

## Introduction

The survival of a company depends on the income it gets from selling its products and services, and the ability to sell is based on the fitness for use. Hence the company functions concerned with quality or achieving fitness for use are known as quality function. It includes a variety of activities. Every one working in the factory or all departments is responsible for the broad quality function. Inspection of raw materials, semi finished products, or finished products is one aspect of quality assurance. Whenever a statistical technique is used to control, improve and maintain the quality, it is termed as Statistical Quality Control. When inspection is for the purpose of acceptance or rejection of a product, based on adherence to a standard, the type of procedure employed is usually called acceptance sampling. Acceptance sampling is one of the major components in the field of Statistical quality control. The probability that a device will function over a specified time period or amount of usage at a stated condition is termed as reliability. A typical application of reliability acceptance sampling is as follows :

A company receives a shipment of products from a vendor. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot, and some quality characteristic of the units in the sample is inspected for a specified period of time. On the basis of the information in this sample, a decision is made regarding the lot's disposition. Usually, this decision is either to accept or to reject the lot. Accepted lots are put into production, rejected lots may be returned to the vendor or may be subjected to some other lot disposition action.

This chapter comprises of sections that consist of basic concepts and review of sampling plans which are relevant to this thesis and enumerated as:

- 1.1. Basic concepts of quality control
- 1.2. Basic concepts of acceptance sampling

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- 1.3. Basic concepts of reliability
  - 1.4. Lifetime distributions
  - 1.5. Notations
  - 1.6. Review of literature
  - 1.7. Thesis Outline and Contribution Overview
  - 1.8. Methodology

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## Section 1.1

### Basic Concepts of Quality Control

#### 1.1.1 Introduction

Quality Control can be defined as the process through which we measure actual quality performance, compare it with the quality standard set and act on the difference in order to improve the product's quality. The reputation of companies depends upon the high reliability of their products. These companies compete with each other on the basis of quality and reliability. Thus quality control has become one of the most important tools to differentiate between competitive enterprises in a global business market. In order to control the quality of the purchased goods, two major alternatives are open to a buyer. One is the complete inspection, in which every single item in the lot is inspected and tested. This is often impractical, uneconomical or impossible. Secondly, the partial inspection in which a sample of the item is taken for inspection and testing and the whole lot is accepted or rejected depending on whether a few or many defective items are found in the sample.

Quality Control is the use of techniques and activities to achieve, sustain, and improve the quality of a product or service. It involves integrating the following related techniques and activities such as specifications of what is needed, design of the product or service to meet the specifications, production or installation to meet the full intent of the specifications, inspection to determine conformance to specifications and review of usage to provide information for the revision of specifications, if needed.

Quality control and management is not very complex at this time, but the craftsman's design of the product to meet the best of his customer's need is very important. The importance of Quality control lies in the following benefits likely to be realized by an effective quality assurance program.

- Improving the quality of product and services
- Increasing productivity

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- Reducing tangible and intangible costs
  - Reducing production and delivery lead times
  - Improving the marketability of products and services
  - Reducing the prices of products and services to customers
  - Detecting the assignable causes and elevating its effects

### **1.1.2 Quality Control**

Quality Control may generally be defined as a system that is used to maintain a desired level of quality in a product or service. This task may be achieved through different measures such as planning, design, use of proper equipment and procedures, inspection, and taking corrective action in case a deviation is observed between the products, service or process output and a specified standard. This general process may be divided into the three main sub processes namely, off-line quality control, statistical process control and acceptance sampling plans.

- An off-line quality control procedure deals with measures to select and choose controllable product and process parameters in such a way that the deviation between the products or process output and the standard will be minimized.
- Statistical Process Control involves comparing the output of a product or service with a standard and taking remedial actions in case of a discrepancy between the two. It also involves determining whether a process can produce a product that meets the desired specifications or requirements.
- Acceptance sampling deals with the inspection of the product or service. When 100 percent inspection of all items is not feasible, a decision has to be made on how many items should be sampled or whether the batch should be sampled at all. The information obtained from the sample is used to decide whether to accept or reject the entire batch, A plan that determines the number of items to be sampled and the acceptance criteria of the lot, based on meeting certain stipulated conditions is known as an acceptance sampling plan.

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### **1.1.3 Quality Improvement**

Quality improvement is defined as the reduction of variability in process and product where variations are measured by statistical methods. An efficient quality improvement program can be instrumental in increasing productivity at a reduced cost. As a result of increasing customer quality requirements and development of new product technology, many existing quality assurance practices and techniques need modifications. The need for statistical and analytical techniques in quality assurance is rapidly increasing owing to stiff competition in the industry. To improve the quality of products and services, it is customary to modernize the quality practices and simultaneously reduce the cost of quality. The three important factors that affect the quality of a product are

- Quality of design
- Quality of conformance
- Quality of performance

#### **Quality of Design**

The quality of design of a product is concerned with the tightness of the specifications for manufacturing the product. A good quality of design must ensure consistent performance over its stipulated life span stated in terms of rated output, efficiency, and overload capacity, continued or intermittent operation for specified application or service. It should also consider the possible modes of failure due to stress, wear, distortion, corrosion, shocks, vibrations high or low temperature, altitude or pressure, environmental conditions etc. However product design and development is a continuous process which results in the evaluation of a product, based on assessed user needs, their feedback after use and development in technology at a given point of time, in a given environment.

