Totally $g^{\#}\psi$ - Continuous Functions in Topological Spaces

DEEPIKA S

(17PMA006)

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore - 641 043

In Partial Fulfilment of the Requirements for the Degree of

Master of Science in Mathematics

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Signature of the Head of the Department

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15.04.2019
Signature of the Supervisor

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CONTENT

INTRODUCTION

The topology plays an important role in the field of mathematics. The term topology was introduced by Johann Benedict in 19^{th} century. In 20^{th} century the idea of a topological space was developed and become a major branch of mathematics. Topology developed as a field of study out of geometry and set theory, through analysis of concepts such as space, dimension and transformation. Topology can be formally defined as the study of qualitative properties of certain objects called topological spaces that are invariant under a certain kind of transformation called a continuous map.

The concept of closed and open sets play an important role in the study of topological spaces. Stone (1937) introduced the regular open sets in topological spaces. The notion of semi-open sets was first introduced and investigated by Levine (1963).The study of generalized closed sets was introduced by Levine (1970) in order to extend some important properties of closed sets to a larger family of sets. Veera Kumar (2000) introduced and investigated ψ-closed sets in topological spaces. Using the concept of ψ-closed sets Kanimozhi et al. (2017a) introduced $g^{\#}\psi$-closed sets in topological spaces.

The notion of continuous function is one of the most important concepts in mathematics. Levine (1970) introduced the concept of continuous functions in topology. Balachandran et al. (1991) introduced generalized continuous functions in topological spaces. Veera Kumar (2000) introduced and investigated ψ-continuous functions in topological spaces. Crossley and Hildebrand (1972) introduced the concept of irresolute functions in topological spaces.

The deliberations in the present study include the following topics:

1) Totally $g^{\#}\psi$ - continuous functions and $g^{\#}\psi$ - totally continuous functions in topological spaces.

2) Contra $g^{\#}\psi$ - continuous functions and almost contra $g^{\#}\psi$ - continuous functions in topological spaces.

3) $g^{\#}\psi$ - irresolute functions in topological spaces.

Basic definition and results relevant to the study are presented in the first chapter.

In chapter 2 a new class of continuous functions called totally $g^{\#}\psi$ - continuous functions are introduced. Also $g^{\#}\psi$ - totally continuous functions are introduced and their properties are discussed.

In chapter 3 two new classes of continuous functions namely contra $g^{\#}\psi$- continuous functions and almost contra $g^{\#}\psi$- continuous functions are defined and their interrelations are analyzed.

The concept of $g^{\#}\psi$ - irresolute functions are defined and their properties are studied in the final chapter.

REVIEW OF LITERATURE

Topology is one of the widely studied areas of mathematics emerged through the works of the great mathematician Henri Poincare in the 19^{th} century. Topology plays an important role in pure and applied mathematics. Topological structures are suitable mathematical models for formulation of both qualitative and quantitative data.

Initially the topological spaces were characterized by open sets. Stone (1937) introduced regular open sets. In (1963) Levine introduced the concept of semi-open sets. Levine (1970) introduced the notion of generalized closed (briefly g-closed) sets in topological spaces. Bhattacharyya and Lahiri (1987) introduced semi generalized closed sets in topological spaces. Veera Kumar (2000) investigated a new class of closed sets called ψ - closed sets in topological spaces. Ramya and parvathi (2013) introduced ψg - closed sets in topological spaces and studied their properties. Kanimozhi et al. (2017a) introduced a stronger form of ψg-closed sets called $g^{\#}\psi$ - closed sets in topological spaces.

Continuous functions are an important notion in the study of mathematical sciences. Many generalization of continuous functions have been introduced over years and many interesting results have been obtained. Levine (1970) introduced continuous functions in topological spaces.

Arya and Gupta (1974) introduced completely continuous functions in topological spaces. Balachandran et al.(1991) introduced g - continuous functions in topological spaces. Veera Kumar (2000) introduced ψ - continuous functions in topological spaces. Kanimozhi et al.(2017b) introduced $g^{\#}\psi$ - continuous functions in topological spaces.

Jain (1980) introduced totally continuous functions in topological spaces. Dontchev (1996) introduced the concept of contra continuous functions in topological spaces. Singal and singal (1968) introduced almost continuous functions in topological spaces. Ekici (2004) introduced almost contra continuous functions in topological spaces.

Crossley and Hildebrand (1972) investigated irresolute function in topological spaces and proved that irresolute functions are stronger than semi continuous functions but are independent of continuous functions. Various forms of irresolute functions were defined by various researchers over the years. Ramya and parvathi (2013) introduced ψg - irresolute functions in topological spaces.

CHAPTER - 1

PRELIMINARIES

Definition 1.1 [14]

A subset A of a topological space (X,τ) is called **regular open** if A = int (cl(A)) and **regular-closed** if A = cl(int(A)).

Definition 1.2 [10]

A subset A of a topological space (X,τ) is called **semi-open** if A $\subseteq$ cl(int(A)) and **semi-closed** if int (cl(A)) $\subseteq$ A.

Definition 1.3 [11]

A subset A of a topological space (X,τ) is called **generalized closed** if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

Definition 1.4 [3]

A subset A of a topological space (X,τ) is called **semi - generalized closed** if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi open in (X,τ).

Definition 1.5 [15]

A subset A of a topological space (X,τ) is called **ψ - closed** if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg - open in (X,τ).

Definition 1.6 [8]

A subset A of a topological space (X,τ) is called **$g^{\#}\psi$ - closed** if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ψ - open in (X,τ).

Definition 1.7 [9]

A subset A of a topological space (X,τ) is called **$g^{\#}\psi$ - clopen** if it is both $g^{\#}\psi$ - open and $g^{\#}\psi$ - closed in (X,τ).

Result 1.8

1. Every closed (open) subset in (X,τ) is $g^{\#}\psi$ - closed ($g^{\#}\psi$ - open).

2. Every clopen subset in (X,τ) is $g^{\#}\psi$ - clopen.

3. Every regular open (regular closed) subset in (X,τ) is open (closed).

Definition 1.9 [13]

A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is said to be **almost continuous** if $f^{-1}(V)$ is closed in (X,τ) for every regular closed set V of (Y,σ).

Definition 1.10 [11]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **continuous** if $f^{-1}(V)$ is closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.11 [1]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **completely continuous** if $f^{-1}(V)$ is regular open in (X,τ) for every open set V of (Y,σ).

Definition 1.12 [7]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **totally continuous** if $f^{-1}(V)$ is clopen in (X,τ) for every open subset V of (Y,σ).

Definition 1.13 [2]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **generalized continuous** if $f^{-1}(V)$ is g - closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.14 [5]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **contra continuous** if $f^{-1}(V)$ is a closed set of (X,τ) for every open set V of (Y,σ).

Definition 1.15 [15]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **ψ - continuous** if $f^{-1}(V)$ is ψ - closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.16 [6]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **almost contra continuous** if f^{-1} (V) is closed in (X,τ) for every regular-open set V of (Y, σ).

Definition 1.17 [8]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **$g^{\#}\psi$ - continuous** if $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.18 [4]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **irresolute** if $f^{-1}(V)$ is semi closed in (X,τ) for every semi closed set V of (Y,σ).

Definition 1.19 [12]

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is said to be **ψg - irresolute** if $f^{-1}(V)$ is ψg - closed in (X,τ) for every ψg - closed set V of (Y,σ).

CHAPTER - 2

Totally $g^{\#}\psi$ - Continuous Functions and $g^{\#}\psi$ - Totally Continuous Functions in Topological Spaces

2.1 Introduction

Levine (1970) introduced the idea of continuous functions in topological spaces. Jain (1980) introduced totally continuous functions in topological spaces. Balachandran et al.(1991) introduced generalized continuous functions in topological spaces. Veera kumar (2000) introduced ψ - continuous functions in topological spaces. Kanimozhi et al. (2017b) introduced $g^{\#}\psi$ - continuous functions in topological spaces.

In this chapter we introduce a new type of totally - continuous functions called totally $g^{\#}\psi$ - continuous functions and $g^{\#}\psi$ - totally continuous functions in topological spaces.

2.2 Totally $g^{\#}\psi$ - continuous functions

In this section, $g^{\#}\psi$ - continuous functions in topological spaces are introduced and studied their properties.

Definition 2.2.1

Let (X,τ) and (Y,σ) be two topological spaces. A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is called **totally $g^{\#}\psi$ - continuous** if $f^{-1}(V)$ is $g^{\#}\psi$ - clopen subset of (X,τ) for every open set V of (Y,σ).

