Certain Studies Relating to Life Test Sampling Plans for New Weibull-Pareto Distribution

Thesis submitted in

Partial Fulfilment of the

Degree of Master of Science (M.Sc.)

By

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(19PMA007)

Department of Mathematics

Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore – 641043

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Signature of the Supervisor **Signature of the Head of the Department**

CERTIFICATE

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I certify that the dissertation entitled "**Certain Studies Relating to Life Test Sampling Plans for New Weibull Pareto Distribution**" submitted for the degree of **Master of Science (M.Sc)** by **NANDHINI A C** is the record of work carried out by her during the period from December 2020 to May 2021 under my guidance and supervision, and that this work has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this University or any other similar Institution of Higher Learning.

Signature of the Supervisor **Signature of the Head of the Department**

DECLARATION

DECLARATION

I declare that the dissertation entitled "**Certain Studies Relating to Life Test Sampling Plans for New Weibull-Pareto Distribution**" submitted by me for the degree of **Master of Science (M.Sc.)** is the record of work carried out by me during the period from December 2020 to May 2021 under the guidance of **Dr. (Tmt.) A. R. SUDAMANI RAMASWAMY,** Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and has not formed the basis for the award of any Degree, Associateship, Fellowship, Titles in this University or any other similar Institution of Higher Learning.

A.C.Nandhini

Signature of the Candidate

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CONTENTS

ABSTRACT

Abstract

This dissertation is devoted to the study of acceptance sampling plans based on truncated life tests assuming that the life time of the items follows New Weibull-Pareto Distribution.

The first chapter deals with basic concepts of quality control, acceptance sampling, reliability, life testing, notations and symbols.

The second chapter deals with New Weibull-Pareto Distribution in Acceptance Sampling Plans based on Truncated Life Tests. In this study, a new acceptance sampling plan based on truncated life test is proposed for a lifetime following a New Weibull-Pareto distribution (NWPD). For various acceptance numbers, confidence levels and values of the ratio of the fixed experiment time to the particular mean lifetime as well as the minimum sample sizes required to assert the specified mean life are found. The operating characteristic function values of the suggested sampling plans and producer's risk are presented. The important tables are presented and the results are explained by a discussing the results of numerical examples.

In the third chapter, special purpose double sampling plan DSP(0,1) for truncated life tests assuming that lifetime of the test units follows New Weibull-Pareto Distribution is developed. The minimum sample size required for ensuring the specified mean life at specified consumer's confidence level, the operating characteristics values for various quality levels are obtained and the results are discussed with the help of tables and examples.

In the fourth chapter, chain sampling plan for truncated life tests assuming that lifetime of the test units follows New Weibull-Pareto Distribution is developed. The minimum sample size required for ensuring the specified mean life at specified consumer's confidence level, the operating characteristics values for various quality levels are obtained. The results are discussed with the help of tables and examples.

CHAPTER 1

Chapter – 1

Introduction

“Quality is the degree of excellence at an acceptable price and the control of variability at an acceptable cost”

- Robert A. Broh

The survival of a company depends on the income it gets from selling its products and services, and the ability to sell is based on the fitness for use. Hence the company functions concerned with quality or achieving fitness for use are known as quality function. It includes a variety of activities. Every one working in the factory or all departments is responsible for the broad quality function. Inspection of raw material, semi finished products, or finished products is one aspect of quality assurance. Whenever a statistical technique is used to control, improve and maintain the quality, it is termed as Statistical Quality Control. When inspection is for the purpose of acceptance or rejection of a product, based on adherence to a standard, the type of procedure employed is usually called acceptance sampling. Acceptance sampling is one of the major components in the field of Statistical quality control. The probability that a device will function over a specified time period or amount of usage at a stated condition is termed as reliability. A typical application of reliability acceptance sampling is as follows:

A company receives a shipment of products from a vendor. This product is often a component or raw material used in the company’s manufacturing process. A sample is taken from the lot, and some quality characteristic of the units in the sample is inspected for a specified period of time. On the basis of the information in this sample, a decision is made regarding the lot’s disposition. Usually, this decision is either to accept or to reject the lot. Accepted lots are put into production, rejected lots may be returned to the vendor or may be subject to some other lot disposition action.

This chapter comprises of sections that consist of basic concepts.

1. Basic concepts of quality control
2. Basic concepts of acceptance sampling
3. Basic concepts of reliability
4. Lifetime distribution
5. Notations

Section - 1.1

Basic Concepts of Quality Control

1.1.1 Introduction

The quality control has become one of the most important tools to differentiate between competitive enterprises in a global business market. Quality control can be defined as the process through which we measure actual quality performance, compare it with the quality standard set and act on the difference in order to improve the product's quality. The reputation of companies depends upon the high reliability of their products. These companies compete with each other on the basis of quality and reliability. In order to control the quality of the purchased goods, two major alternatives are open to a buyer. One is the complete inspection, in which every single item in the lot is inspected and tested. This is often impractical, uneconomical or impossible. Secondly, the partial inspection in which a sample of the item is taken for inspection and testing and the whole lot is accepted or rejected depending on whether a few or many defective items are found in the sample.

Quality control is the use of techniques and activities to achieve, sustain, and improve the quality of a product or service. It involves integrating the following related techniques and activities such as specifications of what is needed, design of the product or service to meet the specifications, production or installation to meet the full intent of the specifications, inspection to determine conformance to specifications and review of usage to provide information for the revision of specifications, if needed. Quality control and management is not very complex at this time, but the craftsman's design of the product to meet the best of his customer's need is very important. The importance of quality control lies in the following benefits likely to be realized by an effective quality assurance program.

- Improving the quality of product and services
- Increasing productivity
- Reducing tangible and intangible costs
- Reducing production and delivery lead times
- Improving the marketability of products and services
- Reducing the prices of products and services to customers
- Detecting the assignable causes and elevating its effects

1.1.2 Quality Control

Quality Control may generally be defined as a system that is used to maintain a desired level of quality in a product or service. This task may be achieved through different measures such as planning, design, use of proper equipment and procedures, inspection, and taking corrective action in case a deviation is observed between the product, service or process output and a specified standard. This general area may be divided into the three main sub areas namely, off-line quality control, statistical process control and acceptance sampling plans.

(i) An off-line quality control procedures deal with measures to select and choose controllable product and process parameters in such a way that the deviation between the product or process output and the standard will be minimized.

(ii) Statistical Process Control involves comparing the output of a product or service with a standard and taking remedial actions in case of a discrepancy between the two. It also involves determining whether a process can produce a product that meets desired specification or requirements.

(iii) Acceptance Sampling deals with inspection of the product or service. When 100 percent inspection of all items is not feasible, a decision has to be made on how many items should be sampled or whether the batch should be sampled at all. The information obtained from the sample is used to decide whether to accept or reject the entire batch or not. A plan that determines the number of items to sample and the acceptance criteria of the lot, based on meeting certain stipulated conditions is known as an acceptance sampling plan.

1.1.3 Quality Improvement

Quality improvement is defined as the reduction of variability in process and product where variations are measured by statistical methods. An efficient quality improvement program can be instrumental in increasing productivity at a reduced cost. As a result of increasing customer quality requirements and development of new product technology, many existing quality assurance practices and techniques need modifications. The need for statistical and analytical techniques in quality assurance is rapidly increasing owing to stiff competition in the industry. To improve the quality of products and services, it is customary to modernize the quality practices and simultaneously reduce the cost of quality. The three important factors that affect the quality of a product are

- Quality of design
- Quality of conformance
- Quality of performance

Quality of Design

The quality of design of a product is concerned with the tightness of the specifications for manufacture of the product. A good quality of design must ensure consistent performance over its stipulated life span stated in terms of rated output, efficiency and overload capacity, continued or intermittent operation for specified application or service.

It should also consider the possible modes of failure due to stress, wear, distortion, corrosion, shocks, vibrations, high or low temperature, altitude or pressure, environmental conditions etc. However product design and development is a continuous process which results into evaluation of a product, based on assessed user needs, their feedback after use and development in technology at a given point of time in a given environment.

Factors Controlling Quality of Design

- The most important factor that controls the quality of design is the type of customers in the market. This can be analyzed in detail with the help of a market survey. A market survey is the study of three main factors; the first and particularly important one is the consuming habits of people. Secondly it is the prices they are willing to pay for various products and services. Third is the choice of design of the product which means the needs of the customers. Thus the quality of the design depends upon the type of customers to provide the intended function with the greatest overall economy.

- In case of capital goods, the decision is usually governed by such considerations as environmental conditions, reliability, importance of continuity of service, maintainability, etc.

- Profit is an important factor for the producers. Thus considering profit as a factor is very essential for a company to succeed monetarily. It is very difficult for a company to give 100 percent quality products and in quality control 100 percent quality from a company is also not essential. Instead, a market segment to which the management desires to cater should be considered. Profit can be maximized by producing products in different grades to suit different types of customers.

- Environmental conditions also play an important role in deciding the quality of a design. A high end car model that can perform extremely well in a normal temperature and normal conditions cannot give the same performance in different temperatures and conditions.
- Another factor is the special importance of the product. Greater the requirements for strength, fatigue resistance, life interchangeability of the manufacture of an item, closer should be the tolerances to give better quality goods.

Quality of Conformance

The quality of conformance means comparing the manufactured product with the quality design to see how well the manufactured product conforms to the quality of design. It is responsible for the production, planning and manufacturing to obtain a high level of quality of conformity when a design is established.

Factors Controlling Quality of Conformance

For good quality of conformance with the design, an organization should ensure the following.

- Incoming raw materials are adequate
- Machines/tools for the job and the measuring instruments are adequate
- Proper selection of the process and process control operators should be well trained, experienced and motivated
- Proper storage for finished goods
- Proper inspection program
- Obtaining feedback from both the internal inspection and the customers for taking corrective action

- Quality control techniques should be used to control variability in manufacturing process
- Higher quality of design usually costs more, higher quality of conformance

Usually costs less, by reducing the number of defective products produced

Quality of Performance

The quality of performance is concerned with how well the manufactured product gives its performance. It depends on both quality of design and conformance. Even in a best designed product, a poor conformance control can cause poor performance; conversely the best performance control cannot make the product function correctly, if the design itself is not right.

1.1.4 Objectives of Quality Control

A proper quality control aims to improve the income of the producers by making the product more acceptable in the market. It tries to fulfill the consumer's needs by providing the product a long life, greater usefulness, aesthetic aspects, maintainability, etc.

- It tries to reduce the company's purchasing cost by reducing the losses due to defects.
- The aim of quality control is to achieve interchangeability of manufactured goods in large scale production.
- It helps to produce optimum quality at minimum price.
- It ensures satisfaction of customers with products or services of high quality level, to build customer's goodwill, confidence and reputation of manufacturer.
- It ensures quality control at proper stages by proper inspection to make sure that non-defective products are produced.
- It judges the conformity of the process to the established standards and takes suitable action when there are deviations.
- It improves quality and productivity by process control, experimenatation and customer's feedback.
- It develops procedure for good vendor – vendee relations.

- It develops quality consciousness in the organization.

1.1.5 Cost of Quality

Every company has to satisfy the quality needs of their customers. The cost that a company spends for carrying out the quality functions is known as costs of quality. These include

- Market survey cost to discover the quality needs of the customers.
- Product research and development costs.
- Design costs of translating the product concept into information which permits planning for manufacture.
- Cost of manufacturing planning in order to meet required quality specifications.
- Cost of inspection and test.
- Cost of defect prevention.
- Cost of scrap, quality failures.
- Cost of quality assurance.
- Cost for field service and such other factors attributed to the quality improvement.

According to the American Society for Quality Control (1987), the quality costs is be defined in four categories.

Cost of Prevention – It consists of the cost associated with personnel engaged in designing, implementing, and maintaining the quality system. This includes the cost associated with creating overall quality planning, cost of preparation of manuals and procedures needed to communicate these plans, cost associated with implementing quality plans, cost for preparing training programs on quality performance, cost associated with preventing recurring defects, cost of the investigation, analysis and correction of causes of defects by quality control department and engineering department and finally the cost of cost consciousness programs.

Cost of Appraisal – The costs associated with the measuring, evaluating or auditing of products, components, and purchased materials to assure conformance with quality standards and performance requirements are called cost of appraisal. This includes the cost of receiving or incoming, laboratory acceptance testing, checking labor, set up for inspection and test materials, maintenance and calibration of test and inspection equipments, quality audits, review of test and inspection data and evaluation of field stock and spare parts.

Cost of Internal Failures – The cost associated with defective products, components, and materials that fail to meet quality requirements and result in manufacturing losses are called as costs of internal failures. This includes the cost associated with scrap, cost of rework and repair, cost of re-inspection and retest after the defective parts are repaired, cost associated with material review activity, cost to ensure whether non-conforming products are usable for some other work and the costs of process yield lower than the yield that might be attainable by improving controls.

Cost of External Failures – It consists of the costs which are generated because of defective products being shipped to the customers. This includes the cost of processing complaints form the customers, cost of service to the customers who receive defective items, cost of inspecting and repairing the defective items returned by the customers, cost of replacing the defective materials or products, cost of concession made to the customers due to substandard products being accepted by them.

