

Intuitionistic fuzzy generalized semi-pre closed mappings

R. Santhi and D. Jayanthi*

Department of Mathematics,

NGM College, Pollachi, Tamil Nadu, India.

E-mails: *santhifuzzy@yahoo.co.in*, *jayanthimaths@rediffmail.com*

Abstract:

In this paper we introduce intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings. We investigate some of their properties. We also introduce intuitionistic fuzzy M -generalized semi-pre closed mappings as well as intuitionistic fuzzy M -generalized semi-pre open mappings. We provide the relation between intuitionistic fuzzy M -generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre closed mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings.

Mathematics Subject Classification: 03E72.

1 Introduction

Zadeh [14] introduced the notion of fuzzy sets. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological space. In this paper we introduce the notion of intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings and study some of their properties. We also introduce intuitionistic fuzzy M -generalized semi-pre closed mappings as well as intuitionistic fuzzy M -generalized semi-pre open mappings. We provide the relation between intuitionistic fuzzy M -generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre closed mappings.

2 Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$$

where the functions $\mu_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in X .

Definition 2.2: [1] Let A and B be IFSs of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \} \text{ and } B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X \}.$$

Then

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
- (b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \}$;
- (d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle \mid x \in X \}$;
- (e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in X \}$.

The intuitionistic fuzzy sets $0_{\sim} = \{ \langle x, 0, 1 \rangle \mid x \in X \}$ and $1_{\sim} = \{ \langle x, 1, 0 \rangle \mid x \in X \}$ are respectively the empty set and the whole set of X . For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$.

Definition 2.3: [3] An *intuitionistic fuzzy topology* (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

- (i) $0_{\sim}, 1_{\sim} \in \tau$
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- (iii) $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X . The complement A^c of an IFOS A in IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X .

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* are defined by

$$\text{int}(A) = \cup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\}.$$

$$\text{cl}(A) = \cap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = [\text{int}(A)]^c$ and $\text{int}(A^c) = [\text{cl}(A)]^c$ [13].

Definition 2.5:[5] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

- (i) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$
- (ii) *intuitionistic fuzzy pre closed set* (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$
- (iii) *intuitionistic fuzzy α closed set* (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs.

Definition 2.6:[13] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be

- (i) *intuitionistic fuzzy semi-pre closed set* (IFSPCS for short) if there exists an IFPCS B such that $\text{int}(B) \subseteq A \subseteq B$.
- (ii) *intuitionistic fuzzy semi-pre open set* (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short) B such that $B \subseteq A \subseteq \text{cl}(B)$.

The family of all IFSPCSs (respectively, IFSPoSs) of an IFTS (X, τ) is denoted by IFSPC(X) (respectively IFSPo(X)). Every IFSCS (respectively IFSoS) and every IFPCS (respectively IFPoS) is an IFSPCS (respectively IFSPoS). But the separate converses need not be true in general [13].

Note that an IFS A is an IFSPCS if and only if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ [7].

Definition 2.7:[7] Let A be an IFS in an IFTS (X, τ) . Then the semi-pre interior and the semi-pre closure of A are defined as

$$\text{spint}(A) = \cup \{G \mid G \text{ is an IFSPoS in } X \text{ and } G \subseteq A\}.$$

$$\text{spcl}(A) = \cap \{K \mid K \text{ is an IFSPCS in } X \text{ and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ) , we have $\text{spcl}(A^c) = [\text{spint}(A)]^c$ and $\text{spint}(A^c) = [\text{spcl}(A)]^c$ [7]

Definition 2.8:[10] An IFS A is an

- (i) *intuitionistic fuzzy regular closed set* (IFRCS for short) if $A = \text{cl int}(A)$
- (ii) *intuitionistic fuzzy generalized closed set* (IFGCS for short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS.

Definition 2.9:[7] An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy generalized semi-pre closed set* (IFGSPCS for short) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) .

Every IFCS, IFSCS, IFPCS, IFRCS, IF α CS, IFSPCS is an IFGSPCS but the separate converses may not be true in general.[7] The family of all IFGSPCSs of an IFTS (X, τ) is denoted by IFGSPC(X).

Definition 2.10:[7] The complement A^c of an IFGSPCS A in an IFTS (X, τ) is called an *intuitionistic fuzzy generalized semi-pre open set* (IFGSPOS for short) in X .

Every IFOS, IFSoS, IFPoS, IFROs, IF α OS, IFSPoS is an IFGSPOS but the separate converses may not be true in general.[7] The family of all IFGSPOSs of an IFTS (X, τ) is denoted by IFGSPO(X).

Definition 2.11:[5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be *intuitionistic fuzzy continuous* (IF continuous for short) mapping if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

Definition 2.12:[7] If every IFGSPCS in (X, τ) is an IFSPCS in (X, τ) , then the space can be called as an *intuitionistic fuzzy semi-pre $T_{1/2}$ space* (IFSPT $_{1/2}$ space for short).

Definition 2.13:[8] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy generalized semi-pre continuous* (IFGSP continuous for short) mapping if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFCS V of (Y, σ) .