Example 2.2.2

Let X=Y={a,b,c}, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$ and $\sigma = \{\phi, \{a,b\}, Y\}$.Let f:(X,τ)→(Y,σ) be a function defined by f(a)=b, f(b) = c , f(c) =a. Then f is totally $g^{\#}\psi$ - continuous.

Theorem 2.2.3

A function f: (X,τ) → (Y,σ) is totally $g^{\#}\psi$ **-** continuous if and only if the inverse image of every closed subset of (Y,σ) is a $g^{\#}\psi$ - clopen subset of (X,τ).

Proof

Necessity: Let f: (X,τ) →(Y,σ) be a totally $g^{\#}\psi$ - continuous function**.** Let V be any closed set in (Y,σ). Then Y-V is open in (Y, σ). Since f is totally $g^{\#}\psi$ - continuous**,** f^{-1} (Y- V) = X - f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ) which implies that f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ).

Sufficiency: Assume that U is any open set in (Y, σ). Then Y - U is closed in (Y,σ). By assumption, f^{-1}(Y - U) = X - f^{-1}(U) is $g^{\#}\psi$ - clopen in (X,τ) which implies that f^{-1}(U) is $g^{\#}\psi$ - clopen in (X,τ). Hence f is totally $g^{\#}\psi$ - continuous.

Proposition 2.2.4

Every totally continuous function is a totally $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any open set in (Y,σ). Since f is totally continuous, f^{-1}(V) is clopen in (X,τ) . By result 1.8 f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ). Hence f is totally $g^{\#}\psi$ - continuous.

Example 2.2.5

Let X=Y={a,b,c}, τ = { ϕ ,{a},{b,c},X} and σ = { ϕ ,{a,},Y}. Let f: (X,τ)→(Y,σ) be a function defined by f(a) = c, f(b) = a ,f(c) =b. Then f is totally $g^{\#}\psi$ - continuous but not totally continuous, since for the open set {a} in (Y,σ), f^{-1}({a})= {b} is $g^{\#}\psi$ - clopen in (X,τ) but not clopen subset in (X,τ).

Proposition 2.2.6

Every totally $g^{\#}\psi$ - continuous function is a $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let f: (X,τ) →(Y,σ) be a totally $g^{\#}\psi$ - continuous function**.** Let V be any open set in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous, f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ). which implies that f^{-1}(V) is $g^{\#}\psi$ - open in (X,τ). Hence f is $g^{\#}\psi$ - continuous.

Example 2.2.7

Let X= Y ={a,b,c},τ = { ϕ ,{a,b},X} and σ = { ϕ ,{a},Y}. Let f: (X,τ) →(Y,σ) be the identity function. Then f is $g^{\#}\psi$ - continuous but not totally $g^{\#}\psi$ - continuous, since for the open set {a} in (Y,σ), f^{1}({a})={a} is $g^{\#}\psi$ - open but not $g^{\#}\psi$ - closed in (X,τ).

Proposition 2.2.8

Every continuous function is independent from totally $g^{\#}\psi$ - continuous function.

Example 2.2.9

Let X=Y= {a,b,c}, τ = { ϕ,{a},{a,b},X} and σ = {ϕ,{a,b},Y}. Let f: (X,τ)→(Y,σ) be the identity function. Then f is continuous but not totally $g^{\#}\psi$ - continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b})={a,b} is open but not $g^{\#}\psi$ - clopen in (X,τ).

Example 2.2.10

Let X=Y={a,b,c}, τ = {ϕ,{a},{b,c},X} and σ = {ϕ,{a},Y}. Let f: (X,τ)→(Y,σ) be defined by f(a)= b, f(b) = a, f(c) = c. Then f is totally $g^{\#}\psi$ - continuous but not continuous, since for the open set {a} in (Y,σ), f^{1}({a}) = {b} is $g^{\#}\psi$ - clopen but not open in (X,τ).

Proposition 2.2.11

If f: (X,τ) →(Y,σ) is totally $g^{\#}\psi$ - continuous and X is $g^{\#}\psi$ - connected, then Y is an indiscrete space.

Proof

Suppose Y is not an indiscrete space. Let V be a non - empty open subset of Y. Since f is totally $g^{\#}\psi$ - continuous, $f^{-1}(V)$ is non - empty $g^{\#}\psi$ - clopen subset of X. Then X = $f^{-1}(V) \cup (f^{-1}(V))^{C}$. Thus X is union of two non - empty disjoint $g^{\#}\psi$ - open sets which is a contradiction to the fact that X is $g^{\#}\psi$ - connected. Therefore Y must be indiscrete space.

Theorem 2.2.12

Let f:(X,τ) →(Y,σ) be any function from discrete space (X,τ) into a topological space (Y,σ). Then f is totally $g^{\#}\psi$ - continuous function if and only if

(i) f is a continuous function

(ii) f is a $g^{\#}\psi$ - continuous function.

Proof

(Necessity): (i) Assume that f is a totally $g^{\#}\psi$ - continuous function. Let V be any open set in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous .Then f^{-1} (V) is $g^{\#}\psi$ - clopen in (X,τ). Since (X,τ) is discrete space, every subset of (X,τ) is open and closed subset in (X,τ) which implies that f^{-1} (V) is open in (X,τ). Therefore f is continuous.

(Sufficiency): Assume that f is a continuous function. Let V be any open set in (Y,σ). Thus f^{-1} (V) is open in (X,τ). Since (X,τ) is discrete space, every subset of (X,τ) is open and closed subset in (X,τ).which implies that f^{-1} (V) is $g^{\#}\psi$ - clopen in (X,τ). Therefore f is totally $g^{\#}\psi$ - continuous.

(Necessity): (ii) Assume that f is a totally $g^{\#}\psi$ - continuous function. Let V be any open set in (Y,σ). Then f^{-1} (V) is $g^{\#}\psi$ - clopen in (X,τ). Since (X,τ) is discrete space, f^{-1} (V) is clopen in (X,τ). By result 1.8, f^{-1} (V) is $g^{\#}\psi$ - open in (X,τ). Therefore f is $g^{\#}\psi$ - continuous.

(Sufficiency): Assume that f is $g^{\#}\psi$ - continuous. Let V be any open set in (Y,σ). Then f^{-1} (V) is $g^{\#}\psi$ - open in (X,τ). Since (X,τ) is discrete space, f^{-1} (V) is clopen in (X,τ). By result 1.8, f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ). Therefore f is totally $g^{\#}\psi$ - continuous.

Proposition 2.2.13

The composition of two totally $g^{\#}\psi$ - continuous functions need not be a totally $g^{\#}\psi$ - continuous function as seen from the following example.

Example 2.2.14

Let X = Y = Z = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X} , σ = {ϕ,{a},{b,c},Y} and η = {ϕ,{a,b},Z}. Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be the identity function and g: $(Y,\sigma)\rightarrow(Z,\eta)$ be the identity function. Then f and g are totally $g^{\#}\psi$ - continuous but their composition g o f : $(X,\tau)\rightarrow(Z,\eta)$ is not totally $g^{\#}\psi$ - continuous , since {a,b} is open in (Z,η), where as $(g \circ f)^{-1}(\{a,b\})$ = {a,b} is not $g^{\#}\psi$ - clopen in (X,τ).

Proposition 2.2.15

If f:$(X,\tau)\rightarrow(Y,\sigma)$ is a totally $g^{\#}\psi$ - continuous function and g:$(Y,\sigma)\rightarrow(Z,\eta)$ is a continuous function, then g o f :$(X,\tau)\rightarrow(Z,\eta)$ is a totally $g^{\#}\psi$ - continuous function .

Proof

Let V be any open set in (Z,η). Since g is continuous, $g^{-1}(V)$ is open in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - clopen in (X,τ).Hence g o f is a totally $g^{\#}\psi$ - continuous function.

Proposition 2.2.16

Let f:$(X,\tau)\rightarrow(Y,\sigma)$ and g:$(Y,\sigma)\rightarrow(Z,\eta)$be two functions. Then g o f: $(X,\tau)\rightarrow(Z,\eta)$ is a totally $g^{\#}\psi$ - continuous function if

(i) f is a totally $g^{\#}\psi$ - continuous function and g is a totally continuous function.

(ii) f and g are totally continuous functions.

Proof

(i) Let V be any open set in (Z,η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is open in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - clopen in (X,τ). Hence g o f is a totally $g^{\#}\psi$ - continuous function.

(ii) Let V be any open set in (Z,η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is open in (Y,σ). Since f is totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.8 $(g \circ f)^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ). Hence g o f is a totally $g^{\#}\psi$ - continuous function.

