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An empirical relation for cluster decay preformation probability

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Empirical relations for the preformation probability of cluster decay process in terms of the Q-value, mass asymmetry (η), mass number of the emitted cluster A_2 is analyzed based on the discrepancy between the calculated and experimental half-lives of the cluster emitters. For the different empirical expressions considered corresponding to different physical quantities the preformation probability for the complete binary breakup of ^{226}Ra is calculated and the obtained results are compared with the preformed cluster model calculation (P_0(PCM)) and another calculation in which the overlapping penetration probability is treated as the preformation probability ($P_0(\mu)$). Our empirical results for the use of Q-value compare well with $P_0(\mu)$. Results due to the use of Q and its powers along with the combination of mass number A_2 of the cluster emitted, and mass asymmetry η, reveal that preformation factor depends strongly on Q-value rather than A_2 and η. Calculated half-lives of different cluster decays for the use of empirical P_0 values are found to be in good agreement with the experimental values.

Keywords: Preformation probability; cluster decay; empirical relation.

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1. Introduction

In nature, an unstable radioactive nucleus comes to a stable state, either by emitting any one of the particles like α or β and/or γ rays, or by emitting heavy-ions which are heavier than alpha particles, generally named as clusters or by spontaneous fission. α emission is one of the decay channels of the medium and heavy nuclei, which conveys important nuclear information such as nuclear mass and structure. Cluster radioactivity is an intermediate phenomenon between α decay and fission, as a result of which a parent nucleus, breaks into two fragments: the emitted cluster and the daughter. The emitted cluster is heavier than alpha particle, but lighter than the lightest of fission fragments. The daughter nuclei observed are always doubly magic nucleus ^{208}Pb or its neighboring nuclei, which implies that cluster decay process associates itself with the closed shell behavior of daughter nucleus.

The cluster emission process was studied extensively by different models[1–7] which generally are based either on collective approach or microscopic approach. These models stem from the ideas of Gamow's model for explaining α-decay. These models either take preformation probability as 1, or consider to have dependence on the size of the cluster, as suggested by Blendowkse and Walliser.[4] In the microscopic approach spherical shell model is used for the many-body wave functions and the preformation probability is defined as the probability of finding the decay channel space in the parent nucleus wave function. However, these calculations are limited only to light clusters $A_2 \leq 16$. Due to this an empirical expression for preformation probability that fits the microscopically calculated spectroscopic factors for α, ^{12}C, ^{14}C and ^{16}O is established and the order of preformation probability varies from 10^{-2} to 10^{-11} for alpha to oxygen. The preformed cluster model (PCM) of Malik and Gupta,[6] is based on the collective model approach, in which the preformation probability of finding the fragments A_1 and A_2 (with fixed charges Z_1 and Z_2, respectively) is defined through the dynamical collective coordinate of mass (and charge) asymmetry of the two fragments (daughter and cluster nuclei). The mass asymmetry and relative path of the fragments are coupled and in a decoupled approximation, the probability is calculated. In unified fission models, the preformation probability is not considered explicitly. However, the equivalence of the unified fission model and the PCM was shown by Poenaru and Greiner.[8] As per their interpretation, the overlapping part of the action integral for the relative separation coordinate is equivalent to the preformation probability. It is to be mentioned here that, in the PCM of Malik and Gupta[6] the action integral is not considered in the overlapping region rather it is evaluated only from the touching configuration of the system.

In literature, there are quite a few number of works on the expression for this preformation factor.[4,9,10] Poenaru and Greiner[8] has given an analytic relation for the preformation probability for the range of clusters known at that time. An empirical preformation factor is defined simply as a measure of the disagreement between the calculated and the experimental decay constant or, the half-life, whereas different theoretical models have different approaches, either on the shell model basis or on the collective model basis, to formulate the expression for preformation factor.

