

4.1 Introduction

Irresolute maps are one of the most important concepts in topological spaces. Crossley and Hildebrand (1972) introduced the concept of irresolute maps and proved that irresolute maps are stronger than semi continuous maps but are independent of continuous maps. Maheshwari and Thakur (1980), Devi et al.(1997), Sundram (1991), Veera Kumar (2000a) and many others have contributed to the study of generalized notions of irresolute maps.

In this chapter, two new classes of irresolute maps namely ψ^* α -irresolute maps and contra ψ^* α -irresolute maps are introduced. Also the relationship between these irresolute maps and the other existing irresolute maps are obtained and their properties are analyzed.

4.2 ψ^* α -irresolute maps

In this section, the strong form ψ^* α -continuous maps, namely ψ^* α -irresolute maps is introduced and its properties are analyzed. It is shown that composition of two ψ^* α -irresolute maps is also a ψ^* α -irresolute map.

Definition 4.2.1 A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **ψ^* α -irresolute** if $f^{-1}(V)$ is ψ^* α -closed in (X, τ) for every ψ^* α -closed set V in (Y, σ) .

Example 4.2.2 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is ψ^* α -irresolute.

Proposition 4.2.3 A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is ψ^* α -irresolute if and only if $f^{-1}(V)$ is ψ^* α -open in (X, τ) for every ψ^* α -open set V in (Y, σ) .

Proposition 4.2.4 If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a ψ^* α -irresolute map, then it is a ψ^* α -continuous map but not conversely.

Proof: Let V be any closed set in (Y, σ) . Since every closed set is $\psi^* \alpha$ -closed set and f is $\psi^* \alpha$ -irresolute, $f^{-1}(V)$ is $\psi^* \alpha$ -closed in (X, τ) . Therefore f is a $\psi^* \alpha$ -continuous map.

Example 4.2.5 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then f is $\psi^* \alpha$ -continuous but not $\psi^* \alpha$ -irresolute, since for the $\psi^* \alpha$ -closed $\{a, c\}$ in (Y, σ) , $f^{-1}(\{a, c\}) = \{a, b\}$ is not $\psi^* \alpha$ -closed in (X, τ) .

Remark 4.2.6 Every $\psi^* \alpha$ -irresolute map $f : (X, \tau) \rightarrow (Y, \sigma)$ is continuous and α -continuous, if (X, τ) is, respectively a $\psi^* \alpha T_c$ -space and a $\psi^* \alpha T_\alpha$ -space.

Theorem 4.2.7 Let (X, τ) be any topological space and (Y, σ) be a $\psi^* \alpha T_c$ -space and $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then the following are equivalent.

- (a) f is $\psi^* \alpha$ -irresolute
- (b) f is $\psi^* \alpha$ -continuous.

Proof: (a) \Rightarrow (b) Let V be a closed set in (Y, σ) . Since every closed set is $\psi^* \alpha$ -closed in (Y, σ) and $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\psi^* \alpha$ -irresolute, $f^{-1}(V)$ is $\psi^* \alpha$ -closed in (X, τ) . Hence f is $\psi^* \alpha$ -continuous.

(b) \Rightarrow (a) Let V be a $\psi^* \alpha$ -closed set in (Y, σ) . Since (Y, σ) is a $\psi^* \alpha T_c$ -space, V is closed in (Y, σ) . Since $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\psi^* \alpha$ -continuous, $f^{-1}(V)$ is $\psi^* \alpha$ -closed in (X, τ) . Hence f is $\psi^* \alpha$ -irresolute.

Proposition 4.2.8 Every quasi $\psi^* \alpha$ -continuous (resp. perfectly $\psi^* \alpha$ -continuous) map is a $\psi^* \alpha$ -irresolute map but not conversely.