2.3 $g^{\#}\psi$ - totally continuous functions

In this section we introduce a new class of continuous functions called $g^{\#}\psi$ - totally continuous functions in topological spaces.

Definition 2.3.1

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **$g^{\#}\psi$ - totally continuous** if $f^{-1}(V)$ is clopen subset of (X,τ) for every $g^{\#}\psi$ - open set V of (Y,σ).

Theorem 2.3.2

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is $g^{\#}\psi$ - totally continuous if and only if the inverse image of every $g^{\#}\psi$ - closed subset of (Y,σ) is a clopen subset of (X,τ).

Proof

(Necessity): Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a $g^{\#}\psi$ - totally continuous function**.** Let V be any $g^{\#}\psi$ - closed set in (Y,σ). Then Y-V is $g^{\#}\psi$ - open in (Y, σ). Since f is $g^{\#}\psi$ - totally continuous**,** f^{-1} (Y- V) = X- f^{-1}(V) is clopen in (X,τ) which implies that f^{-1}(V) is clopen in (X,τ).

(Sufficiency): Assume that U is any $g^{\#}\psi$ - open set in (Y, σ). Then Y - U is $g^{\#}\psi$ - closed in (Y,σ). By assumption, f^{-1}(Y – U) = X - f^{-1}(U) is clopen in (X,τ) which implies that f^{-1}(U) is clopen in (X,τ). Hence f is $g^{\#}\psi$ - totally continuous.

Proposition 2.3.3

If f: $(X,\tau) \rightarrow (Y,\sigma)$ is a $g^{\#}\psi$ - totally continuous function, then

(i) f is a totally continuous function

(ii) f is a totally $g^{\#}\psi$ - continuous function.

Proof

(i) Let V be any open set in (Y,σ). By result 1.8 V is a $g^{\#}\psi$ - open set in (Y, σ). Since f is $g^{\#}\psi$ - totally continuous, f^{-1}(V) is clopen in (X,τ) Therefore f is totally continuous.

(ii) Let V be any open set in (Y,σ). By result 1.8, V is a $g^{\#}\psi$ - open set in (Y, σ). Since f is $g^{\#}\psi$ - totally continuous, f^{-1}(V) is clopen in (X,τ). By result 1.8, f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ). Therefore f is totally $g^{\#}\psi$ - continuous.

The converse of proposition 2.3.3 (i) need not be true in general as seen from the following example.

Example 2.3.4

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},{a,c},X}and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be defined by f(a) = c, f(b) = a, f(c) = b. Then f is totally continuous, but f is not $g^{\#}\psi$ - totally continuous, since for the $g^{\#}\psi$ - open set {a,b} in (Y,σ), f^{-1}({a,b }) = {b,c} is not clopen in (X,τ).

The converse of proposition 2.3.3 (ii) need not be true in general as seen from the following example.

Example 2.3.5

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be defined by f(a) = c, f(b) = b, f(c) = a. Then f is totally $g^{\#}\psi$ - continuous, but f is not $g^{\#}\psi$ - totally continuous, since for the $g^{\#}\psi$ - open set {a} in (Y,σ), f^{-1}({a}) ={c} is not clopen in (X,τ).

Proposition 2.3.6

If f: (X,τ) →(Y,σ) is a $g^{\#}\psi$ - totally continuous function, then

(i) f is a continuous function

(ii) f is a $g^{\#}\psi$ - continuous function.

Proof

(i) Let V be any open set in (Y,σ). By result 1.8, V is a $g^{\#}\psi$ - open set in (Y, σ).Since f is a $g^{\#}\psi$ - totally continuous, f^{-1}(V) is clopen in (X,τ) which implies that f^{-1}(V) is open in (X,τ). Therefore f is continuous.

(ii) Let V be any open set in (Y,σ). By result 1.8, V is a $g^{\#}\psi$ - open set in (Y, σ). Since f is $g^{\#}\psi$ - totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 2.2 $f^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ). Therefore f is $g^{\#}\psi$ - continuous.

The converse of proposition 2.3.6 (i) need not be true in general as seen from the following example.

Example 2.3.7

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},{a,b},Y}. Let f: (X,τ)→(Y,σ) be the identity function. Then f is continuous but f is not $g^{\#}\psi$ - totally continuous, since for the $g^{\#}\psi$ - open set {a} in (Y,σ), f^{1}({a}) = {a} is not clopen in (X,τ).

The converse of proposition 2.3.6 (ii) need not be true in general as seen from the following example.

Example 2.3.8

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X}and σ = {ϕ,{a},Y}.Let f:(X,τ)→(Y,σ) be the identity function. Then f is $g^{\#}\psi$ - continuous but f is not $g^{\#}\psi$ - totally continuous, since for the $g^{\#}\psi$ - open set {a} in (Y,σ), f^{1}({a}) = {a} is not clopen in (X,τ).

Theorem 2.3.9

Let f:(X,τ) →(Y,σ) be any function from discrete space (X,τ) into a topological space (Y,σ), If f is a $g^{\#}\psi$ - totally continuous function then

(i) f is a continuous function

(ii) f is a $g^{\#}\psi$ - continuous function.

Proof

(i) Let V be any open set in (Y,σ).By result 1.8,V is a $g^{\#}\psi$ - open set in (Y, σ). Since f is $g^{\#}\psi$ - totally continuous, f^{-1} (V) is clopen in (X,τ). which implies that f^{-1} (V) is open in (X,τ). Therefore f is continuous.

(ii) Let V be any open set in (Y,σ). By result 1.8,V is $g^{\#}\psi$ - open in (Y, σ). Since f is $g^{\#}\psi$ - totally continuous, f^{-1} (V) is clopen in (X,τ) which implies that f^{-1} (V) is open in (X,τ). By result 1.8 f^{-1} (V) is $g^{\#}\psi$ - open in (X,τ). Therefore f is $g^{\#}\psi$ - continuous..

Proposition 2.3.10

Let f:(X,τ)→(Y,σ) be a $g^{\#}\psi$ - totally continuous function and g:(Y,σ)→(Z,η) be any function. Then g o f :(X,τ) →(Z, η) is a totally $g^{\#}\psi$ - continuous function if

(i) g is a totally $g^{\#}\psi$ - continuous function

(ii) g is a totally continuous function.

Proof

(i) Let V be any open set in (Z, η). Since g is totally $g^{\#}\psi$ - continuous, g^{-1} (V) is $g^{\#}\psi$ - clopen in (Y,σ) which implies that g^{-1} (V) is $g^{\#}\psi$ - open in (Y, σ). Since f is $g^{\#}\psi$ - totally continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.8, $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ). Therefore g o f is totally $g^{\#}\psi$ - continuous.

(ii) Let V be any open set in (Z, η). Since g is totally continuous, g^{-1} (V) is clopen in (Y,σ) which implies that g^{-1} (V) is open in (Y, σ). By result 1.8 g^{-1} (V) is $g^{\#}\psi$ - open in (Y, σ). Since f is $g^{\#}\psi$ - totally continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.8 $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ). Therefore g o f is totally $g^{\#}\psi$ - continuous.

Proposition 2.3.11

Let f:(X,τ) →(Y,σ) be a $g^{\#}\psi$ - totally continuous function and g:(Y,σ)→(Z,η) be any function. Then g o f :(X,τ) →(Z, η) is a totally $g^{\#}\psi$ - continuous function if

(i) g is a continuous function

(ii) g is a $g^{\#}\psi$ - continuous function.

Proof

(i) Let V be any open set in (Z, η). Since g is continuous, g^{-1} (V) is open in (Y,σ) which implies that g^{-1} (V) is $g^{\#}\psi$ - open in (Y, σ) Since f is $g^{\#}\psi$ - totally continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.8, $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ). Therefore g o f is totally $g^{\#}\psi$ - continuous.

(ii) Let V be any open set in (Z, η). Since g is $g^{\#}\psi$ - continuous, g^{-1} (V) is $g^{\#}\psi$ - open in (Y,σ). Since f is $g^{\#}\psi$ - totally continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.8 $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ). Therefore g o f is totally $g^{\#}\psi$ - continuous.

Proposition 2.3.12

Let f: (X,τ) →(Y,σ) and g: (Y,σ)→(Z, η) be any two functions. If g is a $g^{\#}\psi$ - totally continuous function and if

(i) f is a totally continuous function, then g o f :(X,τ) →(Z, η) is a $g^{\#}\psi$ - totally continuous function.