In Gamow's picture, the probability of preformation for α particle was assumed to be 1, however later when this idea was extended to cluster decay phenomenon, the concept of preformation probability was not implemented properly. But it is a fact that when a large number of nucleons assembling themselves as a cluster, containing as many as 30 nucleons, may not have a maximum probability of its existence, before it travels through the confining nuclear interaction barrier. The treatment of taking $P_0 = 1$ for α decay is due to the fact that α particle is $N = Z$ system possessing larger binding energy, hence such system has the highest probability. However, experimentally measured clusters from carbon to silicon are all $N \neq Z$ systems and some of which are radioactive like ^{14}C. Hence, these unstable $N \neq Z$ systems may not have larger probability, to exist inside the parent nucleus. Hence the applicability of Unified Fission Model (UFM) considering a direct penetration approach with $P_0 = 1$ may not be relevant when preformation probability of cluster decay mode is studied. The aim of the present work is to show, that a simple expression containing the macroscopic physical quantity Q-value and its powers, is sufficient to calculate preformation probability. PCM of Gupta and collaborators is briefly explained in the following section and the results and discussion are presented in Sec. 3.

2. The Methodology

In PCM of Gupta and co-workers, the decay constant is defined as,

$$\lambda = \nu_0 P_0 P. \tag{1}$$

Here P_0 is the preformation probability usually calculated by solving the Schrödinger equation within the concepts of quantum mechanical fragmentation theory. The penetrability P is calculated as a two step process,[11] within the WKB method, starting from the touching point configuration to the point where potential reaches the Q-value given by,

$$P = P_a W_i P_b, \tag{2}$$

where,

$$P_a = \exp\left[-\frac{2}{\hbar}\int_{R_a}^{R_i} \{2\mu[V(R) - V(R_i)]\}^{1/2} dR\right], \tag{3}$$

$$P_b = \exp\left[-\frac{2}{\hbar}\int_{R_i}^{R_b} \{2\mu[V(R) - Q]\}^{1/2} dR\right], \tag{4}$$

with $R_a = R_t = C_1 + C_2$, C_i $(i = 0, 1, 2)$ is the Süssmann Central radii of parent, daughter and emitted α/cluster defined as,

$$C_i = R_i\left(1 - \frac{b^2}{R_i^2}\right) \text{fm}, \tag{5}$$

which accounts for the surface correction to the sharp radius R_i,

$$R_i = 1.28A_i^{\frac{1}{3}} - 0.76 + 0.8A_i^{-\frac{1}{3}} \text{fm}, \tag{6}$$

$b \sim 1$ fm is the diffuseness of the nuclear surface. As shown in Fig. 1, in the first step the penetration probability P_a starts at the touching point distance $R_a(= R_t)$ and crosses the barrier and reaches an intermediate point R_i, where $V(R_i) = V(R_a)$ and at R_i, where the α/cluster de-excites with a probability of $W_i = 1$ and from R_i it proceeds to R_b in the second step with a probability of P_b.