Proof: Let V be any $\psi^* \alpha$ -closed set in (Y, σ) . Since $f : (X, \tau) \rightarrow (Y, \sigma)$ is quasi $\psi^* \alpha$ -continuous (resp. perfectly $\psi^* \alpha$ -continuous), $f^1(V)$ is closed (resp. clopen) in (X, τ) . Since every closed set is $\psi^* \alpha$ -closed, $f^1(V)$ is $\psi^* \alpha$ -closed in (X, τ) . Hence f is a $\psi^* \alpha$ -irresolute map.

Example 4.2.9 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\psi^* \alpha$ -irresolute but not quasi $\psi^* \alpha$ -continuous and not perfectly $\psi^* \alpha$ -continuous, since $\{b\}$ is $\psi^* \alpha$ -closed in (Y, σ) but $f^1(\{b\}) = \{b\}$ is not closed in (X, τ) .

Remark 4.2.10 The following examples show that irresolute maps and $\psi^* \alpha$ -irresolute maps are independent.

Example 4.2.11 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is irresolute but not $\psi^* \alpha$ -irresolute, since for the $\psi^* \alpha$ -closed set $\{a\}$ in (Y, σ) , $f^1(\{a\}) = \{a\}$ is not $\psi^* \alpha$ -closed in (X, τ) .

Example 4.2.12 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\psi^* \alpha$ -irresolute but not irresolute, since for the semi closed set $\{b\}$ in (Y, σ) , $f^1(\{b\}) = \{b\}$ is not semi closed in (X, τ) .

Remark 4.2.13 The following examples show that α -irresolute maps and $\psi^* \alpha$ -irresolute maps are independent.

Example 4.2.14 Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, X\}$ and $\sigma = \{\phi, \{a, b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = a$, $f(b) = b$, $f(c) = d$, $f(d) = c$. Then f is α -irresolute but not $\psi^* \alpha$ -irresolute, since for the $\psi^* \alpha$ -closed sets $\{a, d\}$, $\{b, d\}$, $\{a, c, d\}$ and $\{b, c, d\}$ in (Y, σ) , $f^1(\{a, d\}) = \{a, c\}$, $f^1(\{b, d\}) = \{b, c\}$, $f^1(\{a, c, d\}) = \{a, c, d\}$ and $f^1(\{b, c, d\}) = \{b, c, d\}$ are not $\psi^* \alpha$ -closed in (X, τ) .

Example 4.2.15 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\psi^* \alpha$ -irresolute but not α -irresolute,

since for the α -closed sets $\{a, c\}$ and $\{b, c\}$ in (Y, σ) , $f^{-1}(\{a, c\}) = \{a, c\}$ and $f^{-1}(\{b, c\}) = \{b, c\}$ are not α -closed in (X, τ) .

Remark 4.2.16 The following examples show that g -irresolute maps and $\psi^* \alpha$ -irresolute maps are independent.

Example 4.2.17 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is g -irresolute but not $\psi^* \alpha$ -irresolute, since for the $\psi^* \alpha$ -closed set $\{a, c\}$ in (Y, σ) , $f^{-1}(\{a, c\}) = \{a, c\}$ is not $\psi^* \alpha$ -closed in (X, τ) .

Example 4.2.18 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\psi^* \alpha$ -irresolute but not g -irresolute, since for the g -closed sets $\{b\}$ and $\{a, b\}$ in (Y, σ) , $f^{-1}(\{b\}) = \{b\}$ and $f^{-1}(\{a, b\}) = \{a, b\}$ are not g -closed in (X, τ) .

Remark 4.2.19 The following examples show that g^* -irresolute maps and $\psi^* \alpha$ -irresolute maps are independent.

Example 4.2.20 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is g^* -irresolute but not $\psi^* \alpha$ -irresolute, since for the $\psi^* \alpha$ -closed set $\{b\}$ in (Y, σ) , $f^{-1}(\{b\}) = \{b\}$ is not $\psi^* \alpha$ -closed in (X, τ) .

Example 4.2.21 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\psi^* \alpha$ -irresolute but not g^* -irresolute, since for the g^* -closed sets $\{c\}$ and $\{a, c\}$ in (Y, σ) , $f^{-1}(\{a\}) = \{a\}$ and $f^{-1}(\{a, c\}) = \{a, c\}$ are not g^* -closed in (X, τ) .