(ii) f is a continuous function , then g o f :(X,τ) →(Z, η) is a continuous function

(iii) f is a $g^{\#}\psi$ - continuous function, then g o f :(X,τ) →(Z, η) is a $g^{\#}\psi$ - continuous function.

Proof

(i) Let V be any $g^{\#}\psi$ - open set in (Z, η). Since g is $g^{\#}\psi$ - totally continuous, g^{-1} (V) is clopen in (Y,σ) which implies that g^{-1} (V) is open in (Y,σ). Since f is totally continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). Therefore g o f is $g^{\#}\psi$ - totally continuous.

(ii) Let V be any open set in (Z, η). By result 1.8 V is a $g^{\#}\psi$ - open set in (Z, η). Since g is $g^{\#}\psi$ - totally continuous, g^{-1} (V) is clopen in (Y,σ) which implies that g^{-1} (V) is open in (Y,σ). Since f is continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in (X,τ). Therefore g o f is continuous.

(iii) Let V be any open set in (Z, η). By result 1.8 V is a $g^{\#}\psi$ - open set in (Z, η). Since g is $g^{\#}\psi$ - totally continuous, g^{-1} (V) is clopen in (Y,σ) which implies that g^{-1} (V) is open in (Y,σ). Since f is $g^{\#}\psi$ - continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - open in (X,τ). Therefore g o f is $g^{\#}\psi$ - continuous.

Proposition 2.3.13

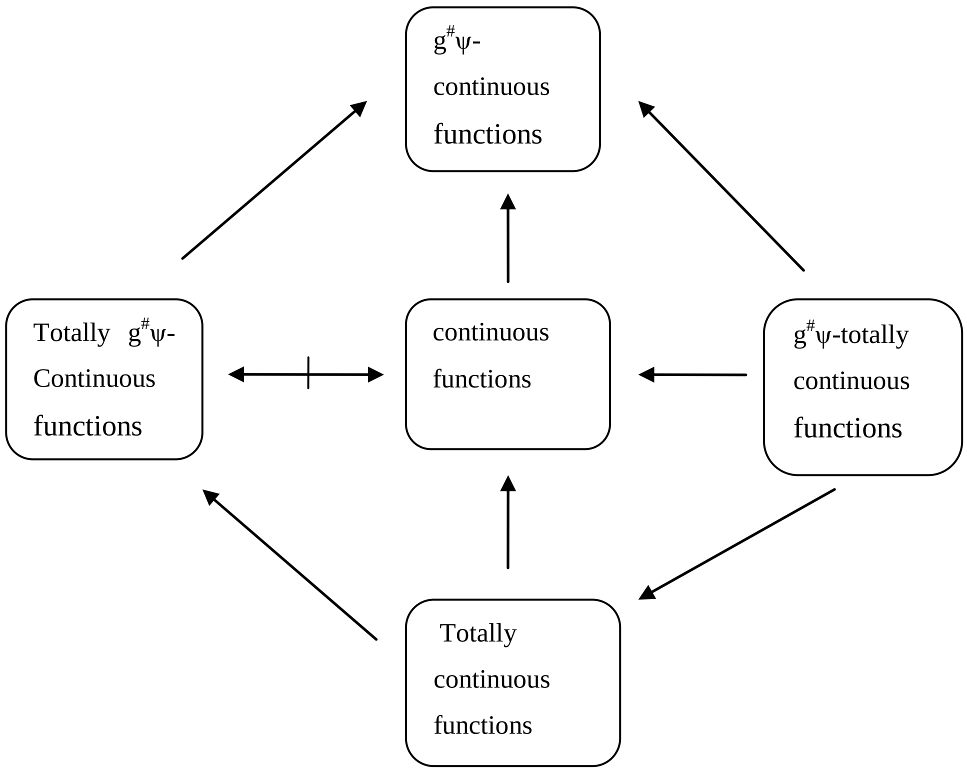
Let f: (X,τ) →(Y,σ) and g: (Y,σ)→(Z, η) be two $g^{\#}\psi$ - totally continuous function, then g o f : (X,τ)→(Z, η) is also a $g^{\#}\psi$ - totally continuous function.

Proof

Let V be any $g^{\#}\psi$ - open set in (Z, η). Since g is $g^{\#}\psi$ - totally continuous, g^{-1} (V) is clopen in (Y,σ) By result 1.8, g^{-1} (V) is $g^{\#}\psi$ - clopen in (Y,σ) which implies that $(g^{-1}(V)$ is $g^{\#}\psi$ - open in (Y,σ). Since f is $g^{\#}\psi$ - totally continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). Therefore g o f is $g^{\#}\psi$ - totally continuous.

Remark 2.3.14

The following diagram exhibits the relationship between newly defined continuous functions with already existing continuous functions.



CHAPTER-3

Contra $g^{\#}\psi$ - Continuous Functions and Almost Contra $g^{\#}\psi$ - Continuous Functions in Topological Spaces

3.1 Introduction

Singal M.K and Singal A.R, (1968) introduced almost continuous mappings in topological spaces. Dontchev (1996) introduced the notion of contra continuous functions in topological spaces. Ekici (2004) introduced almost contra continuous functions in topological spaces.

In this chapter we introduce and study a new type of contra continuous functions called contra $g^{\#}\psi$ - continuous functions and almost contra $g^{\#}\psi$ - continuous functions in topological spaces. Also we obtain the relations between these functions.

3.2 Contra $g^{\#}\psi$ - continuous functions

In this section, contra $g^{\#}\psi$ - continuous functions are introduced and studied their properties.

Definition 3.2.1

A function f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is called **contra $g^{\#}\psi$ - continuous** if $f^{-1}(v)$ is $g^{\#}\psi$ - open in (X,τ) for every closed set V of (Y,σ).

Example 3.2.2

Let X = Y = {a,b,c} , τ = {ϕ,{a},X} and σ = {ϕ,{a,b}Y}. Let f: (X,τ) →(Y,σ) be a function defined by f(a) = c, f(b) = b, f(c) =a. Then f is contra $g^{\#}\psi$ - continuous.

Theorem 3.2.3

A function f**:** (X,τ) → (Y,σ) is contra $g^{\#}\psi$ - continuous if and only if f^{-1}(U) is $g^{\#}\psi$ - closed in (X,τ) for every open set U of (Y,σ).

Proof

(**Necessity**): Let f**:** (X,τ) → (Y,σ) be contra $g^{\#}\psi$ - continuous and U be any open set in (Y,σ). Then Y-U is closed in (Y,σ). Since f is contra $g^{\#}\psi$ - continuous. f^{-1}(Y-U) = X- f^{-1}(U) is $g^{\#}\psi$ - open in (X,τ) which implies that f^{-1}(U) is $g^{\#}\psi$ - closed in (X,τ).

(**Sufficiency):** Assume that V is any closed set in (Y,σ).Then Y-V is open in (Y,σ). By assumption f^{-1}(Y-V) = X- f^{-1}(V) is $g^{\#}\psi$ - closed in (X,τ) which implies that f^{-1}(V) is $g^{\#}\psi$ - open in (X,τ). Hence f is contra $g^{\#}\psi$ - continuous.

Proposition 3.2.4

Every contra continuous function is a contra $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any open set in (Y,σ). Since f is a contra continuous function, f^{-1}(V) is closed in (X,τ). By result 1.8 f^{-1}(V) is $g^{\#}\psi$ - closed in (X,τ). Hence f is contra $g^{\#}\psi$ - continuous.

Example 3.2.5

Let X = Y = {a,b,c} , τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a},Y}. Let f: (X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = a, f(c) =c. Then f is contra $g^{\#}\psi$ - continuous but not contra continuous, since for the open set {a} in (Y,σ), f^{-1}{a}={b} is $g^{\#}\psi$ - closed but not closed in (X,τ).

Proposition 3.2.6

Every totally continuous function is a contra $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any open set in (Y,σ). Since f is totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.8 , $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is contra $g^{\#}\psi$ - continuous.

Example 3.2.7

Let X = Y = {a,b,c}, τ= {ϕ,{a},{a,b}, X} and σ = {ϕ,{a,b},Y}. Let f: (X,τ)→(Y,σ) be a function defined f(a) = c, f(b) = a, f(c) = b. Then f is contra $g^{\#}\psi$ - continuous but not totally continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b}) = {b,c} is $g^{\#}\psi$ - closed but not clopen in (X,τ).

Proposition 3.2.8

Every totally $g^{\#}\psi$ - continuous function is a contra $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any closed set in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous, $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ). Hence f is a contra $g^{\#}\psi$ - continuous function.