1.1.6 Statistical Quality control

Whenever a statistical technique is employed to control, improve and maintain the quality or to solve quality problem it is termed as Statistical Quality Control. The new era of quality control development began during the World War II when statistical quality control was much needed due to mass production. It is used throughout the quality system at various stages of production such as

- Incoming inspection
- Product moving from one stage to other
- In – process
- Machine start – up
- Process monitoring
- Process adjustment

- Final product
- Field surveillance

Statistical quality control is systematic as compared to guess – work of haphazard process inspection, and the mathematical statistical approach neutralizes personal bias and uncovers poor judgment. Statistical quality control consists of three general activities:

- Systematic collection and graphic recording of accurate data.
- Analyzing the data.
- Practical engineering or management or management action, if the information obtained indicates significant deviations from the specified limits.

Statistical quality control plays an important role in total quality control. The following are the statistical tools used generally for the purpose of exercising control, improvement of quality, enhancement of productivity, creation of consumer confidence and development of the industrial economy of the country.

- **Frequency Distribution:** It is a tabulation or tally of the number of times a given quality characteristic occurs within the samples. Graphic representation of frequency distribution will show the average quality, comparison with specific requirements and process capability.
- **Control Chart:** It is a graphical representation of quality characteristics, which indicates whether the process is under control or not.
- **Acceptance Sampling:** It is a popular quality control technique that is applied to discrete lots or batches of product. The term lot refers to a collection of physical units; the term batch is usually applied to chemical materials. The lot or batch should be presented to the inspection department by either a supplier or a production department. The inspection department then inspects a sample from the lot or batch and, based on the results of the inspection, determines the acceptability of the lot or batch.
- **Analysis of the data:** This includes techniques such as analysis of tolerances, correlation, analysis of variances, analysis for engineering design to eliminate cause of troubles.

1.1.7 Advantages of Statistical Quality Control

- Statistical quality control ensures rapid and efficient inspection at a minimum cost.
- It finds out the cause of excessive variability in manufactured products by forecasting trouble before rejections occur and reducing the amount of spoiled work.
- It exerts more effective pressure for quality improvement than that of a 100 percent inspection.
- It easily detects faults. For example using control charts one can easily examine the deterioration in quality by examining whether the points fall above the upper control limits or below the control limits.
- So long as the statistical control continues, specifications can be accurately predicted for the future, by which it is possible to assess whether the production processes are capable of producing the products with the given set of specifications.
- Increases output and reduces wasted machines and materials resulting in higher productivity.
- Better customer relations through general improvement in product and higher share to the market.
- It provides a common language that may be used by designers, production personnel and inspectors.
- Elimination of bottlenecks in the process of manufacturing.
- It says when and where 100 percent inspection is required.
- Create quality awareness in employees.

Section - 1.2

Basic Concepts of Acceptance Sampling

1.2.1 Introduction

Acceptance sampling procedure is an essential tool in statistical quality control. It is a methodology that deals with quality contracting on product order between the producers and consumers and thus allowing the producers to take the decision to accept or reject the manufactured products based on the inspection of samples.

Acceptance sampling is necessary to limit the cost of inspection and is the only available method to appraise the quality in destructive testing. Acceptance sampling itself does not improve quality, but whenever the lot is rejected it indicates the instability of the production process. Acceptance sampling is cost efficient and the only admissible method of efficient tests with quick results.

1.2.2 Importance of Acceptance Sampling

Acceptance sampling is one of the latest aspects of quality assurance and used primarily for the incoming and outgoing lot by lot quality assurance. The most effective use of acceptance sampling is not to "inspect quality into the product", but rather as an audit tool to ensure that the output of a process conforms to requirements.

According to Duncan (1986), an acceptance sampling plan is likely to be implemented

- When the cost of inspection is high and the loss arising from the passing of a non conforming unit is not great; it is also possible in some cases that no inspection at all will be cheapest plan.
- When 100 percent inspection is fatiguing, a carefully worked out sampling plan will produce good or better results. The 100 percent may not mean 100 percent perfect quality, and the percentage of non conforming items passed may be higher than under a scientifically designed sampling plan.
- When inspection is destructive i.e., a situation where inspection is not possible without destroying the article chemically or physically.
- When there are great quantities or areas to be inspected .
- When it is desired to stimulate the maker and/or the buyer.

1.2.3 Major Areas of Acceptance Sampling

According to Dodge (1959), the major areas of acceptance sampling are

- Lot-by-Lot sampling by the method of attributes, in which each unit in a sample is inspected on a go-not-go basis for one or more characteristics.
- Lot-by-Lot sampling by the method of variables, in which each unit in a sample is measured for a single characteristic such as weight or strength.
- Continuous sampling of a flow of units by the method of attributes.
- Special purpose plans including chain sampling, skip-lot sampling, small sample plans, repetitive group sampling plans etc.

1.2.4 Basic Terminologies and Notations

Sampling Plan

According to the American National Standards Institute/American Society for Quality Control (ANSI/ASQC) Standard A2 (1987), an acceptance sampling plan is a specific plan that states the sampling rules to be used with the associated acceptance and non acceptance criteria.

Sampling Scheme

According to the American National Standards Institute/ American Society for Quality Control (ANSI/ASQC) Standard A2 (1987), a sampling scheme is a specific set of procedures which usually consists of acceptance sampling plans in which lot sizes, sample sizes and acceptance criteria, or the amount of 100 percent inspection and sampling are related. Hill (1962), has also described the difference between sampling plans and the sampling scheme. According to him, a sampling scheme is a whole set of sampling plans and operations included in the standard over-all strategy specifying the way in which sampling plans are to be used.

Sampling System

A sampling system can be described as an assigned grouping of two or more sampling plans with the rules for using these plans for sentencing lots to achieve a blend of the advantageous features of each of the sampling plans. Tightened – Normal – Tightened scheme is an example for sampling scheme, ANSY ASQ Z1.4 – 1993 The International organization adopts a single sampling plan and acceptance level for Standardization and designed International Standard IS01 DIS-2859 and MIL-STD-105E. While MIL-STD-105E was developed for government procurement, it has become the standard for attribute inspection for the industry. It is the most widely used acceptance-sampling plan in the world. The American Society made modifications to MIL-STD-105E (1989), for Quality (ASQ) under the designations ANSY ASQ Z1.4.

The standard is applicable, but not limited, to attribute inspection of the following

- End items
- Components and raw materials
- Operations
- Materials in process
- Supplies in storage
- Maintenance operations
- Data or records
- Administrative procedures

Sampling plans of this standard are intended to be used for a continuing series of lots.

Cumulative and Non – Cumulative Sampling Plans

Cumulative results sampling inspection is one which uses the current and past information from the process in making a decision about the process. The Chain sampling plan of Dodge (1955), is an example for cumulative sampling plan. The non – cumulative sampling plan is defined as one which uses the current sample information from the process or current product entity in making decisions about the process or product quality. Single and Double sampling plans are examples of non – cumulative sampling.

Inspection

ANSI / ASQC Standard A2 (1987), defines the term inspection as ‘activities’, such as measuring examining, testing, gauging one or more characteristics of a product and/or service and comparing these with specified requirements to determine conformity. A sampling scheme or sampling system may contain three types of inspections namely normal, tightened and reduced inspection.

Normal Inspection: Inspection that is used in accordance with an acceptance scheme when a process is considered to be operating at its acceptance quality level or slightly better than its acceptance quality level is termed as normal inspection.

Tightened Inspection: A feature of a sampling scheme using stricter acceptance criteria than those used in normal inspection is known as tightened inspection.

Reduced Inspection: A feature of a sampling scheme permitting smaller sample sizes than used in normal inspection is known as reduced inspection.

Acceptance Quality Level (AQL)

The AQL is a percent defective that is the base line requirement for the quality of the producer's product. The producer would like to design a sampling plan such that there is a high probability of accepting a lot that has a defect level less than or equal to the AQL.

Lot Tolerance Percent Defective (LTPD)

The LTPD is a designated high defect level that would be unacceptable by the consumer. The consumer would like the sampling plan to have a low probability of accepting a lot with a defect level as high as the LTPD.

Operating Characteristic (OC) Curve

Every sampling plan is associated with an operating characteristic curve, familiarly known as OC curve of the plan. This curve when referred to two axes, the axis of p-proportion nonconforming of the material offered for inspection and the axis of $P_a(p)$ – probability of acceptance of a lot or process, is the locus of $(p, P_a(p))$. The OC curve gives the practical performance of a sampling plan.

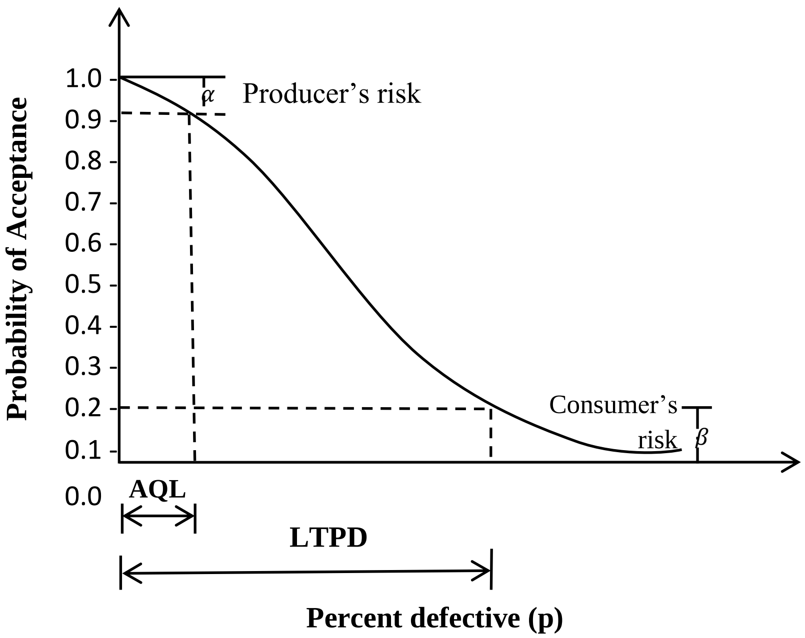

Figure 1.2.4.1: OC Curve

OC curves are generally classified as Type A and Type B OC curves. ANSI/ASQC Standard A2 (1987), defines the two terms as follows:

- Type A OC curve for isolated or unique lots or a lot from an isolated sequence. For a given sampling plan, a type A curve shows the probability of accepting a lot as a function of the lot quality.
- Type B OC curve for a continuous stream of lots. For a given sampling plan, a type B curve shows the probability of accepting a lot as a function of the process average.

For continuous sampling plans the OC curve is a curve showing the long run percentage of the product accepted during the sampling phase(s) as a function of the quality level of the process. For special purpose plans the OC curve is a curve showing, for a specified sampling plan, the probability of continuing to permit the process to continue without adjustment as a function of the process quality.

In sampling schemes or systems, one will have a composite OC curve which gives the steady state probability of acceptance under the switching rules of the scheme or system as a function of process quality.

Under Type A situations (when sampling an attribute characteristic from a finite lot without replacement) hyper geometric distribution is exact to calculate L(p). Under Type B situations (when sampling from the conceptually infinite output of units that the process would turn out under the same essential conditions) binomial model will be exact for the case of non conforming units to calculate L(p). Binomial model is also exact in the case of sampling from a finite lot with replacement. Poisson model is exact in calculating L(p) which specifies a given number of non conformities per unit (or per hundred units). Variable sampling plans use normal distribution for calculating L(p). Binomial, Poisson, Hyper geometric and Normal distributions are the most frequently used distributions in acceptance sampling.

Average Outgoing Quality (AOQ) and Average Outgoing Quality Level (AOQL)

ANSI/ASQC Standard A2 (1987), defines AOQ as the expected quality of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality. AOQL is defined as the maximum AOQ over all possible levels of incoming quality.

Average Sample Number (ASN)

ANSI/ASQC Standard A2 (1987), defines ASN as the average number of sample units per lot used for making decisions (acceptance or non-acceptance). A plot of ASN against p is called the ASN curve.

Type I Error (Producer's Risk)

The producer's risk is the probability of rejecting a good lot which otherwise would have been accepted. The symbol α is used for Type I error.

Type II Error (Consumer's Risk)

The consumer's risk is the probability of defective lots being accepted which otherwise would have been rejected. The symbol β is used for Type II error.

Single Sampling Plan

Sampling inspection in which the decision to accept or not to accept a lot is based on the inspection of a single of size ‘n’.

Double Sampling Plan

Sampling inspection in which the inspection of the first sample of size n_1 leads to a decision to accept a lot, not to accept it or to take a second sample of size n_2 and the inspection of the second sample then leads to a decision to accept or not to accept the lot.

Chain Sampling Plan

Sampling inspection in which the criteria for acceptance and non-acceptance of the lot depend in part on the results of the inspection of the immediately preceding lots.

1.2.5 Designing Sampling Plans

In designing a sampling plan, one has to accomplish a number of different purposes. According to Hamaker (1960), the important ones are:

- To strike a proper balance between the consumer’s requirements, the producer’s capabilities, inspector’s capacity.
- To separate bad lots form good.
- Simplicity of procedures and administration.
- Economy in number of observations.
- To reduce the risk of wrong decisions with increasing lot size.
- To use accumulated sample data as a valuable source of information.
- To exert pressure on the procedure or supplier when the quality of the lots received is unreliable or not up to the standard.
- To reduce sampling when the quality is reliable and satisfactory.