Remark 4.2.22 The following examples show that \hat{g} -irresolute maps and $\psi^* \alpha$ -irresolute maps are independent.

Example 4.2.23 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is \hat{g} -irresolute but not $\psi^* \alpha$ -irresolute, since for the $\psi^* \alpha$ -closed set $\{b\}$ in (Y, σ) , $f^{-1}(\{b\}) = \{b\}$ is not $\psi^* \alpha$ -closed in (X, τ) .

Example 4.2.24 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\psi^* \alpha$ -irresolute but not \hat{g} -irresolute, since for the \hat{g} -closed sets $\{b\}$ and $\{c\}$ in (Y, σ) , $f^{-1}(\{b\}) = \{b\}$ and $f^{-1}(\{c\}) = \{c\}$ are not \hat{g} -closed in (X, τ) .

Remark 4.2.25 The following examples show that αg -irresolute maps and $\psi^* \alpha$ -irresolute maps are independent.

Example 4.2.26 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is αg -irresolute but not $\psi^* \alpha$ -irresolute, since for the $\psi^* \alpha$ -closed set $\{a, c\}$ in (Y, σ) , $f^{-1}(\{a, c\}) = \{a, c\}$ is not $\psi^* \alpha$ -closed in (X, τ) .

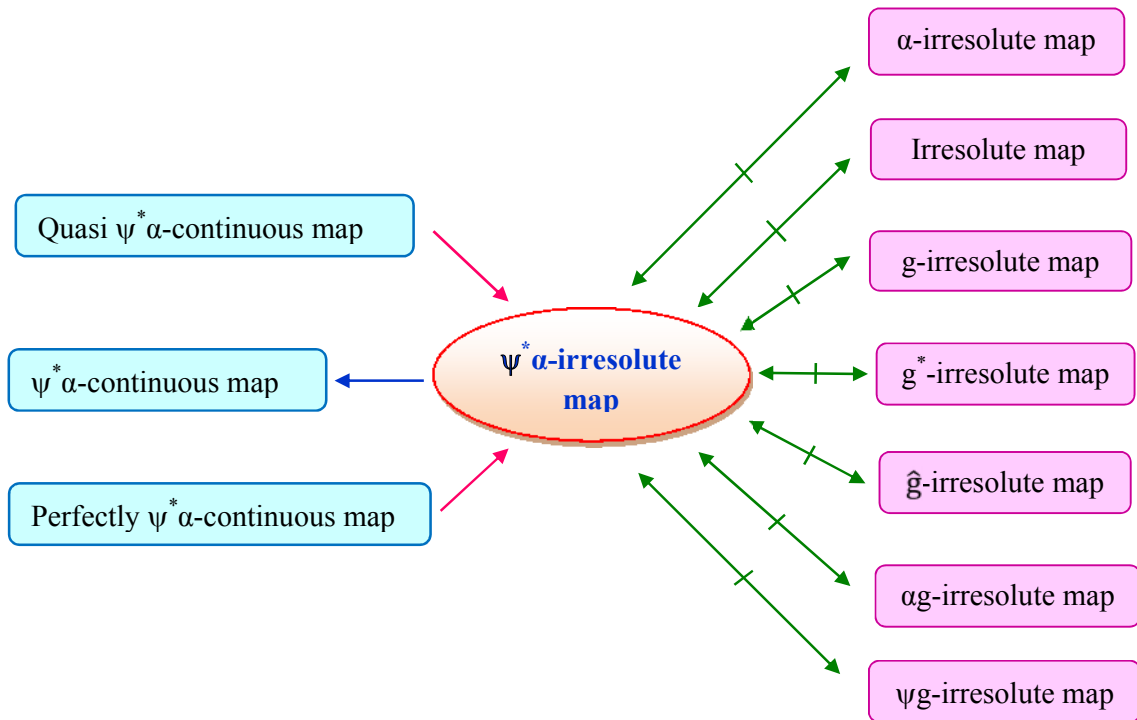
Example 4.2.27 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\psi^* \alpha$ -irresolute but not αg -irresolute, since for the αg -closed set $\{a, b\}$ in (Y, σ) , $f^{-1}(\{a, b\}) = \{a, b\}$ is not αg -closed in (X, τ) .