The frequency with which the α/cluster hits the potential barrier is

$$\nu_0 = \frac{1}{2R_0}\sqrt{\frac{2E}{\mu}}, \tag{7}$$

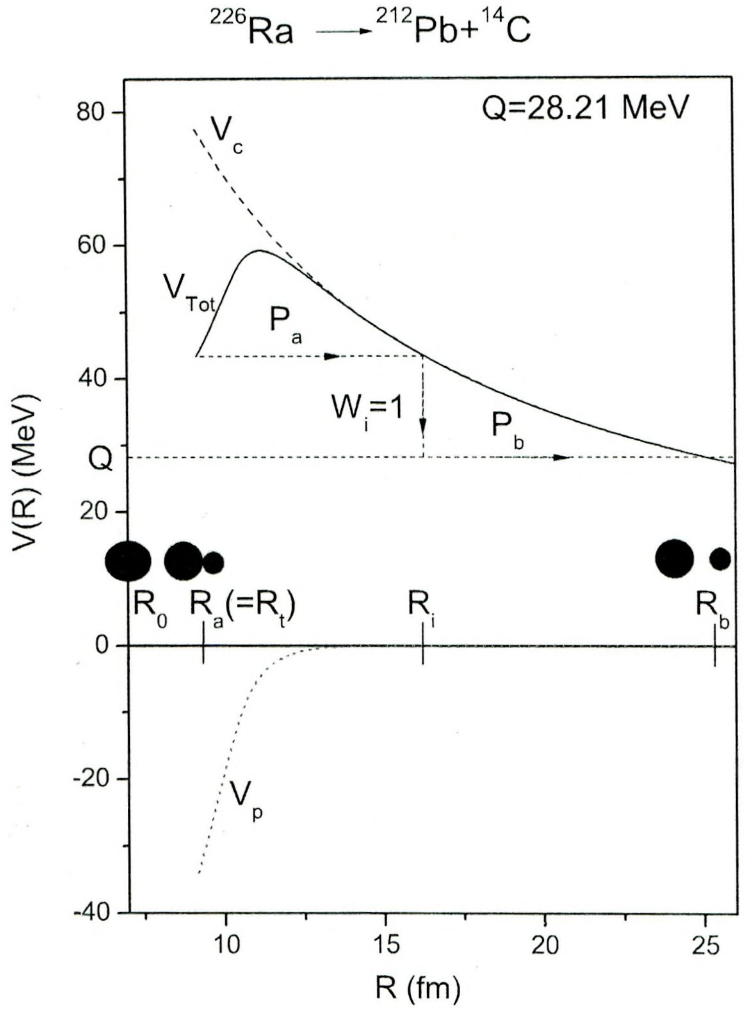

Fig. 1. (Color online) The potential $V_{\text{tot}} = V_c + V_p = V(R)$ is plotted as a function of relative separation R of the ^{14}C decay from the parent nucleus ^{226}Ra. The two-step decay path is marked with the arrows. R_0 is the parent nucleus radius. The touching point radius $R_a = R_t = C_1 + C_2$ is the first turning point. R_b is the second turning point at which $V(R)$ equals Q value. R_i is the intermediate value at which the first decay path de-excites with the probability of $W_i = 1$. The Q-value is also labeled.

where R_0 is the radius of the parent nucleus, where Süssmann correction is applied. The decay half-life $T_{1/2}$ of the parent nucleus $(A,\ Z)$ into an α particle and/or a cluster and a daughter $(A_d,\ Z_d)$ nucleus is given by,

$$T_{1/2} = \frac{\ln 2}{\lambda}. \tag{8}$$

The potential for the post-scission region is the sum of the Coulomb potential and proximity potential as given by

$$V(R) = \frac{Z_1 Z_2 e^2}{R} + V_p \quad \text{for} \quad R_t \leq R \leq R_b, \tag{9}$$

where Z_i's are the charge numbers of the fragments (with $i = 1$ and 2) and $e^2 = 1.44$ MeV fm. In the above equation, the proximity potential due to Blocki *et al.*[13] is used.

3. Results and Discussion

The half-lives of experimentally measured 15 cluster emitters, from the heavy element region $A \geq 200$ are calculated by considering $P_0 = 1$ in Eq. (1) with the Q-values taken from.[16] For the same parent nuclei with $P_0 = 1$ in Eq. (1), the alpha decay half-lives are also calculated. The Q-values for α decay are calculated using mass table of Audi *et al.*[14] The calculated α decay half-lives and cluster decay half-lives are compared with the respective experimental values and are presented in Fig. 2. In Fig. 2(a), the calculated half-lives are in reasonable agreement with the experimental values.[15] Though the structural agreement is there, a difference of nearly five orders in magnitude is noted between the experimental and calculated half-lives. The standard deviation (SD) between the calculated and experimental half-lives of α decay for the cases presented in Fig. 2(a) is found to be 5.785. Further, the calculations with $P_0 = 1$ for cluster decay though reproduces the structural variations with respect to experimental values,[16] are underestimating the half-lives

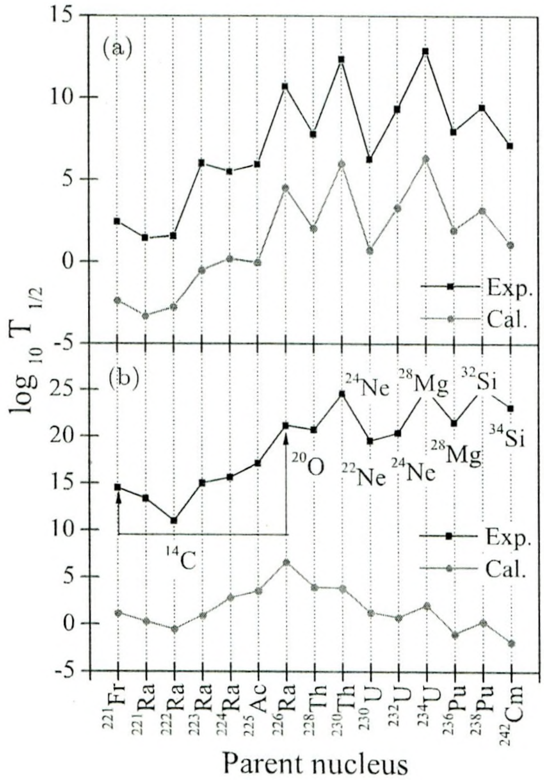

Fig. 2. (Color online) The calculated (a) α decay and (b) cluster decay half-lives for different cluster emitters compared with the experimental values of α decay from Ref. 15 and for cluster decay from Ref. 16.