#### **Factors controlling quality of design**

- The most important factor that controls the quality of design is the type of customers in the market. This can be analyzed in detail with the help of a market survey. A market survey is the study of three main factors; the first and particularly important one is the consuming habits of people. Secondly it

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is the prices they are willing to pay for various products and services. Third is the choice of design of the product which means the needs of the customers. Thus the quality of the design depends upon the type of customers to provide the intended function with the greatest overall economy.

- In case of capital goods, the decision is usually governed by such considerations as environmental conditions, reliability, importance of continuity of service, maintainability etc.
- Profit is an important factor for the producers. Thus considering profit as a factor is very essential for a company to succeed monetarily. It is very difficult for a company to give 100 percent quality products and in quality control 100 percent quality from a company is also not essential. Instead a market segment to which the management desires to cater should be considered. Profit can be maximized by producing products in different grades to suit different types of customers.
- Environmental conditions also play an important role in deciding the quality of a design. A high end car model that can perform extremely well in a normal temperature and normal conditions cannot give the same performance in different temperatures and conditions.
- Another factor is the special importance of the product. Greater the requirements for strength, fatigue resistance, life interchangeability of the manufacture of an item, closer should be the tolerances to give better quality goods.

## **Quality of Conformance**

The quality of conformance means comparing the manufactured product with the quality design to see how well the manufactured product conforms to the quality of design. It is responsible for production, planning and manufacturing to obtain a high level of quality of conformity when a design is established.

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## **Factors controlling quality of conformance**

For good quality of conformance with the design, an organization should ensure the following :

- Incoming raw materials are adequate.
- Machines / tools for the job and the measuring instruments are adequate.
- Proper selection of the process and process control
- Operators should be well trained, experienced and motivated.
- Proper storage for finished goods.
- Proper inspection program
- Obtaining feedback from both the internal inspection and the customers for taking corrective action.
- Quality control techniques should be used to control variability in manufacturing process.
- Higher quality of design usually costs more, higher quality of conformance usually costs less, by reducing the number of defective products produced.

## **Quality of Performance**

The quality of performance is concerned with how well the manufactured product gives its performance. It depends on both the quality of design and conformance. Even in the best designed product, a poor conformance control can cause poor performance, conversely the best conformance control cannot make the product function correctly, if the design itself is not right.

### **1.1.4 Objectives of quality control**

- A proper quality control aims to improve the income of the producers by making the product more acceptable in the market. It tries to fulfill the consumer's needs by providing the product a long life, greater usefulness, aesthetic aspects, maintainability etc.

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- It tries to reduce the company's purchasing cost by reducing the losses due to defects.
  - The aim of quality control is to achieve interchangeability of manufacture in large scale production.
  - It helps to produce optimum quality at minimum price.
  - It ensures satisfaction of customers with products or services of high quality level, to build customer's goodwill, confidence and reputation of manufacturer.
  - It ensures quality control at proper stages by proper inspection to make sure that non – defective products are produced.
  - It judges the conformity of the process to the established standards and takes suitable action when there are deviations.
  - It improves quality and productivity by process control, experimentation and customer's feedback.
  - It develops procedure for good vendor – vendee relations.
  - It develops quality consciousness in the organization.

### **1.1.5 Costs of Quality**

Every company has to satisfy the quality needs of their customers. The cost that a company spends for carrying out the quality functions is known as costs of quality. This includes

- Market survey cost to discover the quality needs of the customers.
- Product research and development costs.
- Design costs of translating the product concept into information which permits planning for manufacture.
- Cost of manufacturing planning in order to meet required quality specifications
- Cost of inspection and test.
- Cost of defect prevention.
- Cost of scrap, quality failures.
- Cost of quality assurance.
- Cost for field service and such other factors attributed to the quality improvement.

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According to the American Society for Quality Control, the quality costs is be defined in four categories

- **Cost of Prevention:** It consists of the cost associated with personnel engaged in designing, implementing, and maintaining the quality system. This includes the cost associated with creating overall quality planning, cost of preparation of manuals and procedures needed to communicate these plans, cost associated with implementing quality plans, cost for preparing training programs on quality performance, cost associated with preventing recurring defects, cost of the investigation, analysis and correction of causes of defects by quality control department and engineering department and finally the cost of cost consciousness programs.
- **Cost of Appraisal :** The costs associated with the measuring, evaluating or auditing of products, components and purchased materials to assure conformance with quality standards and performance requirements are called cost of appraisal. This includes the cost of receiving or incoming, laboratory acceptance testing, checking labor, set up for inspection and test materials, maintenance and calibration of test and inspection equipments, quality audits, review of test and inspection data and evaluation of field stock and spare parts.
- **Cost of Internal Failures :** The cost associated with defective products, components, and materials that fail to meet quality requirements and result in manufacturing losses are called as costs of internal failures. This includes the cost associated with scrap, cost of rework and repair, cost of re-inspection and retest after the defective parts are repaired, cost associated with material review activity, cost to ensure whether non – conforming products are usable for some other work and the costs of process yield lower than the yield that might be attainable by improving controls.
- **Costs of External Failures :** It consists of the costs which are generated because of defective products being shipped to the customers. This includes

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the cost of processing complaints from the customers, cost of service to the customers who receive defective items, cost of inspecting and repairing the defective items returned by the customers, cost of replacing the defective materials or products, cost of concession made to the customers due to substandard products being accepted by them.