Definition 2.14:[9] A map $f: X \rightarrow Y$ is called an *intuitionistic fuzzy closed mapping* (IFCM for short) if $f(A)$ is an IFCS in Y for each IFCS A in X .

Definition 2.15:[5] A map $f: X \rightarrow Y$ is called an

- (i) *intuitionistic fuzzy semi-open mapping* (IFSOM for short) if $f(A)$ is an IFSOS in Y for each IFOS A in X .
- (ii) *intuitionistic fuzzy α -open mapping* (IF α OM for short) if $f(A)$ is an IF α OS in Y for each IFOS A in X .
- (iii) *intuitionistic fuzzy preopen mapping* (IFPOM for short) if $f(A)$ is an IFPOS in Y for each IFOS A in X .

Definition 2.16:[13] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy pre regular closed mapping* (IFPRCM for short) if $f(V)$ is an IFRCS in (Y, σ) for every IFRCS V of (X, τ) .

Definition 2.17: [10] The IFS $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta} \rangle$ where $\alpha \in (0, 1]$, $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an *intuitionistic fuzzy point* (IFP for short) in X .

Note that an IFP $c(\alpha, \beta)$ is said to belong to an IFS $A = \langle x, \mu_A, \gamma_A \rangle$ of X denoted by $c(\alpha, \beta) \in A$ if $\alpha \leq \mu_A$ and $\beta \geq \gamma_A$.

Definition 2.18:[9] Let $c(\alpha, \beta)$ be an IFP of an IFTS (X, τ) . An IFS A of X is called an *intuitionistic fuzzy neighborhood* (IFN for short) of $c(\alpha, \beta)$ if there exists an IFOS B in X such that $c(\alpha, \beta) \in B \subseteq A$.

Definition 2.19:[8] Let $c(\alpha, \beta)$ be an IFP in (X, τ) . An IFS A of X is called an *intuitionistic fuzzy semi neighborhood* (IFSN for short) of $c(\alpha, \beta)$ if there is an IFSPoS B in X such that $c(\alpha, \beta) \in B \subseteq A$.

Theorem 2.20: Let (X, τ) be an IFTS where X is an IFSPT $_{1/2}$ space. An IFS A is an IFGSPOS in

X if and only if A is an IFSN of $c(\alpha, \beta)$ for each IFP $c(\alpha, \beta) \in A$.

Proof: Necessity: Let $c(\alpha, \beta) \in A$. Let A be an IFGSPOS in X . Since X is an IFSPT $_{1/2}$ space, A is an IFSPoS in X . Then clearly A is an IFSN of $c(\alpha, \beta)$.

Sufficiency: Let $c(\alpha, \beta) \in A$. Since A is an IFSN of $c(\alpha, \beta)$, there is an IFSPoS B in X such that $c(\alpha, \beta) \in B \subseteq A$. Now

$$A = \cup \{ c(\alpha, \beta) \mid c(\alpha, \beta) \in A \} \subseteq \cup \{ B_{c(\alpha, \beta)} \mid c(\alpha, \beta) \in A \} \subseteq A.$$

This implies $A = \cup \{ B_{c(\alpha, \beta)} \mid c(\alpha, \beta) \in A \}$. Since each B is an IFSPoS, A is an IFSPoS and hence an IFGSPOS in X .

Theorem 2.21: For any IFS A in an IFTS (X, τ) where X is an IFSPT $_{1/2}$ space, $A \in \text{IFGSPO}(X)$ if and only if for every IFP $c(\alpha, \beta) \in A$, there exists an IFGSPOS B in X such that

$$c(\alpha, \beta) \in B \subseteq A.$$

Proof: Necessity: If $A \in \text{IFGSPO}(X)$, then we can take $B = A$ so that $c(\alpha, \beta) \in B \subseteq A$ for every IFP $c(\alpha, \beta) \in A$.

Sufficiency: Let A be an IFS in X and assume that there exists $B \in \text{IFGSPO}(X)$ such that $c(\alpha, \beta) \in B \subseteq A$. Since X is an IFSPT $_{1/2}$ space, B is an IFSPoS of X . Then

$$A = \cup_{c(\alpha, \beta) \in A} \{ c(\alpha, \beta) \} \subseteq \cup_{c(\alpha, \beta) \in A} B \subseteq A.$$

Therefore $A = \cup_{c(\alpha, \beta) \in A} B$ is an IFSPoS [13] and hence A is an IFGSPOs in X . Thus $A \in \text{IFGSPO}(X)$.

3 Intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings.

In this section we introduce intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings. We study some of their properties

Definition 3.1: A map $f: X \rightarrow Y$ is called an *intuitionistic fuzzy generalized semi-pre closed mapping* (IFGSPCM for short) if $f(A)$ is an IFGSPCS in Y for each IFCS A in X .

For the sake of simplicity, we shall use the notation $A = \langle x, (\mu_a, \mu_b), (v_a, v_b) \rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/v_a, b/v_b) \rangle$ in the following examples. Similarly we shall use the notation

$B = \langle y, (\mu_u, \mu_v), (v_u, v_v) \rangle$ instead of $B = \langle y, (u/\mu_u, v/\mu_v), (u/v_u, v/v_v) \rangle$ in the following examples.