by several orders of magnitude. The SD between the calculated and experimental half-lives of cluster decay for the cases presented in Fig. 2(b) is found to be 18.161. This large discrepancy supports the importance of cluster preformation probability in the calculations which were introduced in the PCM[6] for cluster decays. Without any model dependence, if one needs to calculate the preformation probability of clusters, the discrepancy between the calculated and experimental half-life values of cluster decay can be treated as preformation probability. In Fig. 3, the discrepancy and/or difference between the calculated and experimental half-lives, taken as $\log_{10} P_0$ for α (solid circles) and cluster (open circles) decay, is presented. In the case of α decay, the deviation for different parents is found to lie between 10^{-5} to 10^{-7}. However, for the cluster decay process this value is found to decrease as the size of the emitted cluster increases, clearly indicating the size dependence of the preformation probability. This difference between the calculated and experimental half-lives is considered, to find an empirical preformation probability. This is supported by the empirical calculations due to Blendowske and Walliser[4] represented by open square and empirical expression using PCM by Satish Kumar and Gupta[17] represented by solid square in Fig. 3. In the PCM calculations,[17] the empirical expression for P_0 depends on the emitted cluster; hence the value remains same for the cluster ^{14}C from different parents and empirical P_0 values for a few clusters such as ^{20}O, ^{24}Ne, ^{22}Ne matches well with that due to P_0(PCM) values.[17] The structure in P_0 values between the calculations due to Blendowske and Walliser[4] and the present one is found to be similar with a small difference in magnitude.

We have analyzed the relationship between this discrepancy considered as $\log_{10} P_0$ and various characteristic quantities such as $Q^{1/2}$, Q, $Q^{3/2}$ and combination

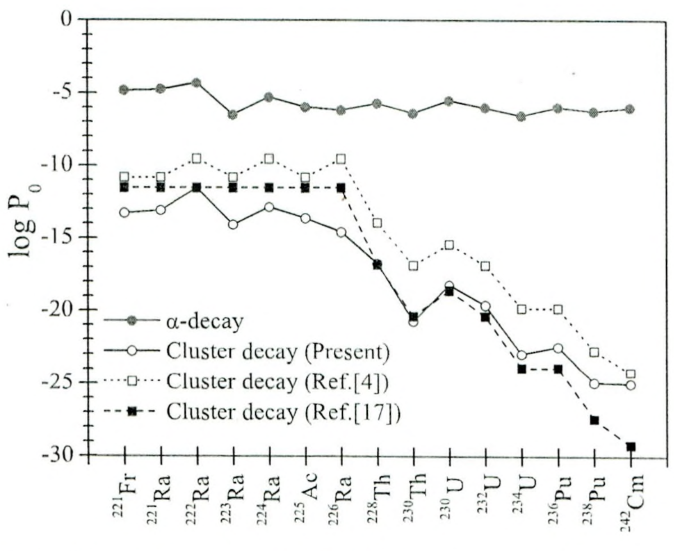

Fig. 3. (Color online) Calculated $\log_{10} P_0$ values for α decay (solid circle) and cluster decays (open circle) for different cluster emitters. Calculations due to Blendowske[4] (open square) and PCM[17] (solid square) are also shown.

of the mass asymmetry (η) with Q dependence, combination of mass number of the cluster (A_2) with Q dependence. These quantities are considered based on the following facts. Q-dependence with powers of 0.5, 1 and 1.5 is considered since the Q-value characterizes the decay process and also is different for different clusters emitted from the same parent nucleus. The size dependence of the cluster A_2 is also an important factor as far as the preformation probability is considered. However, if one considers only A_2 for fitting, the resulting, preformation probability values for the complete mass asymmetry will be linearly decreasing with an increase in size of the cluster without any structural variation. Hence, to have structural variation as well, we have considered the product of A_2 with Q-dependence. Like A_2, the mass asymmetry also characterizes each binary breakup of a parent nucleus taking into consideration all the mass numbers involved in the breakup namely A, A_1 and A_2. The mass asymmetry values vary between 0 to 1. The value of 0 describes symmetric breakup and 1 describes the neutron/proton breakup. This factor is also considered with Q-dependence in order to have structural variation.