He further noted that these aims are partly conflicting and all of them cannot be simultaneously realized.

The design methodologies of acceptance sampling may be categorized as follows:

	Risk Based	Economical Based
Non – Bayesian	1	2
Bayesian	3	4

Risk based sampling plans are traditional in nature, drawing upon procedure and consumer type of risks as depicted by the OC curve. Economically based sampling plans explicitly consider such factors as costs of inspections, accepting a non conforming unit and rejecting a conforming unit in an attempt to design a cost-effective plan. Bayesian plan design takes into account the past history of similar lots submitted previously for inspection purposes. Non – Bayesian plan design is not explicitly based upon the past lot history.

According to Peach (1947), the following are some of the major types of designing the plans, which are classified according to the types of protection.

The plan is specified by requiring the OC curve to pass through (or nearly through) two fixed points. In some cases it may be possible to impose certain additional conditions. The two points generally selected are (p_1, 1-α) and (p_2, β) where

- p_1 or $p_{1-\alpha}$ – the quality level that is considered to be good so that the procedure expects lots of p_1 quality to be accepted most of the time.
- p_2 or p_β – the quality level that is considered to be poor so that the consumer expects lots of p_2 quality to be rejected most of the time.
- α – the producer's risk of rejecting p_1 quality.
- β – the consumer's risk of accepting p_2 quality.

The tables provided by Cameron (1952), are an example for this type of designing. Schilling (1980), considered the term p_1 as the Producer's Quality Level (PQL) and p_2 as the Consumer's Quality Level (CQL). Earlier literature calls p_1 as the Acceptance Quality Level (AQL) and p_2 as the Limiting Quality Level (LQL) or Rejectable Quality Level (RQL) or Lot Tolerance Proportion Defectives (LTPD). Peach (1947), have defined the ratio p_2/p_1 associated with specified values of α and β as the 'Operating Ratio'. Traditionally the values of α and β are assumed to take 0.05 and 0.10 respectively.

- The plan is specified by fixing one point only, through which the OC curve is required to pass and setting up one or more conditions, not explicitly in terms of the OC curve. Dodge and Romig (1959), LTPD tables is an example for this type of design.
- The plan is specified by imposing upon the OC curve two or more independent conditions none of which explicitly involves the OC curve. Dodge and Romig (1959), AOQL tables is an example of this type of design.

Section - 1.3

Basic Concepts of Reliability

1.3.1 Introduction

The concept of reliability has been known for a number of years, but it has assumed greater significance and importance during the past decade, particularly due to the impact of automation, development in complex missile and space programmes. The manufacture of highly complex equipment has served to focus greater attention on reliability. However, reliability is only one of the tools like quality control and design of experiments for the solution of problems of quality and cost. Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered. Reliability of a product is the measure of the ability of a product to function successfully, when required, for the period required, in the specified environment.

The study of reliability is important because it is related to the quality of a product. Reliability of a product is more important because it is common for a person to think that, what is the use of buying a product that does not satisfy the customer needs and fails within a short period. Thus the effectiveness of a system is understood to mean the suitability of the system for the fulfillment of the intended tasks and the efficiency of utilizing the means put into it. The suitability of performing definite tasks is primarily determined by the quality of the system.

1.3.2 Historical Background of Reliability

The growth and development of reliability has strong links with quality control and its developments in the first quarter of the 1900s. In the 1920's a team of workers at Bell Telephone Laboratories, developed statistical methods to solve some of their quality control problems. They provided the basis for the further developments in the area of statistical quality control (SQC). Subsequently, the American society for Testing and Mechanical Engineers joined Bell Laboratories in popularizing the quality control techniques. However, the rate of adoption of these techniques among the enterprises was very slow till World War II broke out in 1939. The important of reliability and quality control was born out of the demands of modern technology used in the World War II. Complexity and automation of equipments used in the war resulted in severe problems of maintenance and repair. Failures of equipments and components, particularly, electronic tubes, were more than expected. During the war the army and navy in the USA set up a joint committee known as the Vacuum Tube Development Committee in 1943 to study the failure of vacuum tubes which was considered to be one of the focal points of trouble. Quantitative techniques for reliability measurement were evolved and introduced.

During the decade following the war many research laboratories and universities initiated studies on failures of equipments and components. Bell Laboratories and Aeronautical Radio, Inc were the two leading organizations among those who contributed heavily in this area. Practicing engineers and mathematicians took interest in the study of life testing and reliability problems. The first major committee on reliability was set up by the US Department of Defence in 1950. This was later called the Advisory Group on Reliability of Electronic Equipment (AGREE). The AGREE published its first report in 1957 which included some reliability specifications such as minimum acceptability limits, reliability test equipments, etc. Since then many new organizations were formed to promote the concepts of reliability and quality among both manufacturers and users. During the 1950s other countries such as Britain and Japan began to take keen interest in the application of reliability principles to their products. In Britain, The British Productivity Council and the Institute of Production Engineers were independently engaged in promoting quality control concepts. In 1961, The National Council for Quality and Reliability was formed with the main objective of creating an awareness of importance of achieving quality and reliability in the design, manufacture and use of products. The last two decades have seen remarkable progress in the application of reliability principles in industries and government departments in almost all developed and developing countries. Today reliability has become a catchword in many firms and is a very common term in most countries.

1.3.3 Definition of Reliability

Reliability of a unit is the probability that the unit performs its intended function adequately for a given period of time under the stated operating conditions or environment. By a unit we mean an element, a system or a part of a system. If T is the time till the failure of the unit (a random variable) occurs, then the probability that it will not fail in a given environment before time t (or its reliability) is

$$R(t) = P(T > t)$$

Thus, reliability is always a function of time. It also depends on environmental conditions which may or may not vary with time. Since it is a probability, its numerical value is always between one and zero, i.e.,

$$R(0) = 1, R(\infty) = 0$$

and R(t) is a non-increasing function between these limits.

1.3.4 Basic Elements of reliability

The reliability definition stresses five elements mainly

- Numerical value of probability.
- Statement defining successful product performance.
- Statement defining the environment in which the equipment must operate.
- Statement of the required operating time.
- The type of distribution likely to be encountered in reliability measurement.

1.3.5 Design for Reliability

Reliability design is an iterative process that begins with the specification of reliability goals consistent with cost and objectives. This requires considerations of the life-cycle costs of the system and the effect that reliability has on overall costs and system effectiveness. Once there liability goals have been established, these goals must be translate into individual component, subcomponent, and part specifications. This is not necessarily an easy task, and it generally requires a reliability block analysis. After individual component and part requirements have been determined, various design methods can be applied in order to meet the goals. These methods include the proper selection of parts and material, stress-strength analysis, simplification, identification of technologies, and use of redundancy.

Following completion of a preliminary detailed design along with initial development and prototyping, a failure analysis may be performed to determine whether the specifications are being met and also provide a systematic approach for identifying, ranking, and eliminating failure models. This requires the use of reliability testing, including, perhaps a formalized reliability growth testing program. Once reliability goals have been achieved, verification that safety margins are also being met must be made. If either the reliability or safety goals are not met, the design process must continue. This may require reallocating reliability goals among the components if it is not possible to achieve a desired component reliability. The effect of design changes should then be verified through continued use of failure analysis and reliability testing.

Although we are considering reliability as an inherent system or component attribute that can largely be determined during design, we cannot ignore the fact that reliability is influenced throughout a product life cycle by factors external to the product itself.

1.3.6 Achievement of Reliability

There are five effective areas for the achievement of reliability of the product. They are

- Design
- Production
- Measurement and testing
- Maintenance
- Field operation

Design is very important than the other four areas and a greater percentage of causes of unreliability can be traced out in this area.

1.3.7 Failure Pattern

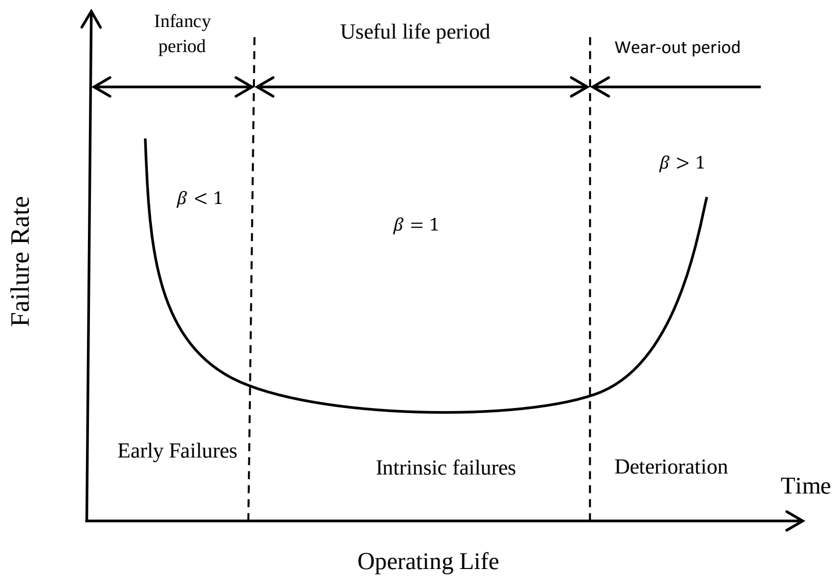

Figure 1.3.7.1: Failure Pattern

The products often follow a familiar pattern of failure. When the failure rate (number of failures per unit time) is plotted against a continuous time scale, the resulting chart is known as "bath tub curve". This curve exhibits three distinct zones. These zones differ from each other in frequency of failure and in the cause of failure pattern. These are follows

1. Infant Morality Period (or burn in or the debugging period): This is characterized by high failure rates. It begins at the first point during manufacture that total equipment operation is possible and continues for such a period of time as permits (through maintenance and repairs), the elimination of marginal parts initially defective though not inoperative and unrecognizable as such until premature failure. Commonly, these are early failures resulting from defect in manufacturing, or other deficiencies which can be detected by debugging, running on or extended testing.
2. The constant failure rate period: Upon replacement of all prematurely failing items, the failure rate will have reached a lower value. From this point the failure rate remains fairly constant. These are chance failures which may result from the limitations inherent in the design plus accidents caused by usage or poor maintenance or hidden defects which escape inspection. This period is the normal operating period in which the average failure rate remains fairly constant.
3. The wear out period: These are failures due to abrasion fatigue, corrosion, vibration etc. Example, the metal becomes embrittled, the insulation dries out. A reduction in failure rate requires preventive replacement of these dying components before they result in catastrophic failure.

Many failure-causing mechanisms give rise to measured distributions of times-to-failure which approximate quite closely to a definite mathematical form, known as probability density functions, or p.d.f. These functions provide mathematical models of failure patterns, which can be used in performance forecasting calculations.

1.3.8 Methods for improving design reliability

Improving the reliability of a product by changing the design is done by designer himself. The following are some of the approaches used by the designers working jointly with reliability engineers to improve the design.

- Review the index selected to define product reliability to make sure that it reflects customer needs.

- Question the function of the unreliable parts with a view of eliminating them entirely if the function is found to be unnecessary.
- Review the selection of any parts which are relatively new.
- Conduct a research and development program to increase the reliability of the parts which are contributing most to the unreliability of the equipments.
- Specify corrective replacement times for unreliable parts and replace the parts before they fail.
- Select the parts which will be subjected to stress which are lower than the parts can normally withstand.
- Control the operating environment so that a part will be operating under conditions which yield a lower failure rate.
- Use redundancy so that if one unit fails a redundant unit will be available to do the job.
- Consider possible trade – offs of reliability with functional performance weight or other parameters.

1.3.9 Life testing

Reliability testing refers to the tests conducted to verify that a product will work satisfactorily for a given time period. Reliability testing therefore consists of functional test, environmental test and life testing. A functional testing involves a test to determine if the product will function at time zone. An environmental test consists of determining the expected environmental levels and then carrying the functional test under the environments under which the product has to operate. The life of the component is the time period during which it retains its quality characteristic. Life tests are carried out to access the working life of a product, its capabilities and hence to form an idea of its quality level. The life test aims to measure the time or period during which the product will retain its desired quality characteristics. This may apply to either shelf life or life during use or both. Life tests are carried out in different manners under different conditions as follows:

- Tests under actual working conditions.
- Tests under intensive conditions.
- Tests under accelerated conditions.

Tests under actual working conditions

This kind of test is a life test of the component under actual working conditions for full durations. This is impractical and do not lend any help in controlling a manufacturing process.

Tests under intensive conditions

Suppose if a product works one hour daily and if it is tested under actual working condition it would be operated only one hour daily and it is just a waste of days. This is impractical and time consuming. Therefore it is worked continuously at rated specifications and thus the life can be estimated in a much shorter duration of time.

Tests under accelerated conditions

These tests are conducted under severe operating conditions to quicken the product failure or break down.

1.3.10 Accelerated life tests

One obtains information on the failure time (actual failure time or an interval containing the failure time) for units that fail and lower bounds for the failure time (also known as the running time or run out time) for units that do not fail.