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle, G_2 = \langle y, (0.3_u, 0.4_v), (0.7_u, 0.6_v) \rangle.$$

Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGSPCM.

Theorem 3.3: Every IFCM is an IFGSPCM but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFCM. Let A be an IFCS in X . Then $f(A)$ is an IFCS in Y . Since every IFCS is an IFGSPCS, $f(A)$ is an IFGSPCS in Y . Hence f is an IFGSPCM.

Example 3.4: In Example 3.2 f is an IFGSPCM but not an IFCM, since $G_1^c = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$ is an IFCS in X , but $f(G_1^c) = \langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v) \rangle$ is not an IFCS in Y , since $\text{cl}(f(G_1^c)) = G_2^c = f(G_1^c)$

Theorem 3.5: Every IF α CM is an IFGSPCM but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IF α CM. Let A be an IFCS in X . Then $f(A)$ is an IF α CS in Y . Since every IF α CS is an IFGSPCS, $f(A)$ is an IFGSPCS in Y . Hence f is an IFGSPCM.

Example 3.6: Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle, G_2 = \langle y, (0.8_u, 0.7_v), (0.2_u, 0.3_v) \rangle.$$

Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGSPCM but not an IF α CM. Since G_1^c is an IFCS in X but $f(G_1^c) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$ is not an IF α CS in Y , since $\text{cl}(\text{int}(\text{cl}(f(G_1^c)))) = 1_- \subseteq f(G_1^c)$

Theorem 3.7: Every IFSCM is an IFGSPCM but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFSCM. Let A be an IFCS in X . Then $f(A)$ is an IFSCS in Y . Since every IFSCS is an IFGSPCS, $f(A)$ is an IFGSPCS in Y . Hence f is an IFGSPCM.

Example 3.8: In Example 3.6, f is an IFGSPCM but not an IFSCM, since G_1^c is an IFCS in X but $f(G_1^c) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$ is not an IFSCS in Y , since

$$\text{int}(\text{cl}(f(G_1^c))) = 1_{\sim} \subseteq f(G_1^c).$$

Theorem 3.9: Every IFPCM is an IFGSPCM but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFPCM. Let A be an IFCS in X . Then $f(A)$ is an IFPCS in Y . Since every IFPCS is an IFGSPCS, $f(A)$ is an IFGSPCS in Y . Hence f is an IFGSPCM.

Example 3.10: Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle, G_2 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v) \rangle.$$

Then $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGSPCM but not an IFPCM, since $f(G_1^c)$ is an IFCS in Y but not an IFPCS in Y , since $\text{cl}(\text{int}(f(G_1^c))) \subseteq G_2^c \subseteq f(G_1^c)$.

Definition 3.11: A mapping $f: X \rightarrow Y$ is said to be an *intuitionistic fuzzy M-generalized semi-pre closed mapping* (IFMGSPCM, for short) if $f(A)$ is an IFGSPCS in Y for every IFGSPCS A in X .

Example 3.12: Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle, G_2 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v) \rangle.$$

Then $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFMGSPCM.

Theorem 3.13 : Every IFMGSPCM is an IFGSPCM but not conversely.

Proof: Let $f: X \rightarrow Y$ be an IFMGSPCM. Let A be an IFCS in X . Then A is an IFGSPCS in X . By hypothesis $f(A)$ is an IFGSPCS in Y . Therefore f is an IFGSPCM.

Example 3.14: Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle,$$

$$G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v) \rangle$$

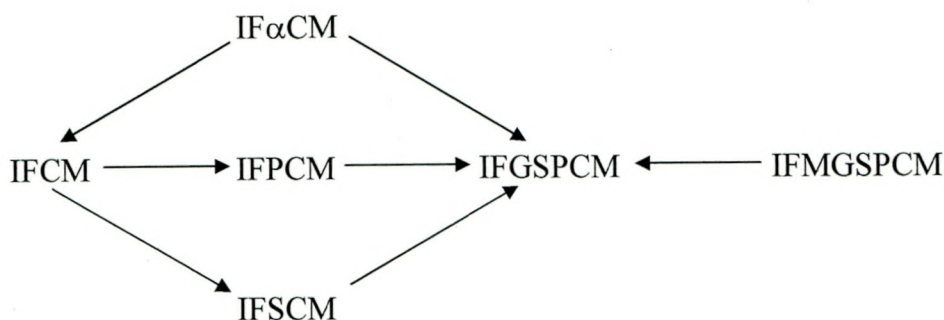
and

$$G_3 = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v) \rangle.$$

Then $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, G_3, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGSPCM but not an IFMGSPCM. Since

$A = \langle x, (0.6_a, 0.8_b), (0.4_a, 0.2_b) \rangle$ is IFGSPCS in X but $f(A) = \langle y, (0.6_u, 0.8_v), (0.4_u, 0.2_v) \rangle$ is not an IFGSPCS in Y , since $f(A) \subseteq G_3$ but $\text{spcl}(f(A)) = 1_{\sim} \subseteq G_3$.

The relation between various types of intuitionistic fuzzy closedness is given in the following diagram.