Example 3.2.9

Let X = Y = {a,b,c} , τ = {ϕ,{a,b},X} and σ = {ϕ,{a},Y}.Let f:(X,τ) →(Y,σ) be a function defined by f(a) = b, f(b) = c, f(c) =a. Then f is contra $g^{\#}\psi$ - continuous but not totally $g^{\#}\psi$ - continuous, since for the closed set {b,c} in (Y,σ), f^{-1}(b,c) = {a,b} is $g^{\#}\psi$ - open but not $g^{\#}\psi$ - closed in (X,τ).

Proposition 3.2.10

Every $g^{\#}\psi$ - totally continuous function is a contra $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any open set in (Y,σ). By result 1.8, V is $g^{\#}\psi$ - open in (Y,σ). Since f is $g^{\#}\psi$ - totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.8, $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is contra $g^{\#}\psi$ - continuous.

Example 3.2.11

Let X = Y = {a,b,c}, τ = {ϕ, {a},{a,b},{a,c},X} and σ = {ϕ, {a},{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined f(a) = c, f(b) = b, f(c) = a. Then f is contra $g^{\#}\psi$ - continuous but not $g^{\#}\psi$ - totally continuous, since for the $g^{\#}\psi$ - open set {a} in (Y,σ), f^{-1}({a }) = {c} is $g^{\#}\psi$ - closed but not clopen in (X,τ).

Remark 3.2.12

Contra $g^{\#}\psi$ - continuous function is independent from $g^{\#}\psi$ - continuous function as seen from the following examples.

Example 3.2.13

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is contra $g^{\#}\psi$ - continuous but not $g^{\#}\psi$ - continuous, since for the closed set $\{c\}$ in (Y,σ), $f^{-1}(\{c\}) = \{a\}$ is $g^{\#}\psi$ - open but not $g^{\#}\psi$ - closed in (X,τ).

Example 3.2.14

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is $g^{\#}\psi$ - continuous but not contra $g^{\#}\psi$ - continuous, since for the closed set $\{b,c\}$ in (Y,σ), $f^{-1}(\{b,c\}) = \{b,c\}$ is $g^{\#}\psi$ - closed but not $g^{\#}\psi$ - open in (X,τ).

Proposition 3.2.15

If $f:(X,\tau) \rightarrow (Y,\sigma)$ is a contra $g^{\#}\psi$ - continuous function and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is a continuous function , then $g \circ f : (X,\tau) \rightarrow (Z, \eta)$ is a contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any closed set in (Z, η). Since g is continuous. $g^{-1}(V)$ is closed in (Y,σ). Since f is contra $g^{\#}\psi$ - continuous. $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.2.16

If f:$(X,\tau) \rightarrow (Y,\sigma)$ is a totally $g^{\#}\psi$ - continuous function and g:$(Y,\sigma)\rightarrow(Z,\eta)$ is a continuous function , then g o f : $(X,\tau) \rightarrow (Z, \eta)$ is a contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any closed set in (Z, η). Since g is continuous. $g^{-1}(V)$ is closed in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous. $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - clopen in (X,τ). By result 1.8 $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.2.17

If f: $(X,\tau) \rightarrow (Y,\sigma)$ is a totally continuous function and g: $(Y,\sigma)\rightarrow(Z, \eta)$ is a continuous function , then g o f : $(X,\tau) \rightarrow (Z, \eta)$ is a contra $g^{\#}\psi$ - continuous function .

Proof

Let V be any closed set in (Z, η). Since g is continuous .$g^{-1}(V)$ is closed in (Y,σ). Since f is totally continuous. $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.8 $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.2.18

If f: $(X,\tau) \rightarrow (Y,\sigma)$ is a $g^{\#}\psi$ - totally continuous function and g:$(Y,\sigma)\rightarrow(Z,\eta)$ is a continuous function , then g o f : $(X,\tau) \rightarrow (Z, \eta)$ is a contra $g^{\#}\psi$ - continuous function .

Proof

Let V be any closed set in (Z, η). Since g is continuous. $g^{-1}(V)$ is closed in (Y,σ). By result 1.8 $g^{-1}(V)$ is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - totally continuous.

$(g \circ f)^{-1}(V) = f^{-1}(g^{1}(V))$ is clopen in (X,τ). By result 1.8 $(g \circ f)^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $(g \circ f)^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.2.19

If f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is contra $g^{\#}\psi$ - continuous and g: $(Y,\sigma)\rightarrow(Z, \eta)$ is totally continuous, then g o f : $(X,\tau) \rightarrow(Z, \eta)$ is a contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any closed set in (Z, η). Since g is totally continuous. $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is closed in(Y,σ). Since f is contra $g^{\#}\psi$ - continuous. $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.2.20

If f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is contra $g^{\#}\psi$ - continuous and g: $(Y,\sigma)\rightarrow(Z, \eta)$ is $g^{\#}\psi$ - totally continuous , then g o f : $(X,\tau) \rightarrow(Z, \eta)$ is a contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any closed set in (Z, η). By result 1.8 V is $g^{\#}\psi$ - closed in (Z, η). Since g is $g^{\#}\psi$ - totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is closed in (Y,σ) . Since f is contra $g^{\#}\psi$ - continuous **.** $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.2.21

If f**:** $(X,\tau) \rightarrow (Y,\sigma)$ is a contra continuous function and g: $(Y,\sigma)\rightarrow(Z, \eta)$ is continuous function, then g o f : $(X,\tau) \rightarrow(Z, \eta)$ is a contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any open set in (Z, η). Since g is continuous. $g^{-1}(V)$ is open in (Y,σ). Since f is contra continuous. $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is closed in (X,τ). By result 1.8 $(g \circ f)^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is contra $g^{\#}\psi$ - continuous function.

Proposition 3.2.22

If f: (X,τ)→(Y,σ) is a totally $g^{\#}\psi$ - continuous function and g: (Y,σ)→(Z, η) is a completely continuous function, then g o f : (X,τ) →(Z,η) is a contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any open set in (Z, η). Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y,σ). By result 1.8 $g^{-1}(V)$ is open in (Y,σ).Since f is a totally $g^{\#}\psi$ - continuous function, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - clopen in(X,τ) which implies that $(g \circ f)^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ) . Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Remark 3.2.23

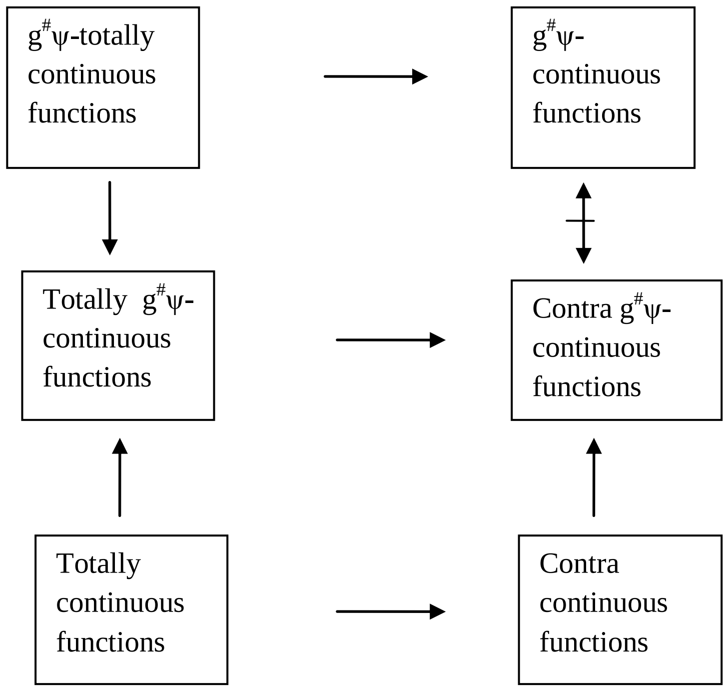
The composition of two contra $g^{\#}\psi$ - continuous functions need not be a contra $g^{\#}\psi$ - continuous function as seen from the following example

Example 3.2.24

Let X = Y = Z ={a,b,c} , τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a,b},Y} and η = {ϕ,{a},Z}. Let f**:** (X,τ) → (Y,σ) be a function defined by f(a) = c, f(b) = b , f(c) =a and g: (Y,σ)→(Z, η) be a function defined by g(a) = c, g(b) = b , g(c) = a. Then the functions f and g are contra $g^{\#}\psi$ - continuous but their composition g o f : (X,τ) →(Z, η) is not contra $g^{\#}\psi$ - continuous. Since for the closed set {b,c}in (Z, η) , $(g \circ f)^{-1}$ ({b,c}) = {b,c} is not $g^{\#}\psi$ - open in (X,τ).