Accelerated life test models

Most popular accelerated life test models have the following two components.

- A parametric distribution for the life of a population of units at a particular level(s) of an experimental variable or variables. It might be possible to avoid this parametric models (e.g., Weibull and lognormal) to provide important practical advantages for most applications.
- A relationship between one (or more) of the distribution parameters and the acceleration will effect that the variables like temperature, voltage, humidity and specimen or unit size will have on the failure-time distribution. This part of the accelerated life model should be based on a physical model such as one relating the accelerating variable to degradation, on a well established empirical relationship, or some combinations.

1.3.11 Acceptance sampling plans based on life tests

The sampling techniques and control charts are very important tools which analysis the data of life (destructive) tests. It is not necessary to subject all the sample pieces to destructive testing; the results in such case can be concluded from the time of first and middle failure. However, the potential capability of the product can be determined only through destructive testing. Recent times many authors have developed tables which shows, for a life test, the relationship between the sample size, probability and percent of units which will fail before their shortest life for different parametric models.

Section – 1.4

Lifetime Distribution

1.4.1 Introduction

The use of parametric distributions complements non-parametric techniques and provides the following advances:

- Parametric models can be described concisely with just few parameters, instead of having to report an entire curve.
- It is possible to use a parametric model to extrapolate (in time) to the lower or upper tail of a distribution.
- Parametric models provide smooth estimates of failure-time distributions.

1.4.2 New Weibull-Pareto Distribution

Many distributions have been used as lifetime models. Many lifetime data used for statistical analysis follows a particular statistical distribution. Knowledge of the appropriate distribution that any phenomenon follows, greatly improves the sensitivity, power and efficiency of the statistical tests associated with it. Several distributions exist for modeling these lifetime data however, some of these lifetime data do not follow these existing distributions or are inappropriately described by them. This therefore creates room for developing new distributions which could better describe some of these phenomena and therefore provide greater flexibility in the modeling of lifetime data. As a result, umpteen of distributions have been developed and studied by researchers.

Gupta (1998) developed the exponentiated exponential distribution, Mudholkar (1995) proposed the exponentiated-Weibull distribution, Akinsete (2008) developed the beta-Pareto distribution, Alzaatreh (2012) developed the gamma-Pareto distribution and Alzaatreh (2013) developed the Weibull-Pareto distribution. Also, Merovci and Puka (2014) developed the transmuted Pareto distribution while Kareema and Boshi (2013), developed the Exponential Pareto distribution. And Suleman Nasiru and Albert Luguterah (2015) present another form of the Weibull-Pareto distribution called the New Weibull-Pareto Distribution (NWPD). Amer Ibrahim Al-Omari, Agustín Santiago, Jose M. Sautto, Carlos N. Bouza (2018) developed New Weibull-Pareto Distribution.

Let x denotes a random variable form a Pareto Distribution with p.d.f

$$f(x;\theta,k) = \frac{k\theta^k}{x^{k+1}} \tag{1.1}$$

where $\theta > 0$ is a scale parameter and $k > 0$ is the shape parameter.

Let x denotes a random variable form a Weibull Distribution with p.d.f

$$f(x;\alpha,\lambda) = \alpha\lambda(\lambda x)^{\alpha-1}e^{-(\lambda x)^{\alpha}} \tag{1.2}$$

where $x > 0$, $\alpha > 0$, and $\lambda > 0$.

The probability density function of New Weibull-Pareto Distribution with a random variable x is defined as

$$g(x;\delta,\theta,\beta) = \frac{\delta\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\delta\left(\frac{x}{\theta}\right)^{\beta}} \tag{1.3}$$

where $x > 0$, $\delta > 0$, $\beta > 0$ and $\theta > 0$.

The corresponding Hazard function H(x) of New Weibull-Pareto Distribution is defined by

$$H(x;\delta,\theta,\beta) = \frac{\delta\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} \tag{1.4}$$

where δ is a scale parameter and β is a shape parameter.

The mean survival time and the cumulative distribution function of the New Weibull-Pareto Distribution are given by

$$\mu = E(x) = \theta\delta^{\frac{-1}{\beta}}\Gamma\left(\frac{1}{\beta}+1\right) \tag{1.5}$$

and

$$g(x;\delta,\theta,\beta) = 1 - e^{-\delta\left(\frac{x}{\theta}\right)^{\beta}} \tag{1.6}$$

According to Aljarrah at al., (2015) the value of p_0 is given for New Weibull-Pareto Distribution

$$p_0 = 1 - e^{-\delta\left(\frac{x}{\theta}\right)^{\beta}} \quad \text{where} \quad \theta = \frac{\mu\delta^{\frac{1}{\beta}}}{\Gamma\left(\frac{1}{\beta}+1\right)} \tag{1.7}$$

Substituting the value of θ and $\mu = \mu_0$, we get

$$p_0 = 1 - e^{-\delta\left(\left(\frac{t}{\mu_0}\right)\frac{1}{\delta^{\frac{1}{\beta}}}\Gamma\left(\frac{1}{\beta}+1\right)\right)^{\beta}} \tag{1.8}$$

Section – 1.5

Notations

n	-	Sample size
n_1	-	Size of the first sample
n_2	-	Size of the second sample
i	-	Number of lots
k	-	Number of lots
d	-	Number of defectives in the sample
$1-P^*$	-	Consumer's risk
t_0	-	Termination time
δ	-	Scale parameter
β	-	Shape parameter
p	-	Probability of failure before time t
$L(p)$	-	Probability of acceptance of lot
μ	-	Mean life
μ_0	-	Specified mean life
t/μ_0	-	Time termination life
μ/μ_0	-	Mean and specified life ratio or Mean ratio
$P^*=1-\beta$	-	Confidence limit

CHAPTER 2

Chapter – 2

New Weibull-Pareto Distribution in Acceptance Sampling Plans Based on Truncated Life Tests

In this chapter, New Weibull-Pareto Distribution in Acceptance Sampling Plans Based on Truncated Life Tests (2018) by Amer Ibrahim Al-Omari, Agustín Santiago, Jose M. Sautto, Carlos N. Bouza has been reviewed. In this paper, they defined the probability density function (pdf) and cumulative distribution function of the New Weibull-Pareto distribution as well as some other statistical properties.

Weibull-Pareto distribution arises from the combination of the Pareto distribution, initiated by Vilfraddo Pareto (1896), and the family of distributions known as Weibull-G, which is a special case proposed by Alzaatreh et al. (2013b). Tair et al. (2015) makes a broad discussion of its properties and applications, and then their aim is to apply acceptance sampling plans to truncated life tests, assuming that the data fit a Weibull-Pareto distribution.

2.1 New Weibull-Pareto Distribution

The probability density function of New Weibull-Pareto Distribution with a random variable x is defined as

$$g(x;\delta,\theta,\beta) = \frac{\delta\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\delta\left(\frac{x}{\theta}\right)^{\beta}} \qquad (2.1)$$

and its corresponding cumulative distribution function is given by

$$g(x;\delta,\theta,\beta) = 1 - e^{-\delta\left(\frac{x}{\theta}\right)^{\beta}} \quad , \quad x > 0,\ \delta > 0,\ \beta > 0 \text{ and } \theta > 0. \qquad (2.2)$$

The mean and the variance of the New Weibull-Pareto Distribution are

$$E(x) = \theta\delta^{\frac{-1}{\beta}}\Gamma\left(\frac{1}{\beta}+1\right) \qquad (2.3)$$

and $$Var(x) = 2\theta\delta^{\frac{-2}{\beta}}\Gamma\left(\frac{\beta+2}{\beta}\right) - \left[\theta\delta^{\frac{-1}{\beta}}\Gamma\left(\frac{1}{\beta}+1\right)\right]^2 \qquad (2.4)$$

The corresponding hazard rate function (x) of the New Weibull-Pareto random variable is defined by

$$H(x;\delta,\theta,\beta) = \frac{\delta\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} \qquad (2.5)$$

where δ is a scale parameter and β is a shape parameter.

2.2 The Suggested Acceptance Sampling Plans

In this section, they explained the suggested acceptance sampling plans based on the New Weibull-Pareto distribution. Acceptance sampling plans based on New Weibull-Pareto distribution have not been studied previously. An acceptance sampling plan based on truncated life tests consists of the following quantities:

(1) The number of units (n) on test.
(2) An acceptance number (c), where if c or less failures happened within the test time (t), the lot is accepted.
(3) The maximum test duration time, t.
(4) The ratio t/μ_0, where μ_0 is the specified average life.

2.2.1 Minimum Sample Size

Assume that the lot size is sufficiently large to be considered infinite to obtain the probability of accepting a lot using the binomial distribution. Here, the problem is to determine the smallest sample size n essential to satisfy the inequality

$$\sum_{i=0}^{c}\binom{n}{i} p^i\,(1-p)^{n-i} \le 1 - P^* \qquad (2.6)$$

up to an acceptance number c for given values of $P^* \in (0,1)$ where $p = F(t;\mu_0)$ is the probability of a failure observed within the time t which depends only on the ratio t/μ_0. If the number of observed failures within the time t is atmost c, then from Inequality (2.6) they confirm with probability P that $F(t;\mu) \le F(t;\mu_0)$, which implies $\mu_0 \le \mu$.

The smallest sample sizes satisfying the Inequality (6) for t/μ_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712, P^* =0.75, 0.9, 0.95, 0.99 and c = 0, 1, 2,…, 10 are presented in Table 2.1. The values of t/μ_0 and P^* considered in this study are the same values which are considered in Gupta and Groll (1961), Baklizi and El Masri (2004), Kantam and Rosaiah (2001), and Al-Nasser and Al-Omari (2013).

2.2.2 Operating Characteristic Function of the Sampling Plan ($\boldsymbol{n}$, $\boldsymbol{c}$, $\boldsymbol{t/\mu_0}$)

The operating characteristic function of the sampling plan (n, c, $/\mu_0$) is the probability of acceptance the lot. The operating characteristic function can be considered as a source for choosing the minimum sample size, n, or the acceptance number, c. It is defined as

$$L(p) = P(Accepting\ a\ lot) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$

$$= 1 - \mathrm{B_p}(c+1, n-c) \qquad (2.7)$$

where $p = F(t; \mu)$ is considered as a function of μ (the lot quality parameter), and $I_p(c + 1, n - c)$ is the incomplete beta function defined as

$$(a, b) = \frac{1}{\mathrm{B(a,b)}} \int_0^{\vartheta} \omega^{a-1} (1-\omega)^{-1}\, d\omega, \qquad a, b > 0, \qquad (2.8)$$

where $B(a, b) = \frac{(a-1)!(b-1)!}{(a+b-1)!}$. The operating characteristic function values as a function of $\mu \geq \mu_0$ for the sampling plan (n, c = 2, $t/\mu 0$) when the parameters of the New Weibull-Pareto Distribution are $\beta = 2$ and $\delta = 3$ are reported in Table 2.2. Also, for fixed time t, the operating characteristic is a decreasing function in the probability p, while p itself is a monotonically decreasing function in $\mu \geq \mu_0$.

2.2.3 Producer's Risk

The probability of rejecting the lot when $\mu > \mu_0$ in known as the producer's risk and it is defined as

$$P(Rejecting\ a\ lot) = \sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

$$= \mathrm{I_p}(c+1, n-c) \qquad (2.9)$$

For a given value of the producer's risk, say λ, under a given sampling plan, one may be interested in knowing what smallest value of μ/μ_0 that will assert the producer's risk is at most λ. The value of μ/μ_0 is the minimum positive number for which $p = F\left(\frac{t}{\mu_0}\frac{\mu_0}{\mu}\right)$ satisfies the inequality

$$P(R(p)) = \sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \leq \lambda \tag{2.10}$$

For a given acceptance sampling plan (n, c, t/μ_0) based on the New Weibull Pareto Distribution at a given confidence level P^* the smallest values of μ/μ_0 satisfying Inequality (2.10) are given in Table 2.3.

2.3 Illustration of the Tables

The smallest sample sizes necessary to ensure that the mean life exceeds μ_0 with probability P^* or greater, and acceptance number c for β = 2 and δ = 3 in the New Weibull-Pareto Distribution are given in Table (2.1). To illustrate the procedure, when P^* = 0.99, t/μ_0 = 1.571 (t = 1571, μ_0 = 1000) and c = 3, the corresponding table value is n = 8 units, which are should be put on test. This implies that out of 8 units, if 3 items fail before the time t, then a 0.99% upper confidence interval for the mean μ is (t/μ_0 = 1.571, ∞). That is if out 8 items, three or less are fail before time t, then the lot can be accepted with probability 0.99. Based on the suggested acceptance sampling plan for the New Weibull-Pareto Distribution it turns out that the minimum samples sizes obtained in this paper are less than that their counterparts in Baklizi and El Masri (2004), and Al-Nasser and Al-Omari (2013).