### **1.1.6 Statistical Quality control**

Whenever a statistical technique is employed to control, improve and maintain quality or to solve quality problem it is termed as Statistical Quality Control. The new era of quality control development began during World War II when statistical quality control was much needed due to mass production. It is used throughout the quality system at various stages of production such as

- Incoming inspection
- Product moving from one stage to other
- In -process
- Machine start-up
- Process monitoring
- Process adjustment
- Final product
- Field surveillance

Statistical quality control is systematic as compared to the guess-work of haphazard process inspection, and also the mathematical and statistical approach neutralizes personal bias and uncovers poor judgment. Statistical quality control consists of three general activities

- Systematic collection and graphic recording of accurate data.
- Analyzing the data.
- Practical engineering / management or management action, if the information obtained indicates significant deviations from the specified limits.

Statistical quality control plays an important role in total quality control. The following are the statistical tools used generally for the purpose of exercising control,

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improvement of quality, enhancement of productivity, creation of consumer confidence and development of the industrial economy of the country.

- Frequency distribution : It is a tabulation or tally of the number of times a given quality characteristic occurs within the samples. Graphic representation of frequency distribution will show the average quality, spread of quality, comparison with specific requirements and process capability.
- Control Chart : It is a graphical representation of quality characteristics, which indicates whether the process is under control or not.
- Acceptance sampling : It is a popular quality control technique that is applied to discrete lots or batches of product. The term lot refers to a collection of physical units; the term batch is usually applied to chemical materials. The lot or batch should be presented to the inspection department by either a supplier or a production department. The inspection department then inspects a sample from the lot or batch and, based on the results of the inspection, determines the acceptability of the lot or batch.
- Analysis of the data: This includes techniques such as analysis of tolerances, correlation, analysis of variances, analysis for engineering design to eliminate cause of troubles.

### **1.1.7 Advantages of Statistical Quality Control**

- Statistical quality control ensures rapid and efficient inspection at a minimum cost
- It finds out the cause of excessive variability in manufactured products by forecasting trouble before rejections occur and reducing the amount of spoiled work.
- It exerts more effective pressure for quality improvement than that of a 100 percent inspection.

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- It easily detects faults. For example using control charts one can easily examine the deterioration in quality by examining whether the points fall above the upper control limits or below the lower control limits.
  - So long as the statistical control continues, specifications can be accurately predicted for the future, by which it is possible to assess whether the production processes are capable of producing the products with the given set of specifications.
  - Increases output and reduces wasted machines and materials resulting in higher productivity.
  - Better customer relations through general improvement in product and higher share of the market.
  - It provides a common language that may be used by designers, production personnel and inspectors.
  - Elimination of bottlenecks in the process of manufacturing.
  - It says when and where 100 percent inspection is required.
  - Creates quality awareness in employees.

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## Section 1.2

### Basic Concepts of Acceptance Sampling

#### 1.2.1 Introduction

Acceptance sampling procedure is an essential tool in statistical quality control. It is a methodology that deals with quality contracting on product order between the producers and consumers thus allowing the producers to take the decision to accept or reject the manufactured products based on the inspection of samples.

Acceptance sampling is necessary to limit the cost of inspection and is the only available method to appraise the quality in destructive testing. Acceptance sampling itself does not improve quality, but whenever the lot is rejected it indicates the instability of the production process. Acceptance sampling is cost efficient and the only admissible method of efficient tests with quick results.

#### 1.2.2 Importance of Acceptance Sampling

Acceptance sampling is one of the latest aspects of quality assurance and used primarily for the incoming and outgoing lot by lot quality assurance. The most effective use of acceptance sampling is not to “inspect quality into the product,” but rather as an audit tool to ensure that the output of a process conforms to requirements.

According to Ducan (1986), an acceptance sampling plan is likely to be implemented

- when the cost of inspection is high and the loss arising from the passing of a non conforming unit is not great; it is also possible in some cases that no inspection at all will be the cheapest plan.
- when 100 percent inspection is fatiguing, a carefully worked out sampling plan will produce good or better results. The 100 percent may not mean 100 percent perfect quality, and the percentage of non conforming items passed may be higher than under a scientifically designed sampling plan.

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- when inspection is destructive i.e., a situation where inspection is not possible without destroying the article chemically or physically.
  - when there are great quantities or areas to be inspected.
  - when it is desired to stimulate the maker and/or the buyer.