Remark 3.2.25

From the above observations we have the following diagram.



3.3 Almost contra $g^{\#}\psi$ - continuous functions

In this section, a weaker form of contra $g^{\#}\psi$ - continuous functions called almost contra $g^{\#}\psi$ - continuous functions is introduced and some of their properties are studied.

Definition 3.3.1

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **almost contra $g^{\#}\psi$ - continuous** if $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ) for every regular open set V of (Y,σ).

Example 3.3.2

Let $X=Y=\{a,b,c\}$, $\tau=\{\phi,\{a\},\{a,b\},X\}$ and $\sigma=\{\phi,\{a\},\{b\},\{a,b\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by f(a)=c, f(b)=b, f(c)=a. Then f is almost contra $g^{\#}\psi$ - continuous.

Theorem 3.3.3

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is almost contra $g^{\#}\psi$ - continuous if and only if the inverse image of every regular open subset of (Y,σ) is $g^{\#}\psi$ - closed in (X,τ) .

Proof

(Necessity): Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be almost contra $g^{\#}\psi$ - continuous. Let V be any regular open set in (Y,σ). Then Y-V is regular closed in (Y,σ). Since f is almost contra $g^{\#}\psi$ - continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ).

(**Sufficiency):** Assume that U is any regular closed set in (Y,σ). Then Y-U is regular open in (Y,σ). By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is $g^{\#}\psi$ - closed in (X,τ) which implies that $f^{-1}(U)$ is $g^{\#}\psi$ - open in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Proposition 3.3.4

Every contra continuous function is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any regular open set in (Y,σ). By result 1.8, V is open in (Y,σ). Since f is contra continuous, $f^{-1}(V)$ is closed in (X,τ). By result 1.8, $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Example 3.3.5

Let $X=Y=\{a,b,c\}$, $\tau = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\},Y\}$. Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a)= b$, $f(b) = a$, $f(c) = c$. Then f is almost contra $g^{\#}\psi$ - continuous but not contra continuous, since for the open set $\{a\}$ in (Y,σ), $f^{-1}(\{a\}) = \{b\}$ is not closed in (X,τ).

Proposition 3.3.6

Every totally continuous function is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any regular open set in (Y,σ). By result 1.8, V is open in (Y,σ). Since f is totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.8, $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Example 3.3.7

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi, \{a\},X\}$ and $\sigma = \{\phi,\{a\},\{b\},\{a,b\},Y\}$ Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = c$, $f(b) =a$, $f(c)= b$. Then f is almost contra $g^{\#}\psi$ - continuous but not totally continuous, since for the open set {a,b} in(Y,σ), f^{-1}({a,b }) = {b,c} is not clopen in (X,τ).

Proposition 3.3.8

Every totally $g^{\#}\psi$ - continuous function is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any regular open set in (Y,σ). By result 1.8, V is open in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous, $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Example 3.3.9

Let X =Y = {a,b,c}, τ = {ϕ, {a},{b},{a,b},{a,c},X} and σ = {ϕ,{a}, {b},{a,b},Y} Let f:(X,τ)→(Y,σ) be a function defined by f(a)=c, f(b)=a, f(c)=b. Then f is almost contra $g^{\#}\psi$ - continuous but not totally $g^{\#}\psi$ - continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b }) = { b,c } is not $g^{\#}\psi$ - clopen in (X,τ).

Proposition 3.3.10

Every $g^{\#}\psi$ - totally continuous function is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any regular open set in (Y,σ). By result 1.8, V is open in (Y,σ) which implies that V is $g^{\#}\psi$ - open in (Y,σ). Since f is $g^{\#}\psi$ - totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.8, $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which Implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Example 3.3.11

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi, \{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\}, \{b\},\{a,b\},Y\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is almost contra $g^{\#}\psi$ - continuous but not $g^{\#}\psi$ - totally continuous, since for the $g^{\#}\psi$ - open set $\{a\}$ in(Y,σ), $f^{-1}(\{a\}) = \{c\}$ is not clopen in (X,τ).

Proposition 3.3.12

If a function $f: (X,\tau) \to(Y,\sigma)$ is contra $g^{\#}\psi$ - continuous, then it is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any regular open set in (Y,σ). By result 1.8, V is open in (Y,σ). Since f is contra $g^{\#}\psi$ - continuous, $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is a almost contra $g^{\#}\psi$ - continuous function.

Example 3.3.13

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi, \{a\},\{b\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\}, x\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by $f(a)=b$, $f(b)=a$, $f(c)=c$. Then f is almost contra $g^{\#}\psi$ - continuous but not contra $g^{\#}\psi$ - continuous, since for the open set $\{a,b\}$ in (Y,σ), $f^{-1}(\{a,b\}) = \{a,b\}$ is not $g^{\#}\psi$ - closed in (X,τ).

Proposition 3.3.14

If $f:(X,\tau)\to(Y,\sigma)$ is a almost contra $g^{\#}\psi$ - continuous function and $g:(Y,\sigma)\to(Z, \eta)$ is a completely continuous function, then $g \circ f :(X,\tau) \to(Z,\eta)$ is a almost contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any regular open set in (Z, η). By result 1.8, V is open in (Z, η). Since g is a completely continuous function, $g^{-1}(V)$ is regular open in (Y,σ). Since f is almost contra $g^{\#}\psi$ - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a almost contra $g^{\#}\psi$ - continuous function.

Proposition 3.3.15

If f:(X,τ)→(Y,σ) is a almost contra $g^{\#}\psi$ - continuous function and g:(Y,σ)→(Z, η) is a completely continuous function, then g o f :(X,τ) →(Z,η) is a contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any open set in (Z, η). Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y,σ). Since f is almost contra $g^{\#}\psi$ - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in(X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.3.16

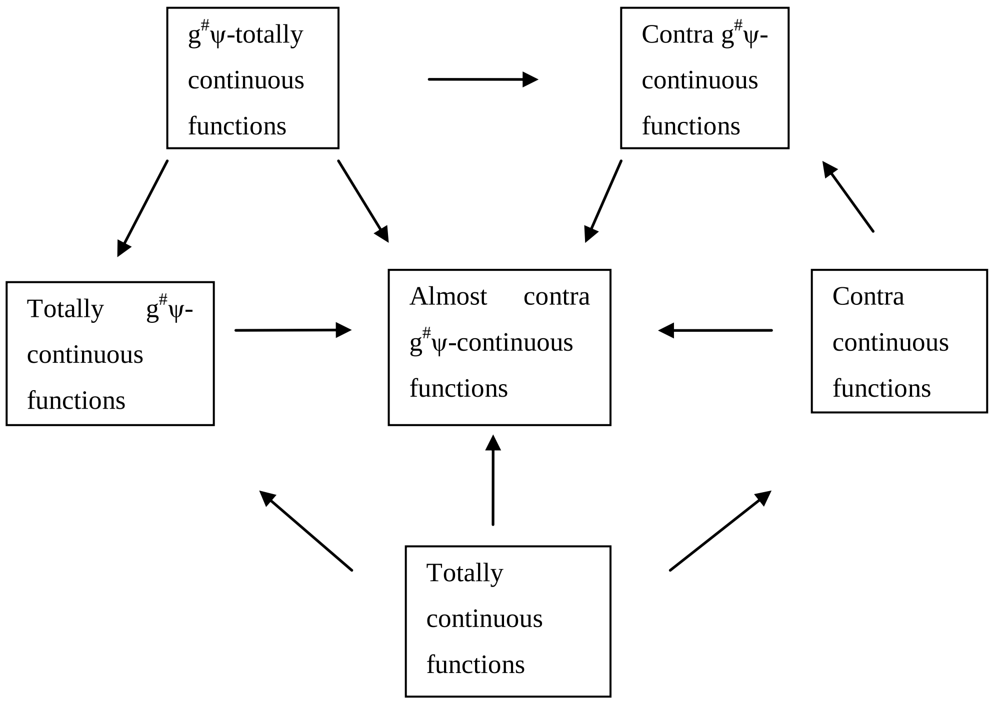
If f:(X,τ)→(Y,σ) is a contra $g^{\#}\psi$ - continuous function and g:(Y,σ)→(Z, η) is a almost continuous function, then g o f :(X,τ) →(Z,η) is a almost contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any regular open set in (Z, η).Since g is almost continuous, $g^{-1}(V)$ is open in (Y,σ). Since f is contra $g^{\#}\psi$ - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a almost contra $g^{\#}\psi$ - continuous function.