Remark 4.2.28 The following examples show that ψg -irresolute maps and $\psi^* \alpha$ -irresolute maps are independent .

Example 4.2.29 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is ψg -irresolute but not $\psi^* \alpha$ -irresolute, since for the $\psi^* \alpha$ -closed set $\{a, c\}$ in (Y, σ) , $f^{-1}(\{a, c\}) = \{a, c\}$ is not $\psi^* \alpha$ -closed in (X, τ) .

Example 4.2.30 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\psi^* \alpha$ -irresolute but not ψg -irresolute, since for the ψg -closed set $\{b\}$ in (Y, σ) , $f^{-1}(\{b\}) = \{b\}$ is not ψg -closed in (X, τ) .

Remark 4.2.31 The above results are depicted in the following diagram.



Proposition 4.2.32 If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\psi^* \alpha$ -irresolute then for every subset A of (X, τ) such that $f(A)$ is $\psi^* \alpha$ -closed in (Y, σ) , $f(\psi^* \alpha \text{cl}(A)) \subseteq \psi^* \alpha \text{cl}(f(A))$.

Proof : Let A be a subset of (X, τ) such that $f(A)$ is $\psi^* \alpha$ -closed in (Y, σ) . Since f is $\psi^* \alpha$ -irresolute, $f^1(\psi^* \alpha \text{cl}(f(A)))$ is $\psi^* \alpha$ -closed in (X, τ) . Now $A \subseteq f^1(f(A)) \subseteq f^1((\psi^* \alpha \text{cl}(f(A))))$. Therefore $\psi^* \alpha \text{cl}(A) \subseteq f^1((\psi^* \alpha \text{cl}(f(A))))$ and hence $f(\psi^* \alpha \text{cl}(A)) \subseteq f(f^1((\psi^* \alpha \text{cl}(f(A)))) \subseteq \psi^* \alpha \text{cl}(f(A))$.

Proposition 4.2.33 If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\psi^* \alpha$ -irresolute then for every $\psi^* \alpha$ -closed set $B \subseteq Y$, $\psi^* \alpha \text{cl}(f^1(B)) \subseteq f^1(\psi^* \alpha \text{cl}(B))$.

Proof: Let B be a $\psi^* \alpha$ -closed set in (Y, σ) . Then $\psi^* \alpha \text{cl}(B)$ is $\psi^* \alpha$ -closed in (Y, σ) . Since f is $\psi^* \alpha$ -irresolute, $f^1(\psi^* \alpha \text{cl}(B))$ is $\psi^* \alpha$ -closed in (X, τ) . Since $B \subseteq \psi^* \alpha \text{cl}(B)$, $f^1(B) \subseteq f^1(\psi^* \alpha \text{cl}(B))$. By definition of $\psi^* \alpha$ -closure, $\psi^* \alpha \text{cl}(f^1(B)) \subseteq f^1(\psi^* \alpha \text{cl}(B))$.

Proposition 4.2.34 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a closed and surjective $\psi^* \alpha$ -irresolute map. If (X, τ) is a $\psi^* \alpha T_c$ -space, then (Y, σ) is also a $\psi^* \alpha T_c$ -space.

Proof: Let V be any ψ^* - α -closed set in (Y, σ) . Since f is a ψ^* - α -irresolute map, $f^{-1}(V)$ is ψ^* - α -closed in (X, τ) . Since (X, τ) is a ψ^* - α - T_c -space, $f^{-1}(V)$ is closed in (X, τ) . Since f is closed and surjective, $f(f^{-1}(V)) = V$ is closed in (Y, σ) . Hence (Y, σ) is a ψ^* - α - T_c -space.