With the above considerations an empirical relation of the form given by

$$\log_{10} P_0 = \gamma X + \beta, \tag{10}$$

is fitted between the discrepancy and the various characteristic quantities denoted as X. γ and β are constants. Using these empirically estimated preformation probability values, corresponding to different fittings, we have calculated the half-lives of the cluster decay. We present in Table 1, the values of the constants γ and β along with the SD between the newly calculated half-lives using the $\log_{10} P_0$ values corresponding to different fittings and the experimental values of cluster decay.

In Fig. 4, for the charge minimized complete binary breakup of ^{226}Ra, the empirical P_0 values are calculated with γ and β values corresponding to Eq. (10) as listed in Table 1. These values are compared with two model calculations (i) the preformation probability calculated within the PCM of Gupta and collaborators

Table 1. Values of γ and β and the SD between the calculated half-lives (for the use of the corresponding $\log_{10} P_0$ values) and the experimental values.

X	γ	β	SD
$Q^{1/2}$	−2.8266	2.3892	1.055
Q	−0.1904	−7.6084	1.089
$Q^{3/2}$	−0.0165	−10.865	1.222
$A_2Q^{1/2}$	−0.0508	−9.5996	0.986
A_2Q	−0.0046	−11.795	1.237
$A_2Q^{3/2}$	−0.0004	−12.993	1.632
$\eta Q^{1/2}$	−5.0729	11.392	1.306
ηQ	−0.2848	−5.7033	1.124
$\eta Q^{3/2}$	−0.0236	−10.048	9.217

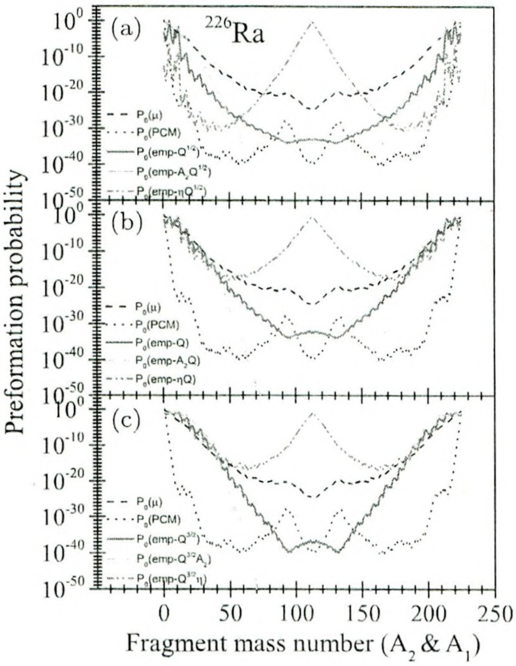

Fig. 4. (Color online) Preformation values for complete binary spectrum of ^{226}Ra for the use of different models.

denoted by dotted lines and labeled as P_0(PCM) and (ii) the preformation probability calculated as the penetration probability of the overlapping region in the scattering potential, an interpretation put forth by Poenaru and Greiner[8] denoted by dashed lines and labeled as $P_0(\mu)$. In this figure the order of preformation probability values as low as 10^{-40} is seen in the calculations based on PCM of Gupta and collaborators for the complete spectrum. But for the clusters of our interest the preformation probability value ranges from 10^{-5} to 10^{-20} only. It is to be mentioned here that in Ref. 12, the dependence of reduced mass and hydrodynamical mass in the overlapping preformation probability is studied by us and the results obtained were compared with the preformation probability values of P_0(PCM).