Remark 3.3.17

The above observations are depicted in the following diagram.



CHAPTER - 4

$g^{\#}\psi$ - Irresolute Functions in Topological Spaces

4.1 Introduction

Irresolute functions are stronger form of continuous functions. Crossley and Hildebrand (1972) introduced the concept of irresolute functions in topological spaces. Ramya and parvathi (2013) introduced the concept of ψg - irresolute functions in topological spaces.

In this chapter we introduce a new type of irresolute functions called $g^{\#}\psi$ - irresolute functions in topological spaces which is stronger than $g^{\#}\psi$ - continuous functions.

4.2 $g^{\#}\psi$ - irresolute functions

In this section $g^{\#}\psi$ - irresolute function in topological spaces are introduced and obtained their relation with various irresolute functions.

Definition 4.2.1

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **$g^{\#}\psi$ - irresolute** if $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ) for every $g^{\#}\psi$ - closed set V of (Y,σ).

Example 4.2.2

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is $g^{\#}\psi$ - irresolute.

Theorem 4.2.3

A function f: (X,τ) →(Y,σ) is $g^{\#}\psi$ - irresolute if and only if $f^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ) for every $g^{\#}\psi$ - open set V of (Y,σ).

Proof

(Necessity): Let f: (X,τ) →(Y,σ) be $g^{\#}\psi$ - irresolute and V be any $g^{\#}\psi$ - open set in (Y,σ). Then Y-V is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $f^{-1}(Y - V) = X - f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ).

(sufficiency): Assume that U is any $g^{\#}\psi$ - closed set in (Y,σ). Then Y-U is $g^{\#}\psi$ - open in (Y,σ). By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is $g^{\#}\psi$ - open in (X,τ) which implies that $f^{-1}(U)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is $g^{\#}\psi$ - irresolute.

Proposition 4.2.4

If a function f:(X,τ)→(Y,σ) is $g^{\#}\psi$ - irresolute then for every subset A of (X,τ) such that f(A) is $g^{\#}\psi$ - closed in (Y,σ), $f(g^{\#}\psi cl(A)) \subseteq g^{\#}\psi cl(f(A))$.

Proof

Let A be a subset of (X,τ) such that f(A) is $g^{\#}\psi$ - closed in (Y,σ). Then $g^{\#}\psi cl(f(A))$ is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $f^{-1}(g^{\#}\psi\ cl(f(A))$ is $g^{\#}\psi$ - closed in (X,τ). Now $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}((g^{\#}\psi cl(f(A)))$. Therefore $g^{\#}\psi cl(A) \subseteq f^{-1}((g^{\#}\psi cl(f(A)))$ and hence $f(g^{\#}\psi cl(A)) \subseteq f(f^{-1}(g^{\#}\psi cl(f(A))) \subseteq g^{\#}\psi cl(f(A))$.

Proposition 4.2.5

If a function f:(X,τ)→(Y,σ) is $g^{\#}\psi$ - irresolute then for every $g^{\#}\psi$ - closed set $B \subseteq Y$, $g^{\#}\psi cl(f^{-1}(B)) \subseteq f^{-1}(g^{\#}\psi cl(B))$.

Proof

Let B be a $g^{\#}\psi$ - closed set in (Y,σ). Then $g^{\#}\psi cl(B)$ is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $f^{-1}(g^{\#}\psi cl(B))$ is $g^{\#}\psi$ - closed in (X,τ). Since $B \subseteq g^{\#}\psi cl(B)$, $f^{-1}(B) \subseteq f^{-1}((g^{\#}\psi cl(B))$. Therefore by definition of $g^{\#}\psi$ - closure, $g^{\#}\psi cl(f^{-1}(B)) \subseteq f^{-1}((g^{\#}\psi cl(B))$.

Proposition 4.2.6

Every $g^{\#}\psi$ - totally continuous function is a $g^{\#}\psi$ - irresolute function but not conversely.

Proof

Let V be any $g^{\#}\psi$ - closed set in (Y,σ). Since f is $g^{\#}\psi$ - totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 1.8, $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is $g^{\#}\psi$ - irresolute.

Example 4.2.7

Let X = Y = {a,b,c}, τ = {ϕ,{a},{a,b},X}and σ = {ϕ,{a},{b},{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = a, f(c) = c. Then f is $g^{\#}\psi$ - irresolute but not $g^{\#}\psi$ - totally continuous, since for the $g^{\#}\psi$ - open set {a} in (Y,σ), $f^{-1}(\{a\}) = \{b\}$ is not clopen in (X,τ).

Proposition 4.2.8

If f: $(X,\tau) \rightarrow (Y,\sigma)$ is a $g^{\#}\psi$ - irresolute function, then it is a $g^{\#}\psi$ - continuous function but not conversely.

Proof

Let V be any closed set in (Y,σ). By result 1.8, V is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is $g^{\#}\psi$ - continuous.

Example 4.2.9

Let X = Y = {a,b,c},τ = {ϕ,{a},X}and σ = {ϕ,{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = c, f(c) = a. Then f is $g^{\#}\psi$ - continuous but not $g^{\#}\psi$ - irresolute, since for the $g^{\#}\psi$ - closed set {b,c} in (Y,σ), f^{-1}({b,c}) = {a,b} is not $g^{\#}\psi$ - closed in (X,τ).

Remark 4.2.10

Irresolute function is independent from $g^{\#}\psi$ - irresolute function as seen from the following examples.

Example 4.2.11

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X}and σ = {ϕ,{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f (a) = c , f(b) = a , f(c) = b. Then f is irresolute but not $g^{\#}\psi$ - irresolute, since for the $g^{\#}\psi$ - closed set {a} in (Y,σ), f^{-1}({a}) ={b} is not $g^{\#}\psi$ - closed in (X,τ).

Example 4.2.12

Let X = Y = {a,b,c},τ = {ϕ,{a},{b,c},X}and σ = {ϕ,{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) =b , f(b) = c , f(c) = a. Then f is $g^{\#}\psi$ - irresolute but not irresolute, since for the semi - closed set {c} in (Y,σ), f^{-1}({c}) = {b} is not semi - closed in (X,τ).

Remark 4.2.13

Totally $g^{\#}\psi$ - continuous function is independent from $g^{\#}\psi$ - irresolute function as seen from the following examples.

Example 4.2.14

Let $X = Y = \{a,b,c\}, \tau = \{\phi, \{a\},\{b\},\{a,b\},\{a,c\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is totally $g^{\#}\psi$ - continuous but not $g^{\#}\psi$ - irresolute, since for the $g^{\#}\psi$ - closed set $\{c\}$ in (Y,σ), $f^{-1}(\{c\}) = \{a\}$ is not $g^{\#}\psi$ - closed in (X,τ).

Example 4.2.15

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a\},\{a,b\},\{a,c\},X\}$ and $\sigma = \{\phi,\{a\},\{a,b\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is $g^{\#}\psi$ - irresolute but not totally $g^{\#}\psi$ - continuous, since for the open set $\{a\}$ in (Y,σ), $f^{-1}(\{a\}) = \{a\}$ is not $g^{\#}\psi$ - clopen in (X,τ).

Remark 4.2.16

ψg - irresolute function is independent from $g^{\#}\psi$ - irresolute function as seen from the following examples.

Example 4.2.17

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a\},\{a,b\},\{a,c\},X\}$ and $\sigma = \{\phi,\{a\},\{b,c\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is ψg - irresolute but not $g^{\#}\psi$ - irresolute, since for the $g^{\#}\psi$ - closed set $\{b,c\}$ in (Y,σ), $f^{-1}(\{b,c\}) = \{a,c\}$ is not $g^{\#}\psi$ - closed in (X,τ).

Example 4.2.18

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},\{b\},\{a,b\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is $g^{\#}\psi$ - irresolute but not ψg - irresolute, since for the ψg - closed set $\{a\}$ in (Y,σ), $f^{-1}(\{a\}) = \{a\}$ is not ψg - closed in (X,τ).

Remark 4.2.19

Contra $g^{\#}\psi$ - continuous function is independent from $g^{\#}\psi$ - irresolute function as seen from the following examples.

Example 4.2.20

Let X = Y = {a,b,c},τ = {ϕ,{a,b},X}and σ = {ϕ, {a},{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = c, f(c) = a. Then f is contra $g^{\#}\psi$ - continuous but not $g^{\#}\psi$ - irresolute, since for the $g^{\#}\psi$ - closed set {b} in (Y,σ), f^{-1}({ b }) = {a} is not $g^{\#}\psi$ - closed in (X,τ).