The reverse implications are not true in general in the above diagram.

Theorem 3.15: Let $f : X \rightarrow Y$ be a mapping. Then the following are equivalent if Y is an IFSPT_{1/2} space

- (i) f is an IFGSPCM
- (ii) $\text{spcl}(f(A)) \subseteq f(\text{cl}(A))$ for each IFS A of X .

Proof: (i) \Rightarrow (ii) Let A be an IFS in X . Then $\text{cl}(A)$ is an IFCS in X . (i) implies that $f(\text{cl}(A))$ is an IFGSPCS in Y . Since Y is an IFSPT_{1/2} space, $f(\text{cl}(A))$ is an IFSPCS in Y . Therefore $\text{spcl}(f(\text{cl}(A))) = f(\text{cl}(A))$. Now $\text{spcl}(f(A)) \subseteq \text{spcl}(f(\text{cl}(A))) = f(\text{cl}(A))$. Hence $\text{spcl}(f(A)) \subseteq f(\text{cl}(A))$ for each IFS A of X .

(ii) \Rightarrow (i) Let A be any IFCS in X . Then $\text{cl}(A) = A$. (ii) implies that

$$\text{spcl}(f(A)) \subseteq f(\text{cl}(A)) = f(A).$$

But $f(A) \subseteq \text{spcl}(f(A))$. Therefore $\text{spcl}(f(A)) = f(A)$. This implies $f(A)$ is an IFSPCS in Y . Since every IFSPCS is an IFGSPCS, $f(A)$ is an IFGSPCS in Y . Hence f is an IFGSPCM.

Theorem 3.16: Let $f : X \rightarrow Y$ be a bijection. Then the following are equivalent if Y is an IFSPT_{1/2} space

- (i) f is an IFGSPCM
- (ii) $\text{spcl}(f(A)) \subseteq f(\text{cl}(A))$ for each IFS A of X
- (iii) $f^{-1}(\text{spcl}(B)) \subseteq \text{cl}(f^{-1}(B))$ for every IFS B of Y .

Proof: (i) \Leftrightarrow (ii) is obvious from Theorem 3.15.

(ii) \Rightarrow (iii) Let B be an IFS in Y . Then $f^{-1}(B)$ is an IFS in X . Since f is onto, $\text{spcl}(B) = \text{spcl}(f(f^{-1}(B)))$ and (ii) implies $\text{spcl}(f(f^{-1}(B))) \subseteq f(\text{cl}(f^{-1}(B)))$. Therefore $\text{spcl}(B) \subseteq f(\text{cl}(f^{-1}(B)))$. Now $f^{-1}(\text{spcl}(B)) \subseteq f^{-1}(f(\text{cl}(f^{-1}(B))))$. Since f is one to one, $f^{-1}(\text{spcl}(B)) \subseteq \text{cl}(f^{-1}(B))$.

(iii) \Rightarrow (ii) Let A be any IFS of X . Then $f(A)$ is an IFS of Y . Since f is one to one, (iii) implies that $f^{-1}(\text{spcl}(f(A))) \subseteq \text{cl}(f^{-1}(f(A))) = \text{cl}(A)$. Therefore $f(f^{-1}(\text{spcl}(f(A)))) \subseteq f(\text{cl}(A))$. Since f is onto $\text{spcl}(f(A)) = f(f^{-1}(\text{spcl}(f(A)))) \subseteq f(\text{cl}(A))$.

Theorem 3.17: Let $f : X \rightarrow Y$ be an IFGSPCM. Then for every IFS A of X , $f(\text{cl}(A))$ is an IFGSPCS in Y .

Proof: Let A be any IFS in X . Then $\text{cl}(A)$ is an IFCS in X . By hypothesis $f(\text{cl}(A))$ is an IFGSPCS in Y .

Theorem 3.18: Let $f : X \rightarrow Y$ be an IFGSPCM where Y is an IFSPT_{1/2}space, then f is an IFCM if every IFSPCS is an IFCS in Y .

Proof: Let f be an IFGSPCM. Then for every IFCS A in X , $f(A)$ is an IFGSPCS in Y . Since Y is an IFSPT_{1/2} space, $f(A)$ is an IFSPCS in Y and by hypothesis $f(A)$ is an IFCS in Y . Hence f is an IFCM.

Theorem 3.19: Let $f : X \rightarrow Y$ be an IFGSPCM where Y is an IFSPT_{1/2} space. Then f is an IFPRCM if every IFSPCS is an IFRCS in Y .

Proof: Let A be an IFRCS in X . since every IFRCS is an IFCS, A is an IFCS in X . By hypothesis $f(A)$ is an IFGSPCS in Y . Since Y is an IFSPT_{1/2} space, $f(A)$ is an IFSPCS in Y and hence is an IFRCS in Y , by hypothesis. This implies that $f(A)$ is an IFPRCM.

Theorem 3.20: If every IFS is an IFCS, then an IFGSPCM is an IFGSP continuous mapping.

Proof: Let A be an IFCS in Y . Then $f^{-1}(A)$ is an IFS in X . Therefore $f^{-1}(A)$ is an IFCS in X . Since every IFCS is an IFGSPCS, $f^{-1}(A)$ is an IFGSPCS in X . This implies that f is an IFGSP continuous mapping.