Theorem 4.2.35 If $f : (X, \tau) \rightarrow (Y, \sigma)$ is bijective, pre ψ g-open and ψ^* - α -continuous from (X, τ) to an α -space (Y, σ) , then f is a ψ^* - α -irresolute map.

Proof: Let A be a ψ^* - α -closed set in (Y, σ) . Let U be any ψ g-open set in (X, τ) such that $f^{-1}(A) \subseteq U$. Then $A \subseteq f(U)$. Since A is ψ^* - α -closed and $f(U)$ is ψ g-open in (Y, σ) , $\alpha\text{cl}(A) \subseteq f(U)$ and $f^{-1}(\alpha\text{cl}(A)) \subseteq U$. Since f is ψ^* - α -continuous and (Y, σ) is an α -space, $f^{-1}(\alpha\text{cl}(A))$ is ψ^* - α -closed in (X, τ) and so $\alpha\text{cl}(f^{-1}(A)) \subseteq \alpha\text{cl}(f^{-1}(\alpha\text{cl}(A))) \subseteq U$. Therefore $f^{-1}(A)$ is ψ^* - α -closed in (X, τ) and hence f is a ψ^* - α -irresolute map.

Theorem 4.2.36 If $f : (X, \tau) \rightarrow (Y, \sigma)$ is bijective, pre ψ g-open and α -irresolute, then f is a ψ^* - α -irresolute map.

Proof: Let A be a ψ^* - α -closed set in (Y, σ) . Let U be any ψ g-open set in (X, τ) such that $f^{-1}(A) \subseteq U$. Then $A \subseteq f(U)$. Since A is ψ^* - α -closed and f is pre ψ g-open, $\alpha\text{cl}(A) \subseteq f(U)$ and $f^{-1}(\alpha\text{cl}(A)) \subseteq U$. Since f is α -irresolute and $\alpha\text{cl}(A)$ is α -closed in (Y, σ) , $f^{-1}(\alpha\text{cl}(A))$ is α -closed in (X, τ) . Thus $\alpha\text{cl}(f^{-1}(A)) \subseteq \alpha\text{cl}(f^{-1}(\alpha\text{cl}(A))) \subseteq U$ and so $f^{-1}(A)$ is ψ^* - α -closed in (X, τ) and hence f is a ψ^* - α -irresolute map.

Theorem 4.2.37 If $f : (X, \tau) \rightarrow (Y, \sigma)$ is bijective, pre α -closed and ψ g-irresolute, then $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is a ψ^* - α -irresolute map.

Proof: Let A be a ψ^* - α -closed set in (X, τ) . Let $(f^{-1})^{-1}(A) = f(A) \subseteq U$, where U is ψ g-open set in (Y, σ) . Then $A \subseteq f^{-1}(U)$ holds. Since $f^{-1}(U)$ is ψ g-open in (X, τ) and A is ψ^* - α -closed in (X, τ) , $\alpha\text{cl}(A) \subseteq f^{-1}(U)$ and hence $f(\alpha\text{cl}(A)) \subseteq U$. Since f is pre α -closed and $\alpha\text{cl}(A)$ is α -closed in (X, τ) , $f(\alpha\text{cl}(A))$ is α -closed in (Y, σ) and so $f(\alpha\text{cl}(A))$ is ψ^* - α -closed in (Y, σ) . Therefore $\alpha\text{cl}(f(A)) \subseteq \alpha\text{cl}(f(\alpha\text{cl}(A))) \subseteq U$. Thus $f(A)$ is ψ^* - α -closed in (Y, σ) and so f^{-1} is a ψ^* - α -irresolute map.

Proposition 4.2.38 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi^* \alpha$ -irresolute map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be a $\psi^* \alpha$ -continuous map. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is a $\psi^* \alpha$ -continuous map.

Proof: Let V be any closed set in (Z, η) . Since g is $\psi^* \alpha$ -continuous, $g^{-1}(V)$ is $\psi^* \alpha$ -closed in (Y, σ) . Since f is $\psi^* \alpha$ -irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^* \alpha$ -closed in X . Hence $g \circ f$ is a $\psi^* \alpha$ -continuous map.