### **1.2.3 Major Areas of Acceptance Sampling**

According to Dodge (1959), the major areas of acceptance sampling are

- Lot-by-Lot sampling by the method of attributes, in which each unit in a sample is inspected on a go-not-go basis for one or more characteristics.
- Lot-by-Lot sampling by the method of variables, in which each unit in a sample is measured for a single characteristic such as weight or strength.
- Continuous sampling of a flow of units by the method of attributes, and
- Special purpose plans including chain sampling, skip-lot sampling, small sample plans, repetitive group sampling plans etc.

### **1.2.4 Basic Terminologies and Notations**

#### **Sampling Plan**

According to the American National Standards Institute / American Society for Quality Control (ANSI / ASQC) Standard A<sub>2</sub> (1987), an acceptance sampling plan is a specific plan that states the sampling rules to be used with the associated acceptance and non acceptance criteria.

#### **Sampling Scheme**

According to the American National Standards Institute / American Society for Quality Control (ANSI / ASQC) Standard A<sub>2</sub> (1987), a sampling scheme is a specific set of procedures which usually consists of acceptance sampling plans in which lot sizes, sample sizes and acceptance criteria, or the amount of 100 percent inspection and sampling are related. Hill (1962), has also described the difference between sampling plans and the sampling scheme. According to him, a sampling scheme is a whole set of sampling plans and operations included in the standard over-all strategy specifying the way in which sampling plans are to be used.

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## **Sampling System**

A sampling system can be described as an assigned grouping of two or more sampling plans with the rules for using these plans for sentencing lots to achieve a blend of the advantageous features of each of the sampling plans. Tightened – Normal – Tightened scheme is an example for sampling scheme.

### **ANSY ASQ Z1.4 – 1993**

The International organization adopts a single sampling plan and acceptance level for Standardization and designed International Standard IS01 DIS-2859 and MIL-STD-105E. While MIL-STD-105E was developed for government procurement, it has become the standard for attribute inspection for the industry. It is the most widely used acceptance-sampling plan in the world. The American Society made modifications to MIL-STD-105E (1989), for Quality (ASQ) under the designations ANSY ASQ Z1.4.

The standard is applicable, but not limited, to attribute inspection of the following

- End items
- Components and raw materials
- Operations
- Materials in process
- Supplies in storage
- Maintenance operations
- Data or records
- Administrative procedures

Sampling plans of this standard are intended to be used for a continuing series of lots.

### **Cumulative and Non – Cumulative Sampling Plans**

The non - cumulative sampling plan is defined as one which uses the current sample information from the process or current product entity in making decisions about the process or product quality. Single and Double sampling plans are

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examples of non-cumulative sampling. Cumulative results sampling inspection is one which uses the current and past information from the process in making a decision about the process. The Chain sampling plan of Dodge (1955), is an example for cumulative sampling plan.

## **Inspection**

ANSI / ASQC Standard A<sub>2</sub> (1987), defines the term inspection as ‘activities’, such as measuring, examining, testing, gauging one or more characteristics of a product and/or service and comparing these with specified requirements to determine conformity. A sampling scheme or sampling system may contain three types of inspections namely normal, tightened and reduced inspection.

- Normal Inspection: Inspection that is used in accordance with an acceptance scheme when a process is considered to be operating at its acceptance quality level or slightly better than its acceptance quality level is termed as normal inspection.
- Tightened Inspection: A feature of a sampling scheme using stricter acceptance criteria than those used in normal inspection is known as tightened inspection.
- Reduced Inspection: A feature of a sampling scheme permitting smaller sample sizes than used in normal inspection is known as reduced inspection.

## **Acceptance Quality Level (AQL)**

The AQL is a percent defective that is the base line requirement for the quality of the producer's product. The producer would like to design a sampling plan such that there is a high probability of accepting a lot that has a defect level less than or equal to the AQL.

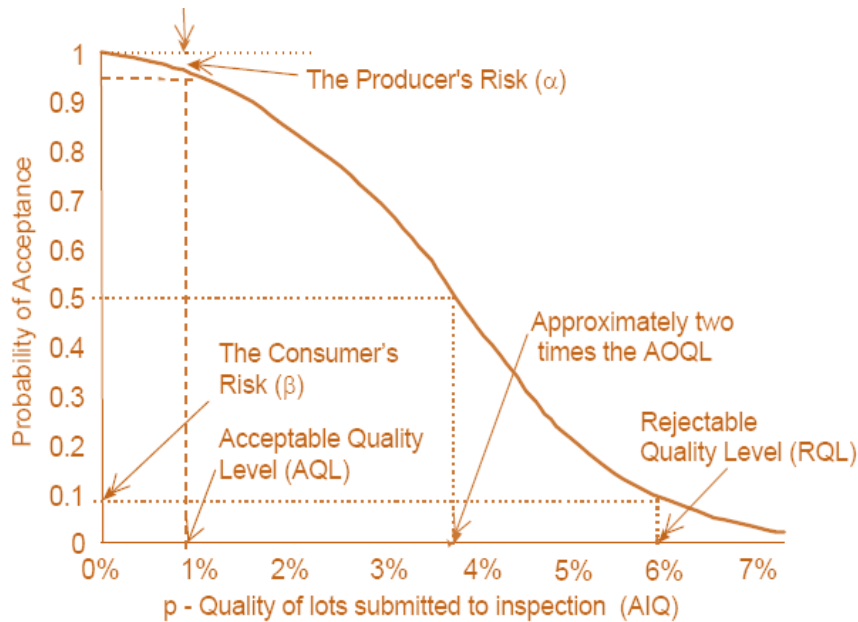
## **Lot Tolerance Percent Defective (LTPD)**