Theorem 3.21: Let A be an IFGCS in X . An onto mapping $f : X \rightarrow Y$ is both IF continuous mapping and IFGSPCM, then $f(A)$ is an IFGSPCS in Y .

Proof: Let $f(A) \subseteq U$ where U is an IFOS in Y , then $A \subseteq f^{-1}(U)$ where $f^{-1}(U)$ is an IFOS in X , by hypothesis. Since A is an IFGCS, $\text{cl}(A) \subseteq f^{-1}(U)$ in X . Hence, $f(\text{cl}(A)) \subseteq f(f^{-1}(U)) = U$. But $f(\text{cl}(A))$ is an IFGSPCS in Y , since $\text{cl}(A)$ is an IFCS in X and f is an IFGSPCM. We have therefore $\text{spcl}(f(\text{cl}(A))) \subseteq U$. Now $\text{spcl}(f(A)) \subseteq \text{spcl}(f(\text{cl}(A))) \subseteq U$. Hence $f(A)$ is an IFGSPCS in Y .

Theorem 3.22: A mapping $f : X \rightarrow Y$ is an IFGSPCM if and only if for every IFS B of Y and for every IFOS U containing $f^{-1}(B)$, there is an IFGSPOS A of Y such that $B \subset A$ and $f^{-1}(A) \subset U$.

Proof: Necessity: Let B be any IFS in Y . Let U be an IFOS in X such that $f^{-1}(B) \subset U$, then U^c is an IFCS in X . By hypothesis $f(U^c)$ is an IFGSPCS in Y . Let $A = (f(U^c))^c$, then A is an IFGSPOS in Y and $B \subset A$. Now $f^{-1}(A) = f^{-1}((f(U^c))^c) = (f^{-1}(f(U^c)))^c \subset U$.

Sufficiency: Let A be any IFCS in X , then A^c is an IFOS in X and $f^{-1}(f(A^c))^c \subset A^c$. By hypothesis there exists an IFGSPOS B in Y such that $f(A^c) \subset B$ and $f^{-1}(B) \subset A^c$. therefore $A \subset (f^{-1}(B))^c$. Hence $B^c \subset f(A) \subset f(f^{-1}(B))^c \subset B^c$. This implies that $f(A) = B^c$. Since B^c is an IFGSPCS in Y , $f(A)$ is an IFGSPCS in Y . Hence f is an IFGSPCM.

Theorem 3.23: If $f : X \rightarrow Y$ is an IFCM and $g : Y \rightarrow Z$ is an IFGSPCM, then $g \circ f$ is an IFGSPCM.

Proof: Let A be an IFCS in X , then $f(A)$ is an IFCS in Y , Since f is an IFCM. Since g is an IFGSPCM, $g(f(A))$ is an IFGSPCS in Z . Therefore $g \circ f$ is an IFGSPCM.

Theorem 3.24: Let $f : X \rightarrow Y$ be a bijective map where Y is an IFSPT_{1/2} space. Then the following are equivalent.

- (i) f is an IFGSPCM
- (ii) $f(B)$ is an IFGSPOS in Y for every IFOS B in X .
- (iii) $f(\text{int}(B)) \subseteq \text{cl}(\text{int}(\text{cl}(f(B))))$ for every IFS B in X .

Proof: (i) \Rightarrow (ii) is obvious.

(ii) \Rightarrow (iii) Let B be an IFS in X , then $\text{int}(B)$ is an IFOS in X . By hypothesis $f(\text{int}(B))$ is an IFGSPOS in Y . Since Y is an IFSP $T_{1/2}$ space, $f(\text{int}(B))$ is an IFSPOS in Y . Therefore $f(\text{int}(B)) = \text{spint}(f(\text{int}(B))) = f(\text{int}(B)) \cap \text{cl}(\text{int}(\text{cl}(f(\text{int}(B))))) \subseteq \text{cl}(\text{int}(\text{cl}(f(\text{int}(B))))) \subseteq \text{cl}(\text{int}(\text{cl}(f(B))))$.

(iii) \Rightarrow (i) let A be an IFCS in X . Then A^c is an IFOS in X . By hypothesis, $f(\text{int}(A^c)) = f(A^c) \subseteq \text{cl}(\text{int}(\text{cl}(f(A^c))))$. That is $\text{int}(\text{cl}(\text{int}(f(A)))) \subseteq f(A)$. This implies $f(A)$ is an IFSPCS in Y and hence an IFGSPCS in Y . Therefore f is an IFGSPCM.

Theorem 3.25: Let $f : X \rightarrow Y$ be a bijective map where Y is an IFSP $T_{1/2}$ space. Then the following are equivalent.

- (i) f is an IFGSPCM
- (ii) $f(B)$ is an IFGSPCS in Y for every IFCS B in X .
- (iii) $\text{int}(\text{cl}(\text{int}(f(B)))) \subseteq f(\text{cl}(B))$ for every IFS B in X .

Proof: (i) \Rightarrow (ii) is obvious.