The panel (a) of Fig. 4 presents empirical P_0 values corresponding to $X = Q^{1/2}$, $A_2Q^{1/2}$ and $\eta Q^{1/2}$ (denoted respectively by solid, dash dotted and dash double dotted lines) and the other two model calculations $P_0(\mu)$ and P_0(PCM) (dashed and dotted lines, respectively). In panel (b) and (c) similar results are compared but corresponding to $X = Q$, A_2Q and ηQ and $X = Q^{3/2}$, $A_2Q^{3/2}$ and $\eta Q^{3/2}$, respectively. In Fig. 4, in the panels (a), (b) and (c), when $X = Q^{1/2}$, Q and $Q^{3/2}$ the magnitude of empirical P_0 values and $P_0(\mu)$ values, compares reasonably well but completely differs with that of P_0(PCM). In particular, when $X = Q^{1/2}$ (panel a), the calculated values lie below $P_0(\mu)$ for the complete breakup, but when the power of Q is raised to 1 and 1.5, (panels b and c), the calculated values overlaps with $P_0(\mu)$ for $X = Q$ and lies slightly above the $P_0(\mu)$ values when $X = Q^{3/2}$ near the very asymmetric region. This overlapping and crossover is compensated near the symmetric region, where the empirical P_0 values decrease with an increase in the power of Q. The magnitude of empirical P_0 values in the symmetric and near symmetric regions are in the comparable range with PCM values but they differ in the structure.

Along with the Q dependence if one considers the mass number A_2 of the cluster emitted and/or the mass asymmetry η, the behavior is completely changed, in particular at symmetric region. When A_2 is multiplied with all the Q dependences ($Q^{1/2}$, Q and $Q^{3/2}$) considered, shown by dash-dotted lines, the magnitude and structure of preformation values compare well with other models, at least up to $A_2 < 50$ and beyond that the role of A_2 merely shows a monotonous decrease in the value of preformation as the size of the cluster increases, without any structural variation. In the case of mass asymmetry η, multiplied with all the Q dependences ($Q^{1/2}$, Q and $Q^{3/2}$) considered, shown by dash double dotted lines, when η becomes 0 corresponding to symmetric breakup, irrespective of Q-value, the preformation values reaches a maximum, making the γ term zero in the Eq. (10). These results imply that the structural variation in preformation probability is mainly due to the Q-value of the decay mode rather than A_2 or η.

For the use of the preformation probability obtained through Eq. (10), the half-lives of 15 clusters from different parent nuclei are calculated. In Table 2, these results are presented along with the experimental values listed under the column (a) and the calculations in which P_0 is considered as 1 as listed under the column (b). The results listed under columns (c), (d) and (e) are, for the use of P_0 values obtained from the empirical expression of Eq. (10), corresponding to $X = Q, Q^{1/2}$

Table 2. Comparison of (a) experimental cluster decay half-lives with the calculated values (b) for the use of P_0 as 1 and (c), (d) and (e) due to different empirical P_0 values corresponding to $X = Q$, $Q^{1/2}$ and $Q^{3/2}$ in Eq. (10).

			$\log_{10} T_{1/2}$				
Parent	Cluster	Q MeV	(a)	(b)	(c)	(d)	(e)
^{221}Fr	^{14}C	31.28	14.52	1.25	14.81	14.67	15.00
^{221}Ra	^{14}C	32.39	13.39	0.30	14.08	14.00	14.21
^{222}Ra	^{14}C	33.05	11.01	−0.53	13.37	13.33	13.47
^{223}Ra	^{14}C	31.85	15.04	0.97	14.64	14.53	14.80
^{224}Ra	^{14}C	30.54	15.68	2.83	16.25	16.06	16.48
^{225}Ac	^{14}C	30.48	17.16	3.55	16.97	16.77	17.20
^{226}Ra	^{14}C	28.21	21.19	6.62	19.60	19.25	19.96
^{228}Th	^{20}O	44.72	20.72	3.97	20.09	20.48	19.77
^{230}Th	^{24}Ne	57.78	24.61	3.90	22.51	23.00	22.02
^{230}U	^{22}Ne	61.4	19.57	1.35	20.65	21.11	20.16
^{232}U	^{24}Ne	62.31	20.40	0.80	20.27	20.72	19.78
^{234}U	^{28}Mg	74.13	25.14	2.16	23.89	24.11	23.56
^{236}Pu	^{28}Mg	79.67	21.52	−0.96	21.81	21.88	21.64
^{238}Pu	^{32}Si	91.21	25.27	0.33	25.30	24.93	25.56
^{242}Cm	^{34}Si	96.53	23.15	−1.86	24.13	23.53	24.66

and $Q^{3/2}$, respectively. All these three results are similar and compares well with the experimental values.