The LTPD is a designated high defect level that would be unacceptable by the consumer. The consumer would like the sampling plan to have a low probability of accepting a lot with a defect level as high as the LTPD.

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## OC Curve

Every sampling plan is associated with an operating characteristic curve, familiarly known as OC curve of the plan. This curve when referred to two axes, the axis of  $p$ -proportion nonconforming of the material offered for inspection and the axis of  $P_a(p)$  – probability of acceptance of a lot or process, is the locus of  $(p, P_a(p))$ . The OC curve gives the practical performance of a sampling plan.



**Figure 1.1 OC Curve**

OC curves are generally classified as Type A and Type B OC curves. ANSI/ASQC Standard A<sub>2</sub> (1987), defines the two terms as follows :

- Type A OC curve for isolated or unique lots or a lot from an isolated sequence. For a given sampling plan, a type A curve shows the probability of accepting a lot as a function of the lot quality.
- Type B OC curve for a continuous stream of lots. For a given sampling plan, a type B curve shows the probability of accepting a lot as a function of the process average.

For continuous sampling plans the OC curve is a curve showing the long run percentage of the product accepted during the sampling phase(s) as a function of

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the quality level of the process. For special purpose plans the OC curve is a curve showing, for a specified sampling plan, the probability of continuing to permit the process to continue without adjustment as a function of the process quality.

In sampling schemes or systems, one will have a composite OC curve which gives the steady state probability of acceptance under the switching rules of the scheme or system as a function of process quality.

Under Type A situations (when sampling an attribute characteristic from a finite lot without replacement) hyper geometric distribution is exact to calculate  $L(p)$ . Under Type B situations (when sampling from the conceptually infinite output of units that the process would turn out under the same essential conditions) binomial model will be exact for the case of non conforming units to calculate  $L(p)$ . Binomial model is also exact in the case of sampling from a finite lot with replacement. Poisson model is exact in calculating  $L(p)$  which specifies a given number of non conformities per unit (or per hundred units). Variable sampling plans use normal distribution for calculating  $L(p)$ . Binomial, Poisson, Hyper geometric and Normal distributions are the most frequently used distributions in acceptance sampling.

### **Average Outgoing Quality (AOQ) and Average Outgoing Quality Level (AOQL)**

ANSI/ASQC Standard A<sub>2</sub> (1987), defines AOQ as the expected quality of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality.

AOQL is defined as the maximum AOQ over all possible levels of incoming quality.

### **Average Sample Number (ASN)**

ANSI/ASQC Standard A<sub>2</sub> (1987), defines ASN as the average number of sample units per lot used for making decisions (acceptance or non- acceptance). A plot of ASN against  $p$  is called the ASN curve.

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## **Type I Error (Producer's Risk)**

The producer's risk is the probability of rejecting a good lot which otherwise would have been accepted. The symbol  $\alpha$  is commonly used for Type I error.

## **Type II Error (Consumer's Risk)**

The consumer's risk is the probability of defective lots being accepted which otherwise would have been rejected. The symbol  $\beta$  is commonly used for Type II error.

## **Single sampling plan**

Sampling inspection in which the decision to accept or not to accept a lot is based on the inspection of a single sample of size 'n'

## **Double sampling plan**

Sampling inspection in which the inspection of the first sample of size  $n_1$  leads to a decision to accept a lot, not to accept it or to take a second sample of size  $n_2$  and the inspection of the second sample then leads to a decision to accept or not to accept the lot.

## **Chain sampling plan**

Sampling inspection in which the criteria for acceptance and non-acceptance of the lot depends in part on the results of the inspection of the immediately preceding lots.

## **Group sampling plan**

Sampling inspection in which the items in a group are tested independently, identically and simultaneously on the different testers for a pre-assigned time.

### **1.2.5 Designing sampling plans**

In designing a sampling plan, one has to accomplish a number of different purposes. According to Hamaker (1960), the important ones are :

- To strike a proper balance between the consumer's requirements, the producer's capabilities, inspector's capacity.

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- To separate bad lots from good.
  - Simplicity of procedures and administration.
  - Economy in number of observations.
  - To reduce the risk of wrong decisions with increasing lot size.
  - To use accumulated sample data as a valuable source of information.
  - To exert pressure on the procedure or supplier when the quality of the lots received is unreliable or not up to the standard.
  - To reduce sampling when the quality is reliable and satisfactory.