Table 2.1: Minimum sample sizes to be tested for a time t to assert the average life exceeds a given value μ_0 with probability P^* and acceptance number c for NWPD with β = 2 and δ = 3

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	5	3	2	1	1	1	1	1
	1	10	5	3	3	2	2	2	2
	2	14	7	5	4	3	3	3	3
	3	19	10	6	5	4	4	4	4
	4	23	12	8	6	5	5	5	5
	5	27	14	10	8	6	6	6	6
	6	31	16	11	9	7	7	7	7
	7	36	18	13	10	8	8	8	8
	8	40	21	14	11	9	9	9	9
	9	44	23	16	12	10	10	10	10
	10	48	25	17	14	11	11	11	11
0.90	0	8	4	2	2	1	1	1	1
	1	14	7	4	3	2	2	2	2
	2	19	9	6	5	3	3	3	3
	3	24	12	8	6	4	4	4	4
	4	29	14	9	7	5	5	5	5
	5	33	17	11	8	6	6	6	6
	6	38	19	13	10	7	7	7	7
	7	42	21	14	11	9	8	8	8
	8	47	24	16	12	10	9	9	9
	9	51	26	17	14	11	10	10	10
	10	56	28	19	15	12	11	11	11

0.95	0	10	5	3	2	1	1	1	1
	1	16	8	5	4	2	2	2	2
	2	22	11	7	5	3	3	3	3
	3	27	13	9	6	5	4	4	4
	4	32	16	10	8	6	5	5	5
	5	37	18	12	9	7	6	6	6
	6	42	21	14	10	8	7	7	7
	7	47	23	15	12	9	8	8	8
	8	52	26	17	13	10	9	9	9
	9	56	28	19	14	11	10	10	10
	10	61	30	20	16	12	11	11	11
0.99	0	15	7	4	3	2	1	1	1
	1	23	11	6	5	3	2	2	2
	2	29	14	8	6	4	3	3	3
	3	35	17	10	8	5	4	4	4
	4	40	19	12	9	6	5	5	5
	5	46	22	14	10	7	6	6	6
	6	51	25	16	12	8	7	7	7
	7	56	27	18	13	9	8	8	8
	8	61	30	19	15	10	9	9	9
	9	66	33	21	16	11	10	10	10
	10	71	35	23	17	13	11	11	11

Table 2.2: Operating characteristic function values for the sampling plan (n, c = 2, t/μ_0) with a given probability P^* under the New Weibull-Pareto Distribution with β = 2 and δ = 3

P^*	n	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	14	0.628	0.920125	0.997850	0.999791	0.999961	0.999990	0.999997
	7	0.942	0.914776	0.997662	0.999772	0.999958	0.999989	0.999996
	5	1.257	0.879749	0.996361	0.999637	0.999933	0.999982	0.999994
	4	1.571	0.840785	0.994663	0.999457	0.999898	0.999973	0.999991
	3	2.356	0.710727	0.986662	0.998543	0.999720	0.999924	0.999974
	3	3.141	0.377024	0.944290	0.992864	0.998544	0.999593	0.999859
	3	3.927	0.140931	0.852288	0.977062	0.994965	0.998543	0.999485
	3	4.712	0.038937	0.710727	0.944262	0.986662	0.995975	0.998543
0.90	19	0.628	0.837615	0.994670	0.999460	0.999899	0.999973	0.999991
	9	0.942	0.839488	0.994737	0.999467	0.999900	0.999973	0.999991
	6	1.257	0.805721	0.993124	0.999293	0.999867	0.999964	0.999988
	5	1.571	0.712980	0.987785	0.998695	0.999751	0.999933	0.999977
	3	2.356	0.710727	0.986662	0.998543	0.999720	0.999924	0.999974
	3	3.141	0.377024	0.944290	0.992864	0.998544	0.999593	0.999859
	3	3.927	0.140931	0.852288	0.977062	0.994965	0.998543	0.999485
	3	4.712	0.038937	0.710727	0.944262	0.986662	0.995975	0.998543
0.95	22	0.628	0.779758	0.991881	0.999159	0.999841	0.999957	0.999986
	11	0.942	0.751208	0.990298	0.998983	0.999807	0.999948	0.999982
	7	1.257	0.724134	0.988629	0.998794	0.999771	0.999938	0.999979
	5	1.571	0.712980	0.987785	0.998695	0.999751	0.999933	0.999977
	3	2.356	0.710727	0.986662	0.998543	0.999720	0.999924	0.999974
	3	3.141	0.377024	0.944290	0.992864	0.998544	0.999593	0.999859
	3	3.927	0.140931	0.852288	0.977062	0.994965	0.998543	0.999485
	3	4.712	0.038937	0.710727	0.944262	0.986662	0.995975	0.998543
0.99	29	0.628	0.635407	0.982543	0.998091	0.999900	0.999900	0.999966
	14	0.942	0.611103	0.980534	0.997850	0.999887	0.999887	0.999961
	8	1.257	0.640210	0.982801	0.998119	0.999638	0.999902	0.999966
	6	1.571	0.581365	0.977621	0.997491	0.999513	0.999867	0.999955
	4	2.356	0.416846	0.956136	0.994669	0.998934	0.999705	0.999898
	3	3.141	0.377024	0.944290	0.992864	0.998544	0.999593	0.999859
	3	3.927	0.140931	0.852288	0.977062	0.994965	0.998543	0.999485
	3	4.712	0.038937	0.710727	0.944262	0.986662	0.995975	0.998543

The operating characteristic function values for the proposed sampling plan for the time truncated acceptance sampling plan calculated from Table (2.1) for various values of t/μ_0 and P^* with acceptance number c = 2. From Table (2.2) the operating characteristic values for the sampling plan (n, c, $/\mu_0$) = (6, 2, 1.571) are as follows:

μ/μ_0	2	4	6	8	10	12
OC	0.58136	0.97762	0.99749	0.99951	0.99986	0.99995

This implies that if the true mean life is twice the specified mean life (μ/μ_0 = 2) the producer's risk is about 0.418635, and the producer's risk is about almost equal zero when the true mean is greater than or equal to 4 ($\mu/\mu_0 \geq 4$) times the specified mean.

From Table 2.3 we can get the value of the minimum ratio of the true mean lifetime to the specified one for various choices of the acceptance c, $t/\mu 0$ such that the producer's risk may not exceed 0.05. Thus, for c = 2, t/μ_0 = 1.571, and P^* = 0.95 the table entry is μ/μ_0 = 3.03. This shows that the product can have an average life of 3.03 times the specified average lifetime of 1000 hours in order that with c = 2, n = 8 the product is accepted with a probability of at least 0.95.

Table 2.3 Minimum ratio of μ/μ_0 for the acceptance of a lot with producer's risk of 0.05 under the New Weibull Pareto Distribution when β = 2, δ = 3 and c = 0, 1, 2, … , 10

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	5.478	6.364	6.934	6.128	9.190	12.251	15.317	18.379
	1	2.867	2.951	2.912	3.639	4.137	5.516	6.896	8.274
	2	2.210	2.242	2.425	2.596	3.071	4.094	5.118	6.141
	3	1.983	2.064	1.974	2.143	2.601	3.468	4.336	5.202
	4	1.809	1.863	1.901	1.886	2.332	3.109	3.886	4.663
	5	1.696	1.733	1.847	1.941	2.154	2.872	3.590	4.308
	6	1.617	1.642	1.693	1.794	2.027	2.702	3.378	4.053
	7	1.583	1.574	1.678	1.684	1.930	2.573	3.216	3.859
	8	1.534	1.570	1.579	1.598	1.853	2.470	3.089	3.706
	9	1.494	1.523	1.577	1.528	1.791	2.387	2.984	3.581
	10	1.462	1.484	1.506	1.588	1.739	2.318	2.898	3.477
0.90	0	6.928	7.349	6.934	8.666	9.190	12.251	15.317	18.379
	1	3.418	3.553	3.465	3.639	4.137	5.516	6.896	8.274
	2	2.602	2.595	2.724	3.030	3.071	4.094	5.118	6.141
	3	2.250	2.299	2.399	2.467	2.601	3.468	4.336	5.202
	4	2.052	2.045	2.064	2.148	2.332	3.109	3.886	4.663
	5	1.894	1.951	1.974	1.941	2.154	2.872	3.590	4.308
	6	1.809	1.827	1.909	1.963	2.027	2.702	3.378	4.053
	7	1.723	1.734	1.772	1.835	2.263	2.573	3.216	3.859
	8	1.677	1.707	1.744	1.735	2.157	2.470	3.089	3.706
	9	1.622	1.646	1.651	1.768	2.072	2.387	2.984	3.581
	10	1.593	1.596	1.639	1.694	2.001	2.318	2.898	3.477

0.95	0	7.746	8.216	8.492	8.666	9.190	12.251	15.317	18.379
	1	3.663	3.819	3.938	4.331	4.137	5.516	6.896	8.274
	2	2.811	2.905	2.991	3.030	3.071	4.094	5.118	6.141
	3	2.396	2.407	2.583	2.467	3.214	3.468	4.336	5.202
	4	2.164	2.211	2.214	2.375	2.828	3.109	3.886	4.663
	5	2.015	2.019	2.094	2.134	2.577	2.872	3.590	4.308
	6	1.910	1.940	2.008	1.963	2.398	2.702	3.378	4.053
	7	1.832	1.833	1.860	1.972	2.263	2.573	3.216	3.859
	8	1.772	1.792	1.820	1.859	2.157	2.470	3.089	3.706
	9	1.708	1.723	1.788	1.768	2.072	2.387	2.984	3.581
	10	1.670	1.665	1.702	1.791	2.001	2.318	2.898	3.477
0.99	0	9.487	9.721	9.806	10.613	12.996	12.251	15.317	18.379
	1	4.414	4.521	4.359	4.922	5.458	5.516	6.896	8.274
	2	3.246	3.314	3.236	3.404	3.893	4.094	5.118	6.141
	3	2.746	2.799	2.755	2.998	3.214	3.468	4.336	5.202
	4	2.436	2.439	2.485	2.580	2.828	3.109	3.886	4.663
	5	2.263	2.268	2.312	2.308	2.577	2.872	3.590	4.308
	6	2.120	2.147	2.190	2.256	2.398	2.702	3.378	4.053
	7	2.014	2.015	2.100	2.097	2.263	2.573	3.216	3.859
	8	1.932	1.951	1.963	2.079	2.157	2.470	3.089	3.706
	9	1.867	1.900	1.914	1.971	2.072	2.387	2.984	3.581
	10	1.813	1.828	1.874	1.882	2.205	2.318	2.898	3.477

2.4 An Application

They explain the suggested acceptance sampling plan using the lifetime (in months) to first failure of 20 small electric carts used for internal transportation and delivery in a large manufacturing facility. The data are 0.9, 1.5, 2.3, 3.2, 3.9, 5.0, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15.0, 16.3, 19.3, 22.6, 24.8, 31.5, 38.1, 53.0, and its descriptive statistics are given in Table 2.4. The same data are investigated by Zimmer et al. (1998) and Lio et al. (2010), and it is asymmetrical distributed.

Table 2.4 Descriptive statistics of the lifetime in months to first failure of 20 small electric carts

n	Mean	Variance	Median	Kurtosis	Skewness	Q_1	Q_2
20	14.675	186.697	10.75	4.27993	1.34871	4.45	20.95

The following criteria is used to fit the data for the New Weibull-Pareto Distribution, which are the Akaike information (AIC), Bayesian information (BIC), consistent Akaike information (CAIC), and Hannan-Quinn information (HQIC), Kolmogorov-Smirnov (K-S), and the Anderson-Darling (A-D) statistic. The values of these criteria are summarized in Table 2.5 and they indicate that the New Weibull-Pareto Distribution fitted these data.

Table 2.5 The AIC, CAIC, BIC, HQIC, W, A, K-S, and -2MLL for the electric carts data

AIC	BIC	CAIC	HQIC	W	A-D	-2MLL	K-S	P-value
153.1055	156.0927	154.6055	153.6886	0.0086	0.0799	73.5528	0.0526	0.9999

The maximum likelihood estimators (MLE) of the New Weibull-Pareto Distribution parameters with standard deviation (Std. Dev.) and the Inf. 95% CI and Sup. 95% CI are given in Table 2.5.

Table 2.6 MLEs, Std. Dev. and the CI for the NWPD parameters based on the electric carts data

Parameter	MLEs	Std. Dev.	Inf. 95% CI	Sup. 95% CI
δ	0.15426	3.75292	-7.20132	7.50985
θ	2.83389	62.1381	-118.95458	124.62236
β	1.10970	0.19310	0.73123	1.48817

And assume that the specified mean life is μ_0 = 6 months and the testing time t_0 = 7.542 months. This leads to the ratio t_0/μ_0 = 1.257. Therefore, from Table (2.1) the sampling plan for P^* = 0.95 is (n = 20, c = 10, t/μ_0 = 1.257). Thus, we accept the lot if only the number of failures before t_0 = 6 months is less than or equal 10. Because the number of failures before t_0 = 6 months is 6, then we can accept the lot.

CHAPTER 3

Chapter – 3

A Time Truncated Special Purpose Double Sampling Plan DSP (0,1) for New Weibull-Pareto Distribution

In this chapter, special purpose double sampling plan DSP (0,1) is developed assuming that lifetime of the test units follow New Weibull-Pareto Distribution and the life test is terminated at a prefixed time. The minimum sample size required for ensuring the specified mean life at specified consumer's confidence level have been determined. The operating characteristics values for various quality levels are obtained and the results are discussed with the help of tables and examples.