(ii) \Rightarrow (iii) Let B be an IFS in X , then $\text{cl}(B)$ is an IFCS in X . By hypothesis $f(\text{cl}(B))$ is an IFGSPCS in Y . Since Y is an IFSP $T_{1/2}$ space, $f(\text{cl}(B))$ is an IFSPCS in Y . Therefore $f(\text{cl}(B)) = \text{spcl}(f(\text{cl}(B))) = f(\text{cl}(B)) \cup \text{int}(\text{cl}(\text{int}(f(\text{cl}(B))))) \supseteq \text{int}(\text{cl}(\text{int}(f(\text{cl}(B))))) \supseteq \text{int}(\text{cl}(\text{int}(f(B))))$.

(iii) \Rightarrow (i) let A be an IFCS in X . By hypothesis, $f(\text{cl}(A)) = f(A) \subseteq \text{int}(\text{cl}(\text{int}(f(A))))$. This implies $f(A)$ is an IFSPCS in Y and hence an IFGSPCS in Y . Therefore f is an IFGSPCM.

Definition 3.26: A mapping $f : X \rightarrow Y$ is said to be an *intuitionistic fuzzy open mapping* (IFOM for short) if $f(A)$ is an IFOS in Y for each IFOS A in X .

Definition 3.27: A mapping $f : X \rightarrow Y$ is said to be an *intuitionistic fuzzy generalized semi-pre open mapping* (IFGSPOM for short) if $f(A)$ is an IFGSPOS in Y for each IFOS in X .

Theorem 3.28: If $f : X \rightarrow Y$ is a mapping. Then the following are equivalent if Y is an IFSP $T_{1/2}$ space

- (i) f is an IFGSPOM
- (ii) $f(\text{int}(A)) \subseteq \text{spint}(f(A))$ for each IFS A of X
- (iii) $\text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{spint}(B))$ for every IFS B of Y .

Proof: (i) \Rightarrow (ii) Let f be an IFGSPOM. Let A be any IFS in X . Then $\text{int}(A)$ is an IFOS in X . (i) implies that $f(\text{int}(A))$ is an IFGSPOS in Y . Since Y is an IFSP $T_{1/2}$ space, $f(\text{int}(A))$ is an IFSPOS in Y . Therefore $\text{spint}(f(\text{int}(A))) = f(\text{int}(A)) \subseteq f(A)$. Now $f(\text{int}(A)) = \text{spint}(f(\text{int}(A))) \subseteq \text{spint}(f(A))$

(ii) \Rightarrow (iii) Let B be any IFS in Y . Then $f^{-1}(B)$ is an IFS in X . (ii) implies that

$$f(\text{int}(f^{-1}(B))) \subseteq \text{spint}(f(f^{-1}(B))) = \text{spint}(B).$$

Now $\text{int}(f^{-1}(B)) \subseteq f^{-1}(f(\text{int}(f^{-1}(B)))) \subseteq f^{-1}(\text{spint}(B))$

(iii) \Rightarrow (i) Let A be an IFOS in X . Then $\text{int}(A) = A$ and $f(A)$ is an IFS in Y . (iii) implies that $\text{int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{spint}(f(A)))$. Now $A = \text{int}(A) \subseteq \text{int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{spint}(f(A)))$. Therefore $f(A) \subseteq f(f^{-1}(\text{spint}(f(A)))) = \text{spint}(f(A)) \subseteq f(A)$. This implies $\text{spint}(f(A)) = f(A)$. Hence $f(A)$ is an IFSPOS in Y . Since every IFSPOS is an IFGSPOS, $f(A)$ is an IFGSPOS in Y . Thus f is an IFGSPOM.

Theorem 3.29: A mapping $f: X \rightarrow Y$ is an IFGSPOM if $f(\text{spint}(A)) \subseteq \text{spint}(f(A))$ for every $A \subseteq X$

Proof: Let A be an IFOS in X . Then $\text{int}(A) = A$. Now $f(A) = f(\text{int}(A)) \subseteq f(\text{spint}(A)) \subseteq \text{spint}(f(A))$, by hypothesis. But $\text{spint}(f(A)) \subseteq f(A)$. Therefore $f(A)$ is an IFSPOS in X . That is $f(A)$ is an IFGSPOM in X . Hence f is an IFGSPOM.

Theorem 3.30: A mapping $f: X \rightarrow Y$ is an IFGSPOM if and only if $\text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{spint}(B))$ for every $B \subseteq Y$, where Y is an IFSPT_{1/2} space.

Proof: Necessity: Let $B \subseteq Y$. Then $f^{-1}(B) \subseteq X$ and $\text{int}(f^{-1}(B))$ is an IFOS in X . By hypothesis, $f(\text{int}(f^{-1}(B)))$ is an IFGSPOM in Y . Since Y is an IFSPT_{1/2} space, $f(\text{int}(f^{-1}(B)))$ is an IFSPOS in Y . Therefore $f(\text{int}(f^{-1}(B))) = \text{spint}(f(\text{int}(f^{-1}(B)))) \subseteq \text{spint}(B)$. This implies $\text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{spint}(B))$.