He further noted that these aims are partly conflicting and all of them cannot be simultaneously realized.

The design methodologies of acceptance sampling may be categorized as follows :

	Risk Based	Economical Based
Non – Bayesian	1	2
Bayesian	3	4

Risk based sampling plans are traditional in nature, drawing upon procedure and consumer type of risks as depicted by the OC curve. Economically based sampling plans explicitly consider such factors as costs of inspections, accepting a non conforming unit and rejecting a conforming unit in an attempt to design a cost-effective plan. Bayesian plan design takes into account the past history of similar lots submitted previously for inspection purposes. Non - Bayesian plan design is not explicitly based upon the past lot history.

According to Peach (1947), the following are some of the major types of designing the plans, which are classified according to the types of protection.

- The plan is specified by requiring the OC curve to pass through (or nearly through) two fixed points. In some cases it may be possible to impose certain additional conditions. The two points generally selected are  $(p_1, 1-\alpha)$  and  $(p_2, \beta)$  where

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- $p_1$  or  $p_{1-\alpha}$  - the quality level that is considered to be good so that the procedure expects lots of  $p_1$  quality to be accepted most of the time.
  - $p_2$  or  $p_\beta$  - the quality level that is considered to be poor so that the consumer expects lots of  $p_2$  quality to be rejected most of the time.
  - $\alpha$  – the producer’s risk of rejecting  $p_1$  quality.
  - $\beta$  – the consumer’s risk of accepting  $p_2$  quality.

The tables provided by Cameron (1952), are an example for this type of designing. Schilling (1980), considered the term  $p_1$  as the Producer’s Quality Level (PQL) and  $p_2$  as the Consumer’s Quality Level (CQL). Earlier literature calls  $p_1$  as the Acceptance Quality Level (AQL) and  $p_2$  as the Limiting Quality Level (LQL) or Rejectable Quality Level (RQL) or Lot Tolerance Proportion Defective (LTPD). Peach (1947), have defined the ratio  $p_2/p_1$  associated with specified values of  $\alpha$  and  $\beta$  as the ‘Operating Ratio’. Traditionally the values of  $\alpha$  and  $\beta$  are assumed to take 0.05 and 0.10 respectively.

- The plan is specified by fixing one point only, through which the OC curve is required to pass and setting up one or more conditions, not explicitly in terms of the OC curve. Dodge and Romig (1959), LTPD tables is an example for this type of design.
- The plan is specified by imposing upon the OC curve two or more independent conditions none of which explicitly involves the OC curve. Dodge and Romig (1959), AOQL tables is an example of this type of design.

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## Section 1.3

### Basic Concepts of Reliability

#### 1.3.1 Introduction

The concept of reliability has been known for a number of years, but it has assumed greater significance and importance during the past decade, particularly due to the impact of automation, development in complex missile and space programmes. The manufacture of highly complex equipment has served to focus greater attention on reliability. However, reliability is only one of the tools of management which must be supplemented by other tools like quality control and design of experiments for the solution of problems of quality and cost. Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered. Reliability of a product is the measure of the ability of a product to function successfully, when required, for the period required, in the specified environment.

The study of reliability is important because it is related to the quality of a product. Reliability of a product is more important because it is common for a person to think that, what is the use of buying a product that does not satisfy the customer needs and fails within a short period. Thus the effectiveness of a system is understood to mean the suitability of the system for the fulfillment of the intended tasks and the efficiency of utilizing the means put into it. The suitability of performing definite tasks is primarily determined by the quality of the system.

#### 1.3.2 Historical Background of Reliability

The growth and development of reliability has strong links with quality control and its developments in the first quarter of the 1900s. In the 1920s a team of workers at Bell Telephone Laboratories, developed statistical methods to solve some of their quality control problems. They provided the basis for the further developments in the area of statistical quality control (SQC). Subsequently, the American Society for Testing and Mechanical Engineers joined Bell Laboratories in popularizing the quality control techniques. However, the rate of adoption of these

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techniques among the enterprises was very slow till World War II broke out in 1939. The importance of reliability and quality control was born out of the demands of modern technology used in the World War II. Complexity and automation of equipments used in the war resulted in severe problems of maintenance and repair. Failures of equipments and components, particularly, electronic tubes, were more than expected. During the war the army and navy in the USA set up a joint committee known as the Vacuum Tube Development Committee in 1943 to study the failure of vacuum tubes which was considered to be one of the focal points of trouble. Quantitative techniques for reliability measurement were evolved and introduced.

During the decade following the war many research laboratories and universities initiated studies on failures of equipments and components. Bell Laboratories and Aeronautical Radio, Inc were the two leading organizations among those who contributed heavily in this area. Practicing engineers and mathematicians took interest in the study of life testing and reliability problems. The first major committee on reliability was set up by the US Department of Defence in 1950. This was later called the Advisory Group on Reliability of Electronic Equipment (AGREE). The AGREE published its first report in 1957 which included some reliability specifications such as minimum acceptability limits, reliability test equipments, etc. Since then many new organizations were formed to promote the concepts of reliability and quality among both manufacturers and users.