In a competitive economy, goods stand in the market if they are of good quality. A consumer wants products of good quality at reasonable and affordable prices. Here the 'quality' can be defined in two different ways. In one sense, goods are said to be of good quality if they meet the expected functional use. Example for second concept, ball-bearings within the specification limits is said to be in 'control production process'. Every unit of production is tested for the standards specified. The units which do not meet the specifications are rejected. The rejected units are said to be of bad quality, as such they are not put to use. Statistical quality control is the procedure for the control of quality by the application of the theory of probability to the results of inspection of samples of the population. Sampling plans are used in the area of quality and reliability analysis. When the quality of product is related to its lifetime, it is called as life test.

In most of the life testing sampling plans a common constraint is the duration of the total time spent on the test. It is usual to terminate a life test by prefixed time and record the number of failures till that time. If the number of observed failures at the end of the fixed time is not greater than the specified acceptance number, then the lot will be accepted. The test may get terminated before the pre specified time is reached when the number of failures exceeds the acceptance number in which case the decision is to reject the lot. Two risks are continually associated to a time truncated acceptance sampling plan. The probability of accepting a bad lot is known as the consumer's risk and the probability of rejecting a good lot is called the producer's risk. For such a truncated life test and the associated decision rule we are interested in obtaining the smallest sample size to arrive at a decision where the life time of an item follows New Weibull-Pareto Distribution.

From Cameron table (1952), one can observe a jump between the operating ratios of single sampling plan with c = 0 and c = 1 and slow reduction of operating ratios for other values of c. It may also be seen that, in between the operating characteristic (OC) curves of single sampling plan with c = 0 and c = 1plans, there is a vast gap to be filled which leads one to assess the possibility of designing plans having OC curves lying between the OC curves of c = 0 and c = 1 plan. To overcome such situation Craig (1981) have proposed Double Sampling Plan with acceptance number 0 and 1 and rejection number 2. Vijayaragavan (1990), has presented tables for the selection of DSP (0,1) plan for attributes under Poisson and Binomial conditions of sampling. Dodge and Romig (1959) have studied the use of DSP (0,1) plan to product characteristics involving costly and destructive testing. Sudamani Ramaswamy and Sutharani (2014), discussed the special purpose double sampling plan of type DSP (0,1) for truncated life test using minimum angle method. Sudamani Ramaswamy and Jaishree (2014) proposed a new approach of designing special purpose double sampling plan of type DSP (0,1) for truncated life test assuming that the experiment is truncated at pre-assigned time, when the lifetime of the items follows different distributions. Double Sampling Plan of type DSP (0,1) for truncated life test is developed, assuming that the experiment is truncated at pre-assigned time when the lifetime of the items follow New Weibull-Pareto Distribution.

Operating procedure of special purpose double sampling plan of type DSP (0,1)

According to Hald (1981), the operating procedure of DSP (0,1) is as follows:

(i) From a lot, select a sample of size n_1, and observe the number of defectives d_1.

(ii) If $d_1 = 0$, accept the lot; If $d_1 > 1$, reject the lot.

(iii) If $d_1 = 1$, select a second sample of size n_2 and observe d_2.

(iv) If $d_2 = 0$, accept the lot, otherwise reject the lot.

Operating procedure for DSP (0,1) sampling plan for truncated life test

(i) From a lot, select a sample of size n_1, and observe the number of defectives d_1, during the time t_0.

(ii) If $d_1 = 0$, accept the lot; If $d_1 > 1$, reject the lot;

(iii) If $d_1 = 1$, select a second sample of size n_2 and observe d_2, during the time t_0.

(iv) If $d_2 = 0$, accept the lot, otherwise reject the lot.

The following is the operating procedure for special purpose double sampling plan for life test in the form of a flow chart.

Flow Chart

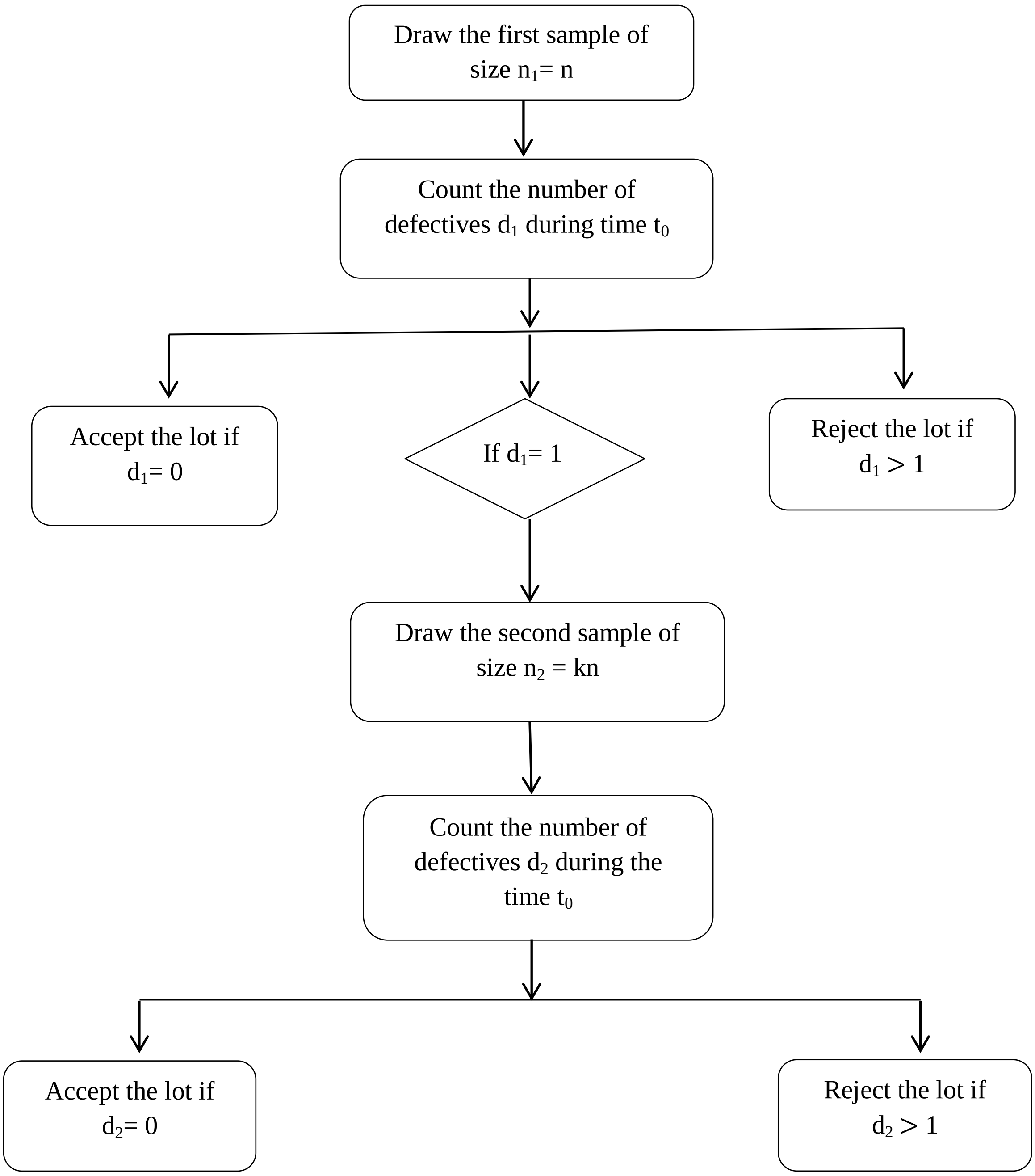

Figure 3.1 Operating procedure for DSP (0,1) sampling plan for truncated life test

Design of the Sampling Plan

It is assumed that the lot size is large enough to use binomial distribution to find the probability of acceptance. The probability of acceptance L(p) for this sampling plan is calculated using the following equation.

$$L(p) = (1-p)^{n_1} + n_1 p (1-p)^{n_1+n_2-1} \quad (3.1)$$

where $n_1 = n$ and $n_2 = kn$ and p is the failure probability.

The required sample size n is the smallest positive integer that satisfies the following inequality.

$$L(p) = (1-p_0)^{n_1} + n_1 p_0 (1-p_0)^{n_1+n_2-1} \leq 1 - P^* \quad (3.2)$$

The minimum values of $n_1 = n$ satisfying inequality (3.2) are obtained and given in Table 3.1 for various values of β and t/μ_0.

By fixing the time termination ratio t/μ_0 as 0.628, 0.942, 1.257, 1.571, 2.356, 3.141 and 4.712, the consumer's risk P^* as 0.75, 0.90, 0.95, 0.99 and the mean ratio μ/μ_0 = 2, 4, 6, 8, 10, 12, we can find the size of the first and the second samples n_1 and n_2 by substituting the failure probability p in the equation (3.1) and satisfying the inequality (3.2) at worst case ($\mu = \mu_0$).

The sample sizes are calculated for the New Weibull-Pareto Distribution and is presented in Table 3.1.

Table 3.1: Minimum Sample Size for DSP (0,1) plan when the life time of the items follows New Weibull-Pareto Distribution with $\beta = 2, \delta = 3$.

P^*	k	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	10	5	3	3	2	2	2	2
	1	6	3	2	2	1	1	1	1
	2	5	3	2	1	1	1	1	1
	3	5	2	2	1	1	1	1	1
	4	5	2	2	1	1	1	1	1
	5	5	2	2	1	1	1	1	1
	6	5	2	2	1	1	1	1	1
	7	5	2	2	1	1	1	1	1
	8	5	2	2	1	1	1	1	1
	9	5	2	2	1	1	1	1	1
	10	5	2	2	1	1	1	1	1
0.90	0	13	6	4	3	2	2	2	2
	1	9	4	3	2	1	1	1	1
	2	8	4	2	2	1	1	1	1
	3	8	4	2	2	1	1	1	1
	4	8	4	2	2	1	1	1	1
	5	8	4	2	2	1	1	1	1
	6	8	4	2	2	1	1	1	1
	7	8	4	2	2	1	1	1	1
	8	8	4	2	2	1	1	1	1
	9	8	4	2	2	1	1	1	1
	10	8	4	2	2	1	1	1	1

0.95	0	16	8	5	3	2	2	2	2
	1	11	5	3	2	1	1	1	1
	2	10	5	3	2	1	1	1	1
	3	10	5	3	2	1	1	1	1
	4	10	5	3	2	1	1	1	1
	5	10	5	3	2	1	1	1	1
	6	10	5	3	2	1	1	1	1
	7	10	5	3	2	1	1	1	1
	8	10	5	3	2	1	1	1	1
	9	10	5	3	2	1	1	1	1
	10	10	5	3	2	1	1	1	1
0.99	0	22	10	6	4	3	2	2	2
	1	15	7	4	3	2	1	1	1
	2	15	7	4	3	2	1	1	1
	3	15	7	4	3	2	1	1	1
	4	15	7	4	3	2	1	1	1
	5	15	7	4	3	2	1	1	1
	6	15	7	4	3	2	1	1	1
	7	15	7	4	3	2	1	1	1
	8	15	7	4	3	2	1	1	1
	9	15	7	4	3	2	1	1	1
	10	15	7	4	3	2	1	1	1

Operating Characteristics (OC) Curve

The OC function of the sampling plan is the probability of accepting a lot and is given by

$$L(p) = (1 - p)^{n_1} + n_1 p(1 - p)^{n_1 + n_2 - 1} \quad (3.3)$$

where $p = F(t, B, \delta)$ is treated as a function of lot quality.

The Operating Characteristic function value for the DSP (0,1) when the parameters of New Weibull-Pareto Distribution are $\beta = 2$, $\delta = 3$ are calculated for different values of μ/μ_0 = 2, 4, 6, 8, 10 and 12, t/μ_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141 and 4.712, and consumer's risk $\beta = 1 - P^*$ = 0.25, 0.10, 0.05, 0.01 are calculated and presented in Table (3.2).