Proposition 4.2.39 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi^* \alpha$ -irresolute map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be a continuous (resp. α -continuous) map. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is a $\psi^* \alpha$ -continuous map.

Proof: Let V be any closed set in (Z, η) . Since g is continuous (resp. α -continuous), $g^{-1}(V)$ is closed (resp. α -closed) in (Y, σ) . Since every closed (resp. α -closed) set is $\psi^* \alpha$ -closed, $g^{-1}(V)$ is $\psi^* \alpha$ -closed. Since f is $\psi^* \alpha$ -irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^* \alpha$ -closed in (X, τ) . Therefore $g \circ f$ is a $\psi^* \alpha$ -continuous map.

Proposition 4.2.40 If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ are $\psi^* \alpha$ -irresolute maps, then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is a $\psi^* \alpha$ -irresolute map.

Proof: Let V be a $\psi^* \alpha$ -closed set in (Z, η) . Since g is $\psi^* \alpha$ -irresolute, $g^{-1}(V)$ is $\psi^* \alpha$ -closed in (Y, σ) . Since f is $\psi^* \alpha$ -irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^* \alpha$ -closed in (X, τ) . Therefore $g \circ f$ is a $\psi^* \alpha$ -irresolute map.

Proposition 4.2.41 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $\psi^* \alpha$ -irresolute map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be a contra $\psi^* \alpha$ -continuous map. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi^* \alpha$ -continuous map.

Proof: Let V be any closed set in (Z, η) . Since g is contra $\psi^* \alpha$ -continuous, $g^{-1}(V)$ is $\psi^* \alpha$ -open in (Y, σ) . Since f is $\psi^* \alpha$ -irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^* \alpha$ -open in (X, τ) . Therefore $g \circ f$ is a contra $\psi^* \alpha$ -continuous map.

4.3 Contra $\psi^* \alpha$ -irresolute maps

In this section, a new class of maps called contra $\psi^* \alpha$ -irresolute maps is defined and some of their properties are discussed. The composition of two contra $\psi^* \alpha$ -irresolute maps need not be a contra $\psi^* \alpha$ -irresolute map is shown by an example.

Definition 4.3.1 A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **contra $\psi^* \alpha$ -irresolute** if $f^{-1}(V)$ is $\psi^* \alpha$ -closed in (X, τ) for every $\psi^* \alpha$ -open set V in (Y, σ) .

Example 4.3.2 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is contra $\psi^* \alpha$ -irresolute.

Proposition 4.3.3 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then the following statements are equivalent

- (i) f is a contra $\psi^* \alpha$ -irresolute map.
- (ii) The inverse image of every $\psi^* \alpha$ -closed set in (Y, σ) is $\psi^* \alpha$ -open in (X, τ) .

Proposition 4.3.4 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a perfectly $\psi^* \alpha$ -continuous map then f is a contra $\psi^* \alpha$ -irresolute map but not conversely.

Proof: Let V be a $\psi^* \alpha$ -open set in (Y, σ) . Since f is perfectly $\psi^* \alpha$ -continuous, $f^{-1}(V)$ is clopen in (X, τ) . Since every closed set is $\psi^* \alpha$ -closed, $f^{-1}(V)$ is $\psi^* \alpha$ -closed in (X, τ) . Hence f is a contra $\psi^* \alpha$ -irresolute map.

Example 4.3.5 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is contra $\psi^* \alpha$ -irresolute but not perfectly $\psi^* \alpha$ -continuous, since for the $\psi^* \alpha$ -open sets $\{b\}$ and $\{a, b\}$ in (Y, σ) , $f^{-1}(\{b\}) = \{c\}$ and $f^{-1}(\{a, b\}) = \{b, c\}$ are closed but not open in (X, τ) .

Proposition 4.3.6 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a strongly $\psi^* \alpha$ -continuous map then f is a contra $\psi^* \alpha$ -irresolute map but not conversely.