During the 1950s other countries such as Britain and Japan began to take keen interest in the application of reliability principles to their products. In Britain, The British Productivity Council and the Institute of Production Engineers were independently engaged in promoting quality control concepts. In 1961, the National Council for Quality and Reliability was formed with the main objective of creating an awareness of importance of achieving quality and reliability in the design, manufacture and use of products. The last two decades have seen remarkable progress in the application of reliability principles in industries and government departments in almost all developed and developing countries. Today reliability has become a catchword in many firms and is a very common term in most countries.

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### 1.3.3 Definition of Reliability

Reliability of a unit is the probability that the unit performs its intended function adequately for a given period of time under the stated operating conditions or environment. By a unit we mean an element, a system or a part of a system.

If  $T$  is the time till the failure of the unit (a random variable) occurs, then the probability that it will not fail in a given environment before time  $t$  (or its reliability) is

$$R(t) = P(T > t)$$

Thus, reliability is always a function of time. It also depends on environmental conditions which may or may not vary with time. Since it is a probability, its numerical value is always between one and zero, i.e.,

$$R(0) = 1, R(\infty) = 0$$

and  $R(t)$  is a non-increasing function between these limits.

### 1.3.4 Basic Elements of Reliability

The reliability definition stresses five elements mainly

- Numerical value of probability.
- Statement defining successful product performance.
- Statement defining the environment in which the equipment must operate.
- Statement of the required operating time.
- The type of distribution likely to be encountered in reliability measurement.

### 1.3.5 Design for Reliability

Reliability design is an iterative process that begins with the specification of reliability goals consistent with cost and objectives. This requires consideration of the life-cycle costs of the system and the effect that reliability has on overall costs and system effectiveness. Once these liability goals have been established, these goals must be translated into individual component, subcomponent, and part specifications. This is not necessarily an easy task, and it generally requires a

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reliability block analysis. After individual component and part requirements have been determined, various design methods can be applied in order to meet the goals. These methods include the proper selection of parts and material, stress-strength analysis, derating, simplification, identification of technologies, and use of redundancy.

Following completion of a preliminary detailed design along with initial development and prototyping, a failure analysis may be performed to determine whether the specifications are being met and also provide a systematic approach for identifying, ranking, and eliminating failure models. This requires the use of reliability testing, including, perhaps a formalized reliability growth testing program. Once reliability goals have been achieved, verification that safety margins are also being met must be made. If either the reliability or safety goals are not met, the design process must continue, This may require reallocating reliability goals among the components if it is not possible to achieve a desired component reliability. The effect of design changes should then be verified through continued use of failure analysis and reliability testing.

Although we are considering reliability as an inherent system or component attribute that can largely be determined during design, we cannot ignore the fact that reliability is influenced throughout a product life cycle by factors external to the product itself.

### **1.3.6 Achievement of Reliability**

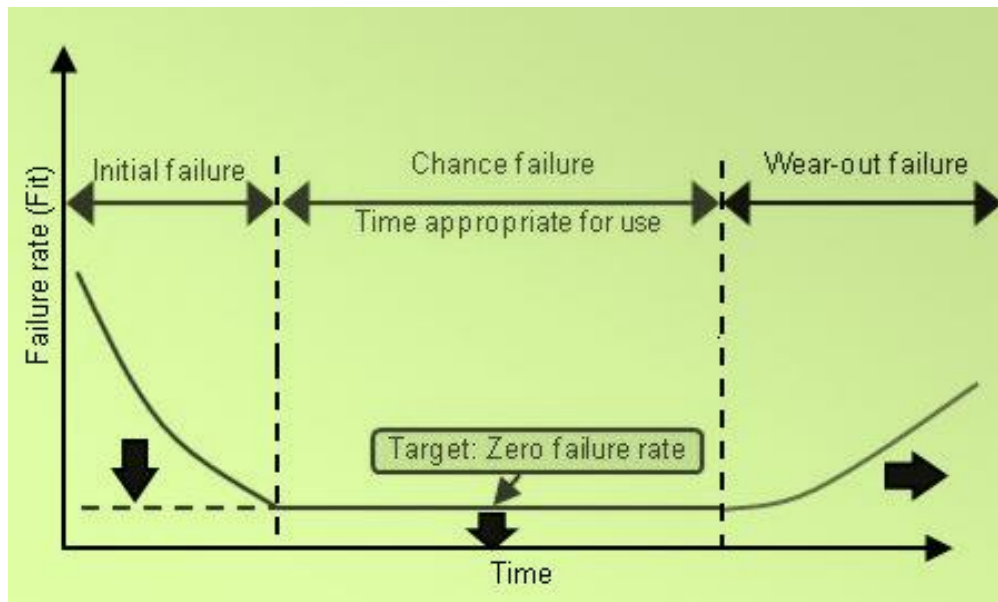
There are five effective areas for the achievement of reliability of the product. They are :

- Design
- Production
- Measurement and testing
- Maintenance
- Field operation

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Design is very important than the other four areas and a greater percentage of causes of unreliability can be traced out in this area.