Table 3.2: Probability of acceptance for DSP (0,1) with k = 1, when the life time of the items follows New Weibull-Pareto Distribution

P^*	n	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	6	0.628	0.748097	0.973095	0.994089	0.998057	0.999190	0.999605
	3	0.942	0.711789	0.967726	0.992838	0.997638	0.999013	0.999519
	2	1.257	0.650607	0.957559	0.990412	0.996817	0.998667	0.999349
	2	1.571	0.447292	0.909281	0.977987	0.992499	0.996819	0.998436
	1	2.356	0.411107	0.899733	0.975449	0.991606	0.996435	0.998246
	1	3.141	0.161750	0.761727	0.932214	0.975458	0.989287	0.994650
	1	3.927	0.050579	0.585712	0.859991	0.945631	0.975441	0.987500
	1	4.712	0.012923	0.411107	0.761657	0.899732	0.952849	0.975449
0.90	9	0.628	0.587488	0.944374	0.987124	0.995688	0.998186	0.999112
	4	0.942	0.592024	0.945652	0.987458	0.995804	0.998236	0.999137
	3	1.257	0.460957	0.912729	0.978890	0.992815	0.996955	0.998504
	2	1.571	0.447292	0.909281	0.977987	0.992499	0.996819	0.998436
	1	2.356	0.411107	0.899733	0.975449	0.991606	0.996435	0.998246
	1	3.141	0.161750	0.761727	0.932214	0.975458	0.989287	0.994650
	1	3.927	0.050579	0.585712	0.859991	0.945631	0.975441	0.987500
	1	4.712	0.012923	0.411107	0.761657	0.899732	0.952849	0.975449
0.95	11	0.628	0.495234	0.921654	0.981239	0.993638	0.997308	0.998679
	5	0.942	0.488009	0.920058	0.980833	0.993498	0.997248	0.998649
	3	1.257	0.460957	0.912729	0.978890	0.992815	0.996955	0.998504
	2	1.571	0.447292	0.909281	0.977987	0.992499	0.996819	0.998436
	1	2.356	0.411107	0.899733	0.975449	0.991606	0.996435	0.998246
	1	3.141	0.161750	0.761727	0.932214	0.975458	0.989287	0.994650
	1	3.927	0.050579	0.585712	0.859991	0.945631	0.975441	0.987500
	1	4.712	0.012923	0.411107	0.761657	0.899732	0.952849	0.975449
0.99	15	0.628	0.349849	0.870593	0.966902	0.988497	0.995076	0.997567
	7	0.942	0.329479	0.861959	0.964366	0.987572	0.994671	0.997365
	4	1.257	0.324082	0.860370	0.963936	0.987420	0.994606	0.997332
	3	1.571	0.257385	0.825034	0.953028	0.983372	0.992820	0.996435
	2	2.356	0.118664	0.701865	0.909343	0.966297	0.985098	0.992505
	1	3.141	0.161750	0.761727	0.932214	0.975458	0.989287	0.994650
	1	3.927	0.050579	0.585712	0.859991	0.945631	0.975441	0.987500
	1	4.712	0.012923	0.411107	0.761657	0.899732	0.952849	0.975449

Example: 3.1

Assume that an experimenter wants to establish that the lifetime of the electrical devices produced in the factory ensures that the true unknown mean life is at least 1000 hours with consumer's risk $\beta = 0.05$. It is desired to stop the experiment at t = 628 hours. It is assumed that k = 1. Based on consumer's risk values and the test termination time, the minimum sample size is determined using the special purpose double sampling plan of type DSP (0,1) for truncated life test. Let the distribution followed be New Weibull-Pareto Distribution, then we get the sample size as 11 from Table 3.1. The lot is accepted, at a given mean ratio μ/μ_0 = 2, during 628 hours, with the plan parameters $(n_1, n_2) = (11, 11)$ satisfying the consumer's risk. From the Table 3.2, one can observe that the probability of acceptance for this sampling, when μ/μ_0 = 2 is 0.495234. For the same measurements and plan parameters the probability of acceptance is 0.998679, when ratio of unknown average life is 12.

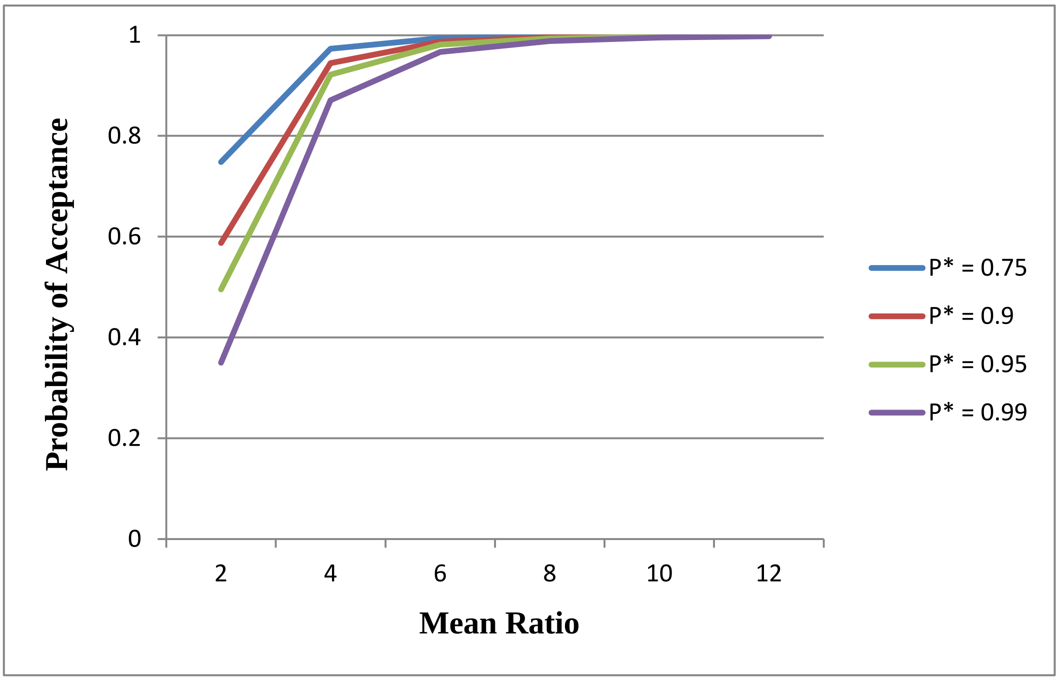

Figure 3.2 OC values vs mean ratio μ/μ_0 with experiment time ratio $t/\mu_0 = 0.628$

It is observed from Figure 3.2 and from Table 3.2 that the Operating Characteristic values of New Weibull-Pareto Distribution increases and it is nearest to unity when μ/μ_0 increases. When there is an increasing in confidence level, the minimum ratio and the sample size are also increases.

For various experiment time ratio, the minimum sample size required to make a decision increases with an increase in the confidence level. This sampling plan can be suggested for the industrial purposes to save time and cost of the life test experiments.

CHAPTER 4

Chapter – 4

A Time Truncated Chain Sampling Plan for New Weibull-Pareto Distribution

In this chapter, Chain sampling plan is developed assuming that life time of the test units follow New Weibull-Pareto Distribution and the life test is terminated at a prefixed time. The minimum sample size required for ensuring the specified mean life at specified consumer's confidence level have been determined. The operating characteristics values for various quality levels are obtained and the results are discussed with the help of Tables and examples.

A single sampling plan having acceptance number zero with a small sample size is often employed in situations involving costly or destructive testing by attributes. The small sample size is warranted due to the costly nature of testing and zero acceptance number arises out of desire to maintain a steep OC curve. But single sampling plan having acceptance number zero has the following disadvantages.

(1) A single defect in the sample calls for rejection of the lot (or for classification of the lot as nonconforming).

(2) The OC curves of all such plans have a uniquely poor shape, in that the probability of acceptance starts to drop rapidly for the smallest values of p.

In contrast, single sampling plan having c = 1 or more, as well as double and multiple sampling plans, lack these undesirable characteristics, but require more sample size. Dodge's (1955) ChSP – 1 plan is an answer to the question whether anything can be done to improve the single sampling plans having c = 0 without increasing the sample size. This ChSP – 1 plan utilize single sampling on an attributes basis with small n and c = 0. The distinguishing feature is that the current lot under inspection can also be accepted if one defective unit is observed in the sample provided that no other defective units were found in the samples from the immediately preceding i lots, i.e., the chain.

Chain sampling plans, in comparison with single sampling plans, have the characteristics of "bowing up" the OC curve for small fraction defective. While having little effect on the end of the curve associated with higher fractions defective. The conditions under which ChSP – 1 plans can be applied and the operating procedure are as follows.

According to Dodge (1955), the conditions for application and operating procedure of ChSP – 1 are as follows.

Conditions for application of ChSP – 1

1. The cost of destructiveness of testing is such that a relatively small sample size is necessary, although other factors make a large sample desirable.
2. The product to be inspected comprises a series of successive lots produced by a continuing process.
3. Normally lots are expected to be of essentially the same quality.
4. The consumer has faith in the integrity of the producer.

Operating Procedure of Chain Sampling Plan

1. For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
2. Accept the lot if d (the observed number of defectiveness) is zero; if $d \geq 2$, reject the lot.
3. Accept the lot if d is one and if no defective units are found in the immediately subceeding i samples of size n.

Operating Procedure of Chain Sampling Plan for truncated life tests

For each lot, select a sample of n units and test each unit for conformance to the specified requirements during the time t_0.

1. Accept the lot if d (the observed number of defectives during the time t_0) is zero in the sample of n units, and reject if d > 1.
2. Accept the lot if d is equal to 1 and if no defectives are found in the immediately subceeding i samples of size n during the time t_0.

The following is the operating procedure for chain sampling plan for life test in the form of a flow chart.

Flow Chart

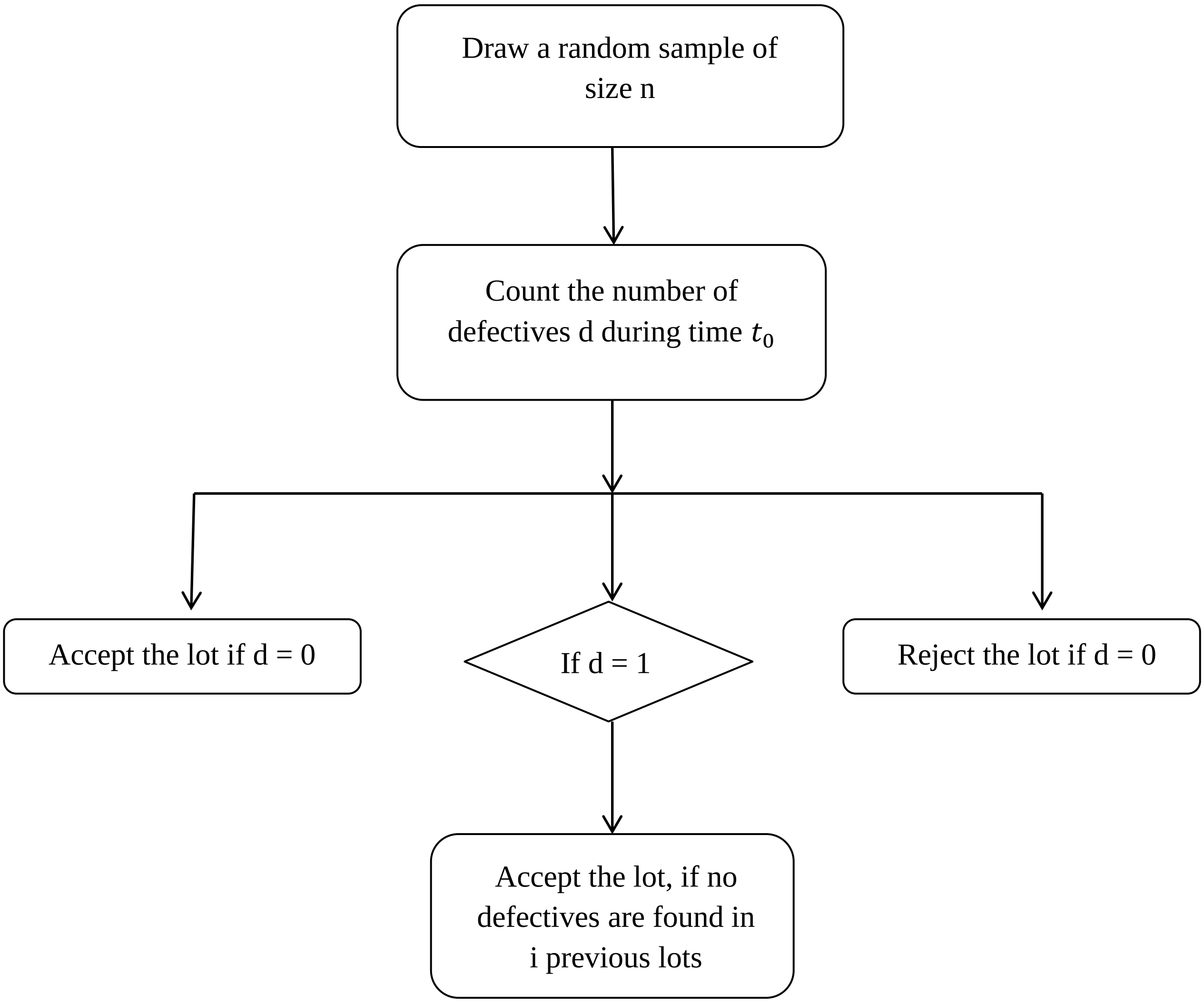

Figure 4.1: Operating Procedure of Chain Sampling Plan for Truncated Life Tests

Design of the sampling plan

It is assumed that the lot size is large enough to use binomial distribution to find the probability of acceptance. The probability of acceptance L(p) for this sampling plan is calculated using the following equation.

$$L(p) = (1-p)^n + np(1-p)^{n-1}(1-p)^{ni} \qquad (4.1)$$

where n is the sample size, i is the number of lots and p is the failure probability.

The required sample size n is the smallest positive integer that satisfies the following inequality.

$$(1-p_0)^n + np_0(1-p_0)^{n-1}(1-p_0)^{ni} \leq 1 - P^* \qquad (4.2)$$

The minimum values of n satisfying inequality (4.2) are obtained and given in Table 4.1 for various values of P^* and t/μ_0.