Proof: Let V be any $\psi^* \alpha$ -open set in (Y, σ) . Since f is strongly $\psi^* \alpha$ -continuous, for every subset V of (Y, σ) , $f^{-1}(V)$ is $\psi^* \alpha$ -clopen in (X, τ) . Therefore $f^{-1}(V)$ is $\psi^* \alpha$ -closed in (X, τ) . Hence f is a contra $\psi^* \alpha$ -irresolute map

Example 4.3.7 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is a contra

ψ^* α -irresolute map but not a strongly ψ^* α -continuous map, since for the subset $\{a, c\}$ in (Y, σ) , $f^{-1}(\{a, c\}) = \{b, c\}$ is ψ^* α -closed but not ψ^* α -open in (X, τ) .

Proposition 4.3.8 Every contra ψ^* α -irresolute map is a contra ψ^* α -continuous map but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is a contra ψ^* α -irresolute map. Let V be any open set in (Y, σ) . Since every open set is ψ^* α -open, V is ψ^* α -open in (Y, σ) . Since f is contra ψ^* α -irresolute, $f^{-1}(V)$ is ψ^* α -closed in (X, τ) . Hence f is a contra ψ^* α -continuous map.

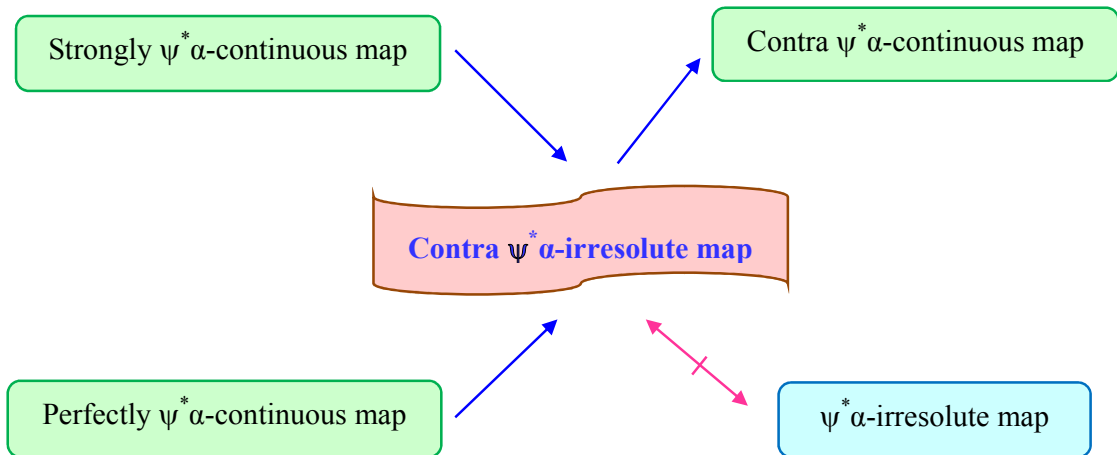
Example 4.3.9 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is contra ψ^* α -continuous but not contra ψ^* α -irresolute, since for the ψ^* α -open set $\{a, c\}$ in (Y, σ) , $f^{-1}(\{a, c\}) = \{a, b\}$ is not ψ^* α -closed in (X, τ) .

Remark 4.3.10 The following examples show that contra ψ^* α -irresolute and ψ^* α -irresolute maps are independent.

Example 4.3.11 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is a ψ^* α -irresolute map but not a contra ψ^* α -irresolute map, since for the ψ^* α -open set $\{a, c\}$ in (Y, σ) , $f^{-1}(\{a, c\}) = \{a, c\}$ is not ψ^* α -closed in (X, τ) .

Example 4.3.12 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is a contra ψ^* α -irresolute map but not a ψ^* α -irresolute map, since for the ψ^* α -closed set $\{b, c\}$ in (Y, σ) , $f^{-1}(\{b, c\}) = \{a, b\}$ is not ψ^* α -closed in (X, τ) .