### 1.3.7 Failure Pattern



**Figure 1.3.7 Failure Pattern**

The products often follow a familiar failure pattern of failure. When the failure rate (number of failures per unit time) is plotted against a continuous time scale, the resulting chart is known as “bath tub curve”. This curve exhibits three distinct zones. These zones differ from each other in frequency of failure and in the cause of failure pattern. These are as follows

1. Infant Mortality Period (Or Burn In Or The Debugging Period): This is characterized by high failure rates. It begins at the first point during manufacture that total equipment operation is possible and continues for such a period of time as permits (through maintenance and repairs), the elimination of marginal parts initially defective though not inoperative and unrecognizable as such until premature failure. Commonly, these are early failures resulting from defect in manufacturing, or other deficiencies which can be detected by debugging, running on or extended testing.

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2. The constant failure rate period : Upon replacement of all prematurely failing items, the failure rate will have reached a lower value. From this point the failure rate remains fairly constant. These are chance failures which may result from the limitations inherent in the design plus accidents caused by usage or poor maintenance or hidden defects which escape inspection. This period is the normal operating period in which the average failure rate remains fairly constant.
  3. The wear out period : These are failures due to abrasion fatigue, corrosion, vibration etc. Example, the metal becomes embrittled, the insulation dries out. A reduction in failure rate requires preventive replacement of these dying components before they result in catastrophic failure.

Many failure-causing mechanisms give rise to measured distributions of times-to-failure which approximate quite closely to a definite mathematical form, known as probability density functions, or p.d.f. These functions provide mathematical models of failure patterns, which can be used in performance forecasting calculations.

### **1.3.8 Methods for improving design reliability**

Improving the reliability of a product by changing the design is done by designer himself. The following are some of the approaches used by the designers working jointly with reliability engineers to improve the design.

- Review the index selected to define product reliability to make sure that it reflects customer needs.
- Question the function of the unreliable parts with a view of eliminating them entirely if the function is found to be unnecessary.
- Review the selection of any parts which are relatively new.
- Conduct a research and development program to increase the reliability of the parts which are contributing most to the unreliability of the equipments.

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- Specify corrective replacement times for unreliable parts and replace the parts before they fail.
  - Select the parts which will be subjected to stress which are lower than the parts can normally withstand.
  - Control the operating environment so that a part will be operating under conditions which yield a lower failure rate.
  - Use redundancy so that if one unit fails a redundant unit will be available to do the job.
  - Consider possible trade-offs of reliability with functional performance weight or other parameters.

### **1.3.9 Life testing**

Reliability testing refers to the tests conducted to verify that a product will work satisfactorily for a given time period. Reliability testing therefore consists of functional test, environmental test and life testing.

A functional testing involves a test to determine if the product will function at time zone. An environmental test consists of determining the expected environmental levels and then carrying the functional test under the environments under which the product has to operate.

The life of the component is the time period during which it retains its quality characteristic. Life tests are carried out to assess the working life of a product, its capabilities and hence to form an idea of its quality level. The life test aims to measure the time or period during which the product will retain its desired quality characteristics. This may apply to either shelf life or life during use or both.

Life tests are carried out in different manners under different conditions as follows

- Tests under actual working conditions
- Tests under intensive conditions
- Tests under accelerated conditions

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## **Tests under actual working conditions**

This kind of test is a life test of the component under actual working conditions for full durations. This is impractical and do not lend any help in controlling a manufacturing process.

## **Tests under intensive conditions**

Suppose if a product works one hour daily and if it is tested under actual working condition it would be operated only one hour daily and it is just a waste of days. This is impractical and time consuming. Therefore it is worked continuously at rated specifications and thus the life can be estimated in a much shorter duration of time.

## **Tests under accelerated conditions**

These tests are conducted under severe operating conditions to quicken the product failure or break down.

### **1.3.10 Accelerated life tests**

One obtains information on the failure time (actual failure time or an interval containing the failure time) for units that fail and lower bounds for the failure time (also known as the running time or run out time) for units that do not fail.

## **Accelerated life test models**

Most popular accelerated life test models have the following two components

- A parametric distribution for the life of a population of units at a particular level(s) of an experimental variable or variables. It might be possible to avoid this parametric models (e.g., Weibull and lognormal) to provide important practical advantages for most applications.
- A relationship between one (or more) of the distribution parameters and the acceleration will effect that the variables like temperature, voltage, humidity and specimen or unit size will have on the failure-time distribution. This part of the accelerated life model should be based on a physical model such as one

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relating the accelerating variable to degradation, on a well established empirical relationship, or some combinations.

### **1.3.11 Acceptance sampling plans based on life tests**

The sampling techniques and control charts are very important tools which analyses the data of life (destructive) tests. It is not necessary to subject all the sample pieces to destructive testing, the results in such case can be concluded from the time of first and middle failure. However, the potential capability of the product can be determined only through destructive testing.

In recent times many authors have developed tables which show, for a life test, the relationship between the sample