By fixing the time termination ratio t/μ_0 as 0.628, 0.942, 1.257, 1.571, 2.356, 3.141 and 4.712, the consumer's confidence level P^* as 0.75, 0.90, 0.95, 0.99 and the mean ratio μ/μ_0 = 2, 4, 6, 8, 10, 12, we can find the sample size (n) by substituting the failure probability p in the equation (4.1) and satisfying the inequality (4.2) at worst case ($\mu = \mu_0$).

The sample size are calculated for the New Weibull-Pareto Distribution and is presented in Table 4.1.

Table 4.1: Minimum sample size (n) for Chain sampling plan when the life time of the items follows New Weibull-Pareto Distribution with $\beta = 2,\ \delta = 3$.

P^*	i	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	10	5	3	3	2	2	2	2
	1	6	3	2	2	1	1	1	1
	2	5	3	2	1	1	1	1	1
	3	5	2	2	1	1	1	1	1
	4	5	2	2	1	1	1	1	1
	5	5	2	2	1	1	1	1	1
	6	5	2	2	1	1	1	1	1
	7	5	2	2	1	1	1	1	1
	8	5	2	2	1	1	1	1	1
	9	5	2	2	1	1	1	1	1
	10	5	2	2	1	1	1	1	1
0.90	0	13	6	4	3	2	2	2	2
	1	9	4	3	2	1	1	1	1
	2	8	4	2	2	1	1	1	1
	3	8	4	2	2	1	1	1	1
	4	8	4	2	2	1	1	1	1
	5	8	4	2	2	1	1	1	1
	6	8	4	2	2	1	1	1	1
	7	8	4	2	2	1	1	1	1
	8	8	4	2	2	1	1	1	1
	9	8	4	2	2	1	1	1	1
	10	8	4	2	2	1	1	1	1

0.95	0	16	8	5	3	2	2	2	2
	1	11	5	3	2	1	1	1	1
	2	10	5	3	2	1	1	1	1
	3	10	5	3	2	1	1	1	1
	4	10	5	3	2	1	1	1	1
	5	10	5	3	2	1	1	1	1
	6	10	5	3	2	1	1	1	1
	7	10	5	3	2	1	1	1	1
	8	10	5	3	2	1	1	1	1
	9	10	5	3	2	1	1	1	1
	10	10	5	3	2	1	1	1	1
0.99	0	22	10	6	4	3	2	2	2
	1	15	7	4	3	2	1	1	1
	2	15	7	4	3	2	1	1	1
	3	15	7	4	3	2	1	1	1
	4	15	7	4	3	2	1	1	1
	5	15	7	4	3	2	1	1	1
	6	15	7	4	3	2	1	1	1
	7	15	7	4	3	2	1	1	1
	8	15	7	4	3	2	1	1	1
	9	15	7	4	3	2	1	1	1
	10	15	7	4	3	2	1	1	1

Operating Characteristic (OC) Curve

The OC function of the sampling plan is the probability of accepting a lot and is given by

$$L(p) = (1-p)^n + np(1-p)^{n-1}(1-p)^{ni} \qquad (4.3)$$

where $p = F(t, B, \delta)$ is treated as a function of lot quality.

The Operating Characteristic function value for the Chain Sampling Plan when the parameters of New Weibull-Pareto Distribution are $\beta = 2$, $\delta = 3$ are calculated for different values of μ/μ_0 = 2, 4, 6, 8, 10 and 12, t/μ_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141 and 4.712, and consumer's risk $\beta = 1 - P^*$ = 0.25, 0.10, 0.05, 0.01 are calculated and presented in Table (4.2).

Table 3.2: Probability of acceptance for Chain Sampling Plan with i = 2, when the lifetime of the units follows New Weibull-Pareto Distribution

P^*	n	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	5	0.628	0.804857	0.980832	0.995863	0.998649	0.999438	0.999727
	3	0.942	0.711789	0.967726	0.992837	0.997638	0.999013	0.999519
	2	1.257	0.650607	0.957559	0.990411	0.996817	0.998667	0.999349
	1	1.571	0.761518	0.975430	0.994644	0.998245	0.999269	0.999644
	1	2.356	0.411106	0.899732	0.975449	0.991606	0.996435	0.998246
	1	3.141	0.161750	0.761726	0.932213	0.975458	0.989287	0.994650
	1	3.927	0.050579	0.585712	0.859991	0.945631	0.975441	0.987500
	1	4.712	0.012922	0.411106	0.761657	0.899732	0.952849	0.975449
0.90	8	0.628	0.638470	0.954757	0.989706	0.996574	5.731912	0.999298
	4	0.942	0.592024	0.945651	0.987458	0.995804	3.165765	0.999137
	2	1.257	0.650607	0.957559	0.990411	0.996817	2.198376	0.999349
	2	1.571	0.447291	0.909280	0.977986	0.992499	1.773877	0.998436
	1	2.356	0.411106	0.899732	0.975449	0.991606	1.334091	0.998246
	1	3.141	0.161750	0.761726	0.932213	0.975458	1.063931	0.994650
	1	3.927	0.050579	0.585712	0.859991	0.945631	0.925580	0.987500
	1	4.712	0.012922	0.411106	0.761657	0.899732	0.849033	0.975449
0.95	10	0.628	0.539687	0.933311	0.984298	0.994709	5.902534	0.998906
	5	0.942	0.488009	0.920057	0.980832	0.993498	3.178866	0.998649
	3	1.257	0.460957	0.912728	0.978890	0.992815	2.215568	0.998504
	2	1.571	0.447291	0.909280	0.977986	0.992499	1.773877	0.998436
	1	2.356	0.411106	0.899732	0.975449	0.991606	1.334091	0.998246
	1	3.141	0.161750	0.761726	0.932213	0.975458	1.063931	0.994650
	1	3.927	0.050579	0.585712	0.859991	0.945631	0.925580	0.987500
	1	4.712	0.012922	0.411106	0.761657	0.899732	0.849033	0.975449
0.99	15	0.628	0.349848	0.870592	0.966902	0.988497	5.533453	0.997567
	7	0.942	0.329479	0.861959	0.964365	0.987572	2.958534	0.997365
	4	1.257	0.324082	0.860370	0.963936	0.987420	2.082411	0.997332
	3	1.571	0.257384	0.825033	0.953028	0.983372	1.604680	0.996435
	2	2.356	0.118663	0.701865	0.909343	0.966297	1.107299	0.992505
	1	3.141	0.161750	0.761726	0.932213	0.975458	1.063931	0.994650
	1	3.927	0.050579	0.585712	0.859991	0.945631	0.925580	0.987500
	1	4.712	0.012922	0.411106	0.761657	0.899732	0.849033	0.975449

Example 4.1

Assume that an experimenter wants to establish that the lifetime of the electrical devices produced in the factory ensures that the true unknown mean life is atleast (say) 1000 hours with consumer's risk 0.05. It is decided to stop the experiment at t = 942 hours. It is assumed that i = 2. Based on consumer's risk values and test termination time, the minimum sample size is determined using the chain sampling plan for truncated life test. Let the distribution followed be New Weibull-Pareto Distribution. If during 942 hours no failures out of 2 are observed then the experimenter can assert, with a confidence level of 0.90 that the average life is at least 1000 hours, if one failure occurs provided i = 2 preceding samples are free from defectives, accept the lot or otherwise reject the lot. Then we get the sample size as 5 from Table 4.1. The lot is accepted, at given mean ratio $\mu/\mu_0 = 2$, during 942 hours, with the sample size n = 5 satisfying the consumer's risk. From Table 4.2, one can observe that the probability of acceptance for this sampling plan when $\mu/\mu_0 = 2$ is 0.488009. For the same measurements and plan parameters, the probability of acceptance is 0.998649, when the ratio of unknown average life is 12.

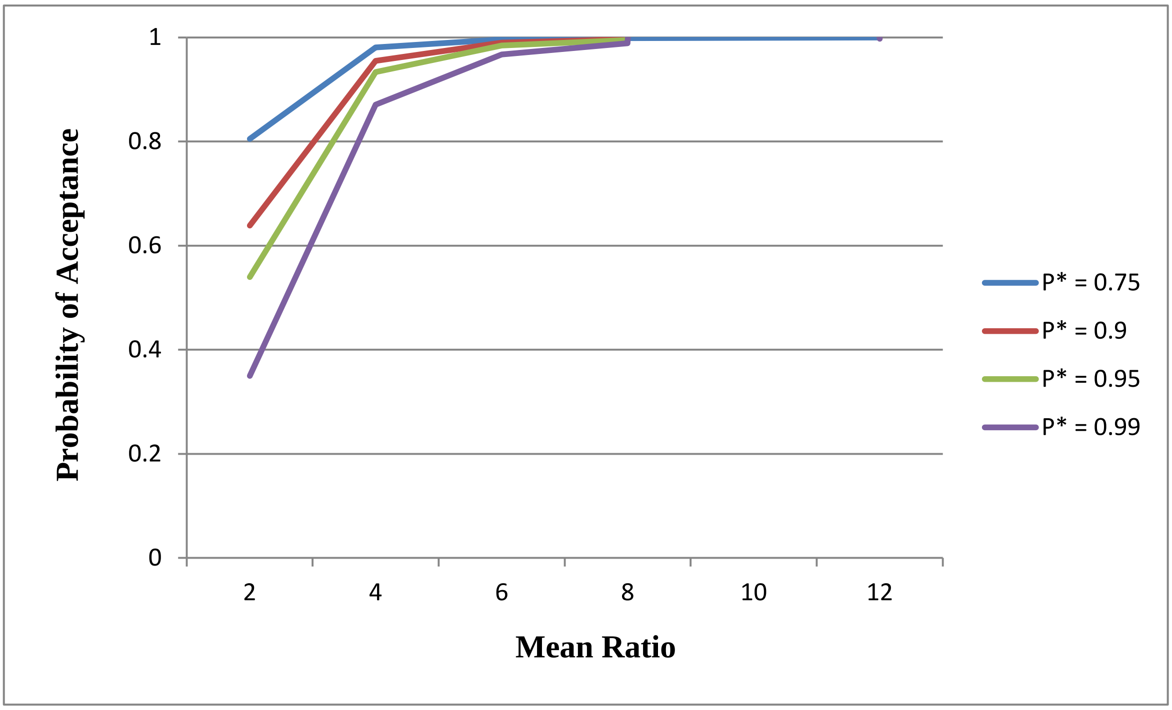

Figure 4.2 OC Values vs mean ratio μ/μ_0 with experiment time ratio $t/\mu_0 = 0.628$

It is observed from Figure 4.2 and from Table 4.2 that the Operating Characteristic values of New Weibull-Pareto Distribution increases and it is nearest to unity when μ/μ_0 increases. When there is an increasing in confidence level, the minimum ratio and the sample size are also increases.

For various experiment time ratio, the minimum sample size required to make a decision increases with an increase in the confidence level. This sampling plan can be suggested for the industrial purposes to save time and cost of the life test experiments.

SUMMARY AND CONCLUSION

Summary and Conclusion

Reliability characteristics, such as probability of survival, mean time to failure, availability, mean down time and frequency of failures are some measures of system effectiveness. Reliability of a product (or a system) is the probability that a given product will successfully perform a required function without break or failure under specified environmental conditions, for a specified period of time. Acceptance sample is the middle ground between hundred percent inspection and no inspection. The important thing in acceptance sampling is to minimize the cost and time required for quality control or reliability test for the decision about the acceptance or rejection of the submitted lot of the products.

The first chapter deals with basic concepts of quality control, acceptance sampling, reliability, life testing, notations and symbols.

In the second chapter, New Weibull-Pareto Distribution in Acceptance Sampling based on Truncated Life Tests has been reviewed. In this paper, Amer Ibrahim Al-Omari, Agustín Santiago, Jose M. Sautto, Carlos N. Bouza proposed the time truncated acceptance sampling plan for the New Weibull-Pareto distribution. The tables for the minimum sample size required to assure a certain average life of the experimental items were derived and are given. The operating characteristic function values and the associated producer's risks are also provided. Practitioners can use the results obtained in this paper and also it can be used for other distributions which can be converted to New Weibull-Pareto distribution.

In the third chapter, special purpose double sampling plan DSP(0,1) for truncated life tests assuming that lifetime of the test units follows New Weibull-Pareto Distribution is developed. The minimum sample size required for ensuring the specified mean life at specified consumer's confidence level, the operating characteristics values for various quality levels are obtained and the results are discussed with the help of tables and examples. It is observed that the operating characteristic values of New Weibull-Pareto Distribution increases and it is nearest to unity when the mean ratio μ/μ_0 increases. For various experiment time ratio time ratio, the minimum sample size required to make a decision increases with an increase in confidence level.

In the fourth chapter, chain sampling plan for truncated life tests assuming that lifetime of the test units follows New Weibull-Pareto Distribution is developed. The minimum sample size required for ensuring the specified mean life at specified consumer's confidence level, the operating characteristics values for various quality levels are obtained. The results are discussed with the help of tables and examples. It is observed that the operating characteristic values of New Weibull-Pareto Distribution increases and it is nearest to unity when the mean ratio μ/μ_0 increases. For various experiment time ratio time ratio, the minimum sample size required to make a decision increases with an increase in confidence level. It is concluded that this sampling plan can be suggested for the industrial purposes to save time and cost of the life test experiments.

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