Example 4.2.21

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X }and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is $g^{\#}\psi$ - irresolute but not contra $g^{\#}\psi$ - continuous, since for the closed set {c} in (Y,σ), f^{-1}({c}) = {c} is not $g^{\#}\psi$ - open in (X,τ)

Proposition 4.2.22

If f:(X,τ)→(Y,σ) is a $g^{\#}\psi$ - irresolute function and g:(Y,σ)→(Z, η) is a $g^{\#}\psi$ - irresolute function, then g o f :(X,τ) →(Z,η) is a $g^{\#}\psi$ - irresolute function.

Proof

Let V be $g^{\#}\psi$ - closed set in (Z, η).Since g is $g^{\#}\psi$ - irresolute, g^{-1}(V) is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, (g o f)$^{-1}$(V) = f^{-1}(g^{-1}(V)) is $g^{\#}\psi$ - closed in (X, τ). Hence g o f is a $g^{\#}\psi$ - irresolute function.

Proposition 4.2.23

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a $g^{\#}\psi$ - irresolute function and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is a $g^{\#}\psi$ - continuous function, then $g \circ f:(X,\tau)\rightarrow(Z,\eta)$ is a $g^{\#}\psi$ - continuous function.

Proof

Let V be any closed set in (Z,η). Since g is $g^{\#}\psi$ - continuous, $g^{-1}(V)$ is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a $g^{\#}\psi$ - continuous function.

Proposition 4.2.24

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a $g^{\#}\psi$ - irresolute function and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is a contra $g^{\#}\psi$ - continuous function, then $g \circ f:(X,\tau)\rightarrow(Z,\eta)$ is a contra $g^{\#}\psi$ - continuous function.

Proof

Let V be any closed set in (Z,η). Since g is contra $g^{\#}\psi$ - continuous, $g^{-1}(V)$ is $g^{\#}\psi$ - open in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 4.2.25

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a $g^{\#}\psi$ - irresolute function and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is a continuous function , then $g \circ f:(X,\tau)\rightarrow(Z,\eta)$ is a $g^{\#}\psi$ - continuous function.

Proof

Let V be any closed set in (Z,η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ). By result 1.8, $g^{-1}(V)$ is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a $g^{\#}\psi$ - continuous function.

Proposition 4.2.26

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a $g^{\#}\psi$-irresolute function and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is a totally $g^{\#}\psi$ - continuous function, then $g \circ f:(X,\tau)\rightarrow(Z,\eta)$ is a $g^{\#}\psi$ - continuous function.

Proof

Let V be any closed set in (Z,η). Since g is totally $g^{\#}\psi$ - continuous, $g^{-1}(V)$ is $g^{\#}\psi$- clopen in (Y,σ) which implies that $g^{-1}(V)$ is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a $g^{\#}\psi$ - continuous function.

Proposition 4.2.27

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a $g^{\#}\psi$ - irresolute function and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is a totally continuous function, then $g \circ f:(X,\tau)\rightarrow(Z,\eta)$ is a $g^{\#}\psi$ - continuous function.

Proof

Let V be any closed set in (Z,η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ). By result 1.8, $g^{-1}(V)$ is $g^{\#}\psi$ - clopen in (Y,σ) which implies that $g^{-1}(V)$ is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a $g^{\#}\psi$ - continuous function.

Proposition 4.2.28

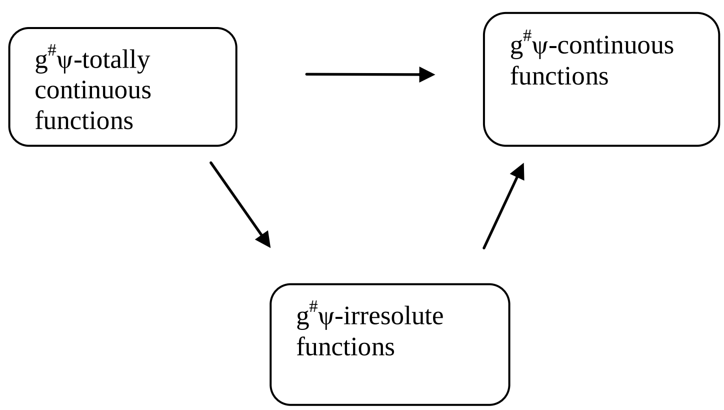
If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a $g^{\#}\psi$ - irresolute function and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is a $g^{\#}\psi$ - totally continuous function, then $g \circ f:(X,\tau)\rightarrow(Z,\eta)$ is a $g^{\#}\psi$ - continuous function.

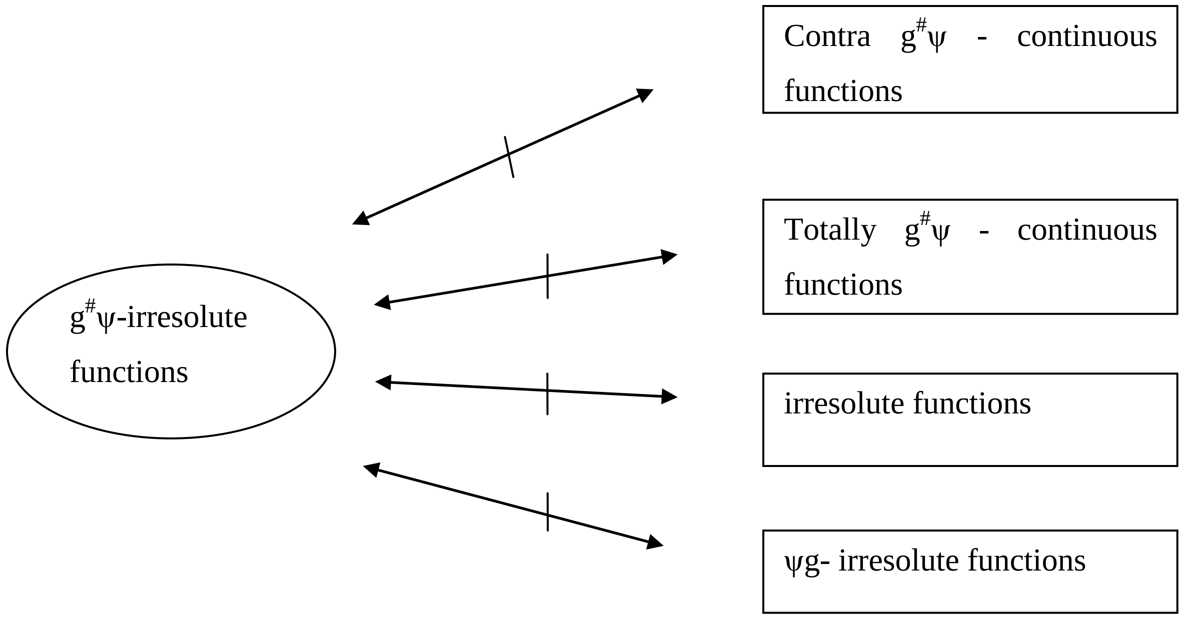
Proof

Let V be any closed set in (Z, η). By result 1.8, V is $g^{\#}\psi$ - closed in (Z, η). Since g is $g^{\#}\psi$ - totally continuous, $g^{-1}(V)$ is clopen in (Y,σ). By result 2.2, $g^{-1}(V)$ is $g^{\#}\psi$ - clopen in (Y,σ) which implies that $g^{-1}(V)$ is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X, τ) .Hence g o f is a $g^{\#}\psi$ - continuous function.

Remark 4.2.29

The interrelation between various forms of irresolute functions are given in the following diagram.




Contra g#ψ - continuous functions
Totally g#ψ - continuous functions
g#ψ-irresolute functions
irresolute functions
ψg- irresolute functions

SUMMARY AND CONCLUSION

Preliminary definitions in topological spaces are listed in chapter one.

In chapter 2 two new types of continuous functions called totally $g^{\#}\psi$ - continuous functions and $g^{\#}\psi$ - totally continuous functions are introduced. Properties and their interrelations are derived.

Contra $g^{\#}\psi$- continuous functions and almost contra $g^{\#}\psi$- continuous functions are defined in chapter three. Relations between the newly defined contra continuous functions with already existing various continuous functions are obtained.

In chapter 4 the stronger form of $g^{\#}\psi$- continuous functions namely $g^{\#}\psi$ - irresolute functions are introduced. Properties and association between $g^{\#}\psi$ - irresolute functions with already existing various irresolute functions are analyzed.

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