Remark 4.3.13 The above results are given in the following diagram



Proposition 4.3.14 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be any map. Then the following results hold.

- (a) If f is a quasi $\psi^* \alpha$ -continuous map and (X, τ) is a discrete space, then f is a contra $\psi^* \alpha$ -irresolute map.
- (b) If f is a totally $\psi^* \alpha$ -continuous map and (Y, σ) is a ${}_{\psi^* \alpha} T_c$ -space, then f is a contra $\psi^* \alpha$ -irresolute map.
- (c) If f is a contra $\psi^* \alpha$ -continuous map and (Y, σ) is a ${}_{\psi^* \alpha} T_c$ -space, then f is a contra $\psi^* \alpha$ -irresolute map.

Proof: (a) Let V be any $\psi^* \alpha$ -open set in (Y, σ) . Since f is quasi $\psi^* \alpha$ -continuous, $f^{-1}(V)$ is open in (X, τ) . Since (X, τ) is a discrete space, $f^{-1}(V)$ is closed in (X, τ) . Since every closed set is $\psi^* \alpha$ -closed, $f^{-1}(V)$ is $\psi^* \alpha$ -closed in (X, τ) . Hence f is a contra $\psi^* \alpha$ -irresolute map.

(b) Let V be any $\psi^* \alpha$ -closed set in (Y, σ) . Since (Y, σ) is a ${}_{\psi^* \alpha} T_c$ -space, V is closed in (Y, σ) . Since f is totally $\psi^* \alpha$ -continuous, $f^{-1}(V)$ is $\psi^* \alpha$ -clopen in (X, τ) . Since $f^{-1}(V)$ is $\psi^* \alpha$ -open, f is a contra $\psi^* \alpha$ -irresolute map.

(c) Let V be any $\psi^* \alpha$ -closed set in (Y, σ) . Since (Y, σ) is a ${}_{\psi^* \alpha} T_c$ -space, V is closed in (Y, σ) . Since f is contra $\psi^* \alpha$ -continuous, $f^{-1}(V)$ is $\psi^* \alpha$ -open in (X, τ) . Hence f is contra $\psi^* \alpha$ -irresolute map.

Proposition 4.3.15 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be two maps, then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is contra $\psi^* \alpha$ -irresolute if

- (a) $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\psi^* \alpha$ -irresolute and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is contra $\psi^* \alpha$ -irresolute.
- (b) $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra $\psi^* \alpha$ -irresolute and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is $\psi^* \alpha$ -irresolute
- (c) $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra $\psi^* \alpha$ -continuous and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is quasi $\psi^* \alpha$ -continuous (resp. perfectly $\psi^* \alpha$ -continuous)

Proof: (a) Let V be any $\psi^* \alpha$ -open set in (Z, η) . Since g is contra $\psi^* \alpha$ -irresolute, $g^{-1}(V)$ is $\psi^* \alpha$ -closed in (Y, σ) . Since f is $\psi^* \alpha$ -irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^* \alpha$ -closed in (X, τ) . Hence $g \circ f$ is a contra $\psi^* \alpha$ -irresolute map.

Proof of (b) and (c) in **Proposition 4.3.15** are similar to the proof of (a)

Remark 4.3.16 The composition of two contra $\psi^* \alpha$ -irresolute maps need not be a contra $\psi^* \alpha$ -irresolute map as seen from the following example.

Example 4.3.17 Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$ and $\eta = \{\emptyset, \{a\}, \{a, b\}, Z\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = c$, $f(b) = b$, $f(c) = a$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be a map defined by $g(a) = b$, $g(b) = c$, $g(c) = a$. Then f and g are contra $\psi^* \alpha$ -irresolute but their composition $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is not contra $\psi^* \alpha$ -irresolute, since $\{a\}$ is $\psi^* \alpha$ -open in (Z, η) , whereas $(g \circ f)^{-1}(\{a\}) = \{a\}$ is not $\psi^* \alpha$ -closed in (X, τ) .