Sufficiency: Let A be an IFOS in X . Therefore $\text{int}(A) = A$. Then $f(A) \subseteq Y$. By hypothesis $\text{int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{spint}(f(A)))$. That is

$$\text{int}(A) \subseteq \text{int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{spint}(f(A))).$$

Therefore $A \subseteq f^{-1}(\text{spint}(f(A)))$. This implies $f(A) \subseteq \text{spint}(f(A)) \subseteq f(A)$. Hence $f(A)$ is an IFSPOS in Y and hence an IFGSPOM in Y . Thus f is an IFGSPOM.

Theorem 3.31: Let $f: X \rightarrow Y$ be an onto mapping where Y is an IFSPT_{1/2} space. Then f is an IFGSPOM if and only if for any IFP $c(\alpha, \beta) \in Y$ and for any IFN B of $f^{-1}(c(\alpha, \beta))$, there is an IFSN A of $c(\alpha, \beta)$ such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B$.

Proof: Necessity: Let $c(\alpha, \beta) \in Y$ and let B be an IFN of $f^{-1}(c(\alpha, \beta))$. Then there is an IFOS C in X such that $f^{-1}(c(\alpha, \beta)) \in C \subseteq B$. Since f is an IFGSPCM, $f(C)$ is an IFGSPOM in Y . Since Y is an IFSPT_{1/2} space, $f(C)$ is an IFSPOS in Y and

$$c(\alpha, \beta) \in f(f^{-1}(c(\alpha, \beta))) \subseteq f(C) \subseteq f(B).$$

Put $A = f(C)$. Then A is an IFSN of $c(\alpha, \beta)$ and $c(\alpha, \beta) \in A \subseteq f(B)$. Thus $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq f^{-1}(f(B)) = B$. That is $f^{-1}(A) \subseteq B$.

Sufficiency: Let $B \subseteq X$ be an IFOS. If $f(B) = 0$, then there is nothing to prove. Suppose that $c(\alpha, \beta) \in f(B)$. This implies $f^{-1}(c(\alpha, \beta)) \in B$. Then B is an IFN of $f^{-1}(c(\alpha, \beta))$. By hypothesis there is an IFSN A of $c(\alpha, \beta)$ such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B$. Therefore there is an IFSPOM C in Y such that

$$c(\alpha, \beta) \in C \subseteq A = f(f^{-1}(A)) \subseteq f(B).$$

Hence $f(B) = \cup\{c(\alpha, \beta) \mid c(\alpha, \beta) \in f(B)\} \subseteq \cup\{C_{c(\alpha, \beta)} \mid c(\alpha, \beta) \in f(B)\} \subseteq f(B)$. Thus $f(B) = \cup\{C_{c(\alpha, \beta)} \mid c(\alpha, \beta) \in f(B)\}$. Since each C is an IFSPOM, $f(B)$ is also an IFSPOM and hence is an IFGSPOM in Y . Therefore f is an IFGSPOM.

Theorem 3.32: If $f: X \rightarrow Y$ is a mapping, then the following are equivalent.

- (i) f is an IFMGSPCM
- (ii) $f(A)$ is an IFGSPCS in Y for every IFGSPCS A in X
- (iii) $f(A)$ is an IFGSPOM in Y for every IFGSPOM A in X

Proof: (i) \Leftrightarrow (ii) is obvious from the Definition 3.11.

(ii) \Rightarrow (iii) Let A be an IFGSPOM in X . Then A^c is an IFGSPCS in X . By hypothesis, $f(A^c)$ is an IFGSPCS in Y . That is $f(A)^c$ is an IFGSPCS in Y and hence $f(A)$ is an IFGSPOM in Y .

(iii) \Rightarrow (i) Let A be an IFGSPCS in X . Then A^c is an IFGSPOS in X . By hypothesis, $f(A^c)$ is an IFGSPOS in Y . That is $f(A)^c$ is an IFGSPOS in Y and hence $f(A)$ is an IFGSPCS in Y . Hence f is an IFMGSPCM.

Theorem 3.33: Let $f: X \rightarrow Y$ be a bijective mapping, where X is an IFSPT_{1/2} space. Then the following are equivalent.

- (i) f is an IFMGSPCM
- (ii) for each IFP $c(\alpha, \beta) \in Y$ and every IFSN A of $f^{-1}(c(\alpha, \beta))$, there exists an IFGSPOS B in Y such that $c(\alpha, \beta) \in B \subseteq f(A)$.
- (iii) for each IFP $c(\alpha, \beta) \in Y$ and every IFSN A of $f^{-1}(c(\alpha, \beta))$, there exists an IFGSPOS B in Y such that $c(\alpha, \beta) \in B$ and $f^{-1}(B) \subseteq A$.

Proof: (i) \Rightarrow (ii) Let $c(\alpha, \beta) \in Y$ and A the IFSN of $f^{-1}(c(\alpha, \beta))$. Then there exists an IFSPOS C in X such that $f^{-1}(c(\alpha, \beta)) \in C \subseteq A$. Since every IFSPOS is an IFGSPOS, C is an IFGSPOS in X . Then by hypothesis, $f(C)$ is an IFGSPOS in Y . Now $c(\alpha, \beta) \in f(C) \subseteq f(A)$. Put $B = f(C)$. This implies $c(\alpha, \beta) \in B \subseteq f(A)$.