4. Summary

In the unified fission models the cluster preformation probability is considered to be 1, whereas in PCM it takes different values depending upon the fragment size. In this work initially by taking preformation probability as 1, in a two-step model (starting from touching configuration of the fragments) we have calculated the half-lives for alpha and cluster decay of 15 cluster emitters. For alpha decay the results are found in reasonable agreement with experimental values, whereas for cluster decay a large discrepancy is seen. Using this discrepancy between the calculated and experimental half-lives for cluster decays, as $\log_{10} P_0$, we have established a relation connecting $\log_{10} P_0$ with different physical quantities such as $Q^{1/2}$, Q, $Q^{3/2}$ and A_2, η and different combinations among them. Using these empirical relations, the preformation factor for the complete binary decay of ^{226}Ra is calculated and compared with the preformed cluster model calculation (P_0(PCM)) and another calculation in which the overlapping penetration probability is treated as the preformation probability ($P_0(\mu)$). It is seen from the results that, for the use of physical quantity Q and $Q^{3/2}$, our empirical P_0 values compare well with $P_0(\mu)$, in magnitude as well as in structure, at least till near symmetric region and it differs in the symmetric region; and for the use of $Q^{1/2}$ the curve lies between the two models $P_0(\mu)$ and P_0(PCM).

When one combines the mass number of the emitted cluster A_2 with Q, $Q^{1/2}$ and $Q^{3/2}$, which results in the reduction of the magnitude of empirical P_0 as A_2 increases, showing structural variations only in the light mass region say, $A_2 < 50$, whereas for the use of η with these Q-dependences, the magnitude as well as structural difference is noted in a different way. When the value of η varies from 1 to 0, from particle emission to symmetric region, structural effect is seen only in the lower mass region and at the symmetric region the probability reaches the maximum value (due to $\eta = 0$). In conclusion, the structural variation in the preformation factor for complete binary decay of any system depends mainly on Q-value of the decay, rather than any other physical quantity.

These values of P_0 are further used to find the half-lives of the cluster decay processes considered. It is found that the introduction of empirical preformation probability in the expression of the decay constant improves the agreement between the calculated and experimental half-lives for the cluster decays.

References

1. A. Săndulescu, D. N. Poenaru and W. Greiner, *Sov. J. Part. Nucl.* **11** (1980) 528.
2. Y. J. Shi and W. J. Swiatecki, *Phys. Rev. Lett.* **54** (1985) 300.
3. G. Shanmugam and B. Kamalaharan, *Phys. Rev. C* **38** (1988) 1377.
4. R. Blendowske and H. Walliser, *Phys. Rev. Lett.* **61** (1988) 1930.
5. B. Buck and A. C. Merchant, *J. Phys. G: Nucl. Part. Phys.* **15** (1989) 615.

6. S. S. Malik and R. K. Gupta, *Phys. Rev. C* **39** (1989) 1992.
7. D. N. Poenaru, R. A. Gherghescu and W. Greiner, *Phys. Rev. C* **85** (2012) 034615.
8. D. N. Poenaru and W. Greiner, *Phys. Scripta* **44** (1991) 427.
9. H. F. Zhang, G. Royer, Y. J. Wang, J. M. Dong, W. Zuo and J. Q. Li, *Phys. Rev. C* **80** (2009) 057301.
10. D. N. Poenaru, R. A. Gherghescu and W. Greiner, *J. Phys. G: Nucl. Part. Phys.* **39** (2012) 015105.
11. M. Greiner and W. Scheid, *J. Phys. G* **12** (1986) L229.
12. N. S. Rajeswari, K. R. Vijayaraghavan and M. Balasubramaniam, *Eur. Phys. J A* **47** (2011) 126.
13. J. Blocki, J. Randrup, W. J. Swiatecki and C. F. Tsang, *Ann. Phys. (N.Y.)* **105** (1977) 427.
14. G. Audi, F. G. Kondev, M. Wang, B. Pfeiffer, X. Sun, J. Blachot and M. MacCormik, *Chin. Phys. C* **36** (2012) 1157.
15. J. Tuli, *Nuclear Wallet Cards* (National Nuclear Data Center, Brookhaven National Laboratory, Upton, New York, U.S., 2011).
16. R. Bonetti and A. Guglielmetti, *Rom. Rep. Phys.* **59** (2007) 301.
17. S. Kumar and R. K. Gupta, *Phys. Rev. C* **55** (1977) 218.