf{u_1}$	$\mathbf{u_2}$	$\mathbf{u_3}$
$\mathbf{d_{11}} \times \mathbf{w_1}$	(0.0358, 0.0448, 0.0537)	(0.0627, 0.0268, 0.0448)	(0.0448, 0.0537, 0.0537)
$\mathbf{d_{12}} \times \mathbf{w_2}$	(0.0637, 0.0637, 0.0764)	(0.0382, 0.0382, 0.0637)	(0.0637, 0.0892, 0.0764)
$\mathbf{d_{13}} \times \mathbf{w_3}$	(0.0421, 0.0280, 0.0982)	(0.0982, 0.0421, 0.0842)	(0.0982, 0.0701, 0.0842)
$\mathbf{d_{21}} \times \mathbf{w_4}$	(0.0392, 0.0294, 0.0588)	(0.0784, 0.0392, 0.0490)	(0.0294, 0.0490, 0.0490)

$\mathbf{d_{22}} \times \mathbf{w_5}$	(0.0458, 0.0343, 0.0687)	(0.0343, 0.0687, 0.0458)	(0.0343, 0.0572, 0.0572)
$\mathbf{d_{23}} \times \mathbf{w_6}$	(0.0272, 0.0181, 0.0635)	(0.0725, 0.0453, 0.0544)	(0.0272, 0.0453, 0.0453)
$\mathbf{d_{31}} \times \mathbf{w_7}$	(0.0460, 0.0345, 0.0690)	(0.0575, 0.0460, 0.0575)	(0.0575, 0.0460, 0.0690)
$\mathbf{d_{32}} \times \mathbf{w_8}$	(0.0603, 0.0362, 0.0724)	(0.0362, 0.724, 0.0482)	(0.0603, 0.0482, 0.0724)
$\mathbf{d_{33}} \times \mathbf{w_9}$	(0.0310, 0.0206, 0.0620)	(0.0517, 0.0517, 0.0620)	(0.0620, 0.0413, 0.0620)

weighted decision matrix

Step 5: Calculation of RPIS and RNIS

From the weighted decision matrix the RPIS (R_G^+) and RNIS (R_G^-)is attained as:

$$R_G^+ \quad \langle (0.0637, 0.0181, 0.0537);\ (0.0982, 0.0268, 0.0448);\ (0.0982, 0.0413, 0.0453) \rangle$$

$$R_G^- \quad \langle (0.0272, 0.0637, 0.0982);\ (0.0343, 0.0724, 0.0842);\ (0.0272, 0.0892, 0.0842) \rangle.$$

Step 6: Calculate the distance measure of the alternative from the RPIS and RNIS,

The distance measures of the alternative from the RPIS are established as:

$$D_{euc}^{1+} = 0.0424, \quad D_{euc}^{2+} = 0.0535, \ D_{euc}^{3+} = 0.0578$$

Likewise, the distance measures of the alternative from the RNIS are established as:

$$D_{euc}^{1-} = 0.0485, \quad D_{euc}^{2-} = 0.0520, \ D_{euc}^{3-} = 0.0550$$

Step 7: Computation of relative closeness co-efficient

Now compute the relative closeness co-efficient ρ_i^* , i = 1, 2, 3

ρ_1^* 0.5332 (0.4667), ρ_2^* 0.4926 (0.5073), ρ_3^* 0.4876 (0.5123)

Step 8: Ranking the alternatives

The ranking order of alternatives with respect to relative closeness coefficient is given as follows:

$C_3 > C_2 > C_1$

Therefore, C_3 is the best choice.

CONCLUSIONS

In this thesis, a new hybrid generalized possibility neutrosophic soft set is introduced and some of its basic properties are discussed. Further we constructed a decision making method based on generalized possibility soft set. Finally a numerical example is represented to depict the importance of the same.

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