(ii) \Rightarrow (iii) Let $c(\alpha, \beta) \in Y$ and A the IFSN of $f^{-1}(c(\alpha, \beta))$. Then there exists an IFSPOS C in X such that $f^{-1}(c(\alpha, \beta)) \in C \subseteq A$. Since every IFSPOS is an IFGSPOS, C is an IFGSPOS in X . Then by hypothesis, $f(C)$ is an IFGSPOS in Y . Now

$$c(\alpha, \beta) \in f(C) \subseteq f(A).$$

Put $B = f(C)$. This implies $c(\alpha, \beta) \in B \subseteq f(A)$. Now $f^{-1}(B) \subseteq f^{-1}(f(A)) \subseteq A$. That is $f^{-1}(B) \subseteq A$.

(iii) \Rightarrow (i) Let A be an IFGSPCS in X . Since X is an IFSPT_{1/2} space, A is an IFSPOS in X . Let $c(\alpha, \beta) \in Y$ and $f^{-1}(c(\alpha, \beta)) \in A$. That is $c(\alpha, \beta) \in f(A)$. This implies A is an IFSN of $f^{-1}(c(\alpha, \beta))$. Then by hypothesis, there exists an IFGSPOS B in Y such that $c(\alpha, \beta) \in B$ and $f^{-1}(B) \subseteq A$. Hence by Theorem 2.21, $f(A)$ is an IFGSPOS in Y . Therefore f is an IFMGSPCM

Theorem 3.34: If $f: X \rightarrow Y$ is a bijective mapping, then the following are equivalent.

- (i) f is an IFMGSPCM
- (ii) $f(A)$ is an IFGSPOS in Y for every IFGSPOS A in X
- (iii) for every IFP $c(\alpha, \beta) \in Y$ and for every IFGSPOS B in X such that $f^{-1}(c(\alpha, \beta)) \in B$, there exists an IFGSPOS A in Y such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B$.

Proof: (i) \Rightarrow (ii) is obvious by Theorem 3.32.

(ii) \Rightarrow (iii) Let $c(\alpha, \beta) \in Y$ and let B be an IFGSPOS in X such that $f^{-1}(c(\alpha, \beta)) \in B$. This implies $c(\alpha, \beta) \in f(B)$. By hypothesis, $f(B)$ is an IFGSPOS in Y . Let $A = f(B)$. Therefore $c(\alpha, \beta) \in f(B) = A$ and $f^{-1}(A) = f^{-1}(f(B)) \subseteq B$.

(iii) \Rightarrow (i) Let B be an IFGSPCS in X . Then B^c is an IFGSPOS in X . Let $c(\alpha, \beta) \in Y$ and $f^{-1}(c(\alpha, \beta)) \in B^c$. This implies $c(\alpha, \beta) \in f(B^c)$. By hypothesis there exists an IFGSPOS A in Y such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B^c$. Put $A = f(B^c)$. Then $c(\alpha, \beta) \in f(B^c)$ and $A = f(f^{-1}(B^c)) \subseteq f(B^c)$. Hence by Theorem 2.21, $f(B^c)$ is an IFGSPOS in Y . Therefore $f(B)$ is an IFGSPCS in Y . Thus f is an IFMGSPCM.

Theorem 3.35: If $f: X \rightarrow Y$ is a bijective mapping, then the following are equivalent.

- (i) f is an IFMGSPCM
- (ii) $f(A)$ is an IFGSPOS in Y for every IFGSPOS A in X

(iii) $f(\text{spint}(B)) \subseteq \text{spint}(f(B))$ for every IFS B in X

(iv) $\text{spcl}(f(B)) \subseteq f(\text{spcl}(B))$ for every IFS B in X .

Proof: $(i) \Rightarrow (ii)$ is obvious.

$(ii) \Rightarrow (iii)$ Let B be any IFS in X . Since $\text{spint}(B)$ is an IFSPPOS, it is an IFGSPOS in X . Then by hypothesis, $f(\text{spint}(B))$ is an IFGSPOS in Y . Since Y is an IFSP $T_{1/2}$ space, $f(\text{spint}(B))$ is an IFSPPOS in Y . Therefore $f(\text{spint}(B)) = \text{spint}(f(\text{spint}(B))) \subseteq \text{spint}(f(B))$.

$(iii) \Rightarrow (iv)$ can easily proved by taking complement in (iii) .

$(iv) \Rightarrow (i)$ Let A be an IFGSPCS in X . By hypothesis, $\text{spcl}(f(A)) \subseteq f(\text{spcl}(A))$. Since X is an IFSP $T_{1/2}$ space, A is an IFSPCS in X . Therefore,

$$\text{spcl}(f(A)) \subseteq f(\text{spcl}(A)) = f(A) \subseteq \text{spcl}(f(A)).$$

Hence $f(A)$ is an IFSPCS in Y and hence an IFGSPCS in Y . Thus f is an IFMGSPCM.

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