Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be UniversityEstd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with 'A++' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

**Continuous Internal Assessment Test- I, February 2025**
**Semester - IV**

**Class : II PG** | **Time : 2 Hours**
**Major : Mathematics** | **Max.Marks : 60**

**23MMAC22 - Functional Analysis**

**Course Outcomes:**

On completion of course the students will able to

CO1: explain the fundamental concepts of functional analysis in applied contexts.
CO1: use the properties of Banach space.
CO1: Use the Hilbert space to construct orthonormal sets.
CO1: identify normal, self adjoint and unitary operators.
CO1: communicate the spectrum of bounded linear operator.

**Part –A** 6 x 1 = 6

**Choose the correct answer**

1. A Banach space is a ______________. CO1K1
   a. linear space b. normed linear space
   c. complete space d.complete normed linear space

2. In a normed linear space, which of the following is true? CO1K1
   a. $\left| \|x\| - \|y\| \right| \le \|x - y\|$ b. $\left| \|x\| + \|y\| \right| \le \|x - y\|$
   c. $\left| \|x\| - \|y\| \right| = \|x - y\|$ d. $\left| \|x\| - \|y\| \right| \ge \|x - y\|$

3. In a Banach algebra $B(N)$, which of the following is true? CO1K2
   a. $\|TT'\| = \|T\| \|T'\|$ b. $\|TT'\| \le \|T\| \|T'\|$
   c. $\|TT'\| \ge \|T\| \|T'\|$ d. $\|TT'\| > \|T\| \|T'\|$

4. A normed linear space (X, ||.||) is called complete if every________ CO2K1
   a. Cauchy sequence is convergent b. sequence is convergent
   c. convergent sequence is Cauchy d. Cauchy sequence is not convergent

5. If B and B′ are Banach spaces and T is a linear transformation of B into B′ then T(Sr) = _____ CO2K2
   a. T(S) b. $T(rS_1)$ c. $rT(S_1)$ d. both (b) and (c)

6. If P is a projection on B into itself, then _____ CO2K2
   a. $P^2 = P$ b. $P^{-1} = P$ c. $P^3 = P$ d. both (a) and (b)

## Part - B 3x6=18

**Answer any two questions**

**Each answer should not exceed 400 words or two pages**

7. a. If $N$ is a NLS and $x_0$ is a non-zero vector in $N$, then there exists a functional $f_0$ in $N^*$ such that $f_0(x_0) = \|x_0\| \text{ and } \|f_0\| = 1$. CO1K4

(or)

7. b. If $M$ is a closed linear subspace of a NLS $N$ and $x_0$ is a vector not in $M$, then there exists a functional $f_0$ in $N^*$ such that $f_0(M) = 0 \text{ and } f_0(x_0) \neq 0$. CO1K4

8. a. State and prove the Hahn-Banach theorem. CO1K3

(or)

8.b . State and prove the open mapping theorem. CO2K3

9.a. State and prove the closed Graph theorem. CO2K3

(or)

9. b. A one-to-one continuous linear transformation of one Banach space onto another is a homeomorphism. In particular, if a one-to-one linear transformation $T$ of a Banach space onto itself is continuous, then its inverse $T^{-1}$ is automatically continuous. CO2K3

## Part - C 3 x12 =36

**Answer any one question**

**Each answer should not exceed 800 words or four pages**

10. a. Let $M$ be a closed linear subspace of a normed linear space $N$. If the norm of a coset $x + M$ in the quotient space $N/M$ is defined by, CO1K5

$$\|x + M\| = \inf\{\|x + m\| : m \in M\},$$

Then prove that $N/M$ is a normed linear space. Further, if $N$ is a Banach space, then so is $N/M$.

(or)

10. b. Let $M$ be a linear subspace of a NLS $N$, and let $f$ be a functional defined on $M$. If $x_0$ is a vector not in $M$, and if $M_0 = M + [x_0]$ is the linear subspace spanned by $M$ and $x_0$, then $f$ can be extended to a functional $f_0$ defined on $M_0$ such that $\|f_0\| = \|f\|$. CO1K5

11. a. If $N$ and $N'$ are NLSs, then the set $B(N, N')$ of all continuous LTs of $N$ into $N'$ is itself a NLS with respect to the pointwise linear operations and the norm defined by $\|T\| = \sup\{\| T(x) \| : \|x\| \leq 1\}$. Further if $N'$ is a BS, then $B(N, N')$ is also a BS. CO1K5

(or)

11. b. If $B$ and $B'$ are Banach spaces, and if $T$ is a continuous linear transformation of $B$ onto $B'$, then the image of each open sphere centered on the origin in $B$ contains an open sphere centered on the origin in $B'$. CO2K5

12. a. If $P$ is a projection on a Banach space $B$, and if $M$ and $N$ are its range and null space, then $M$ and $N$ are closed linear subspaces of $B$ such that $B = M \oplus N$. CO2K4

(or)

12. b. State and prove the uniform boundedness theorem. CO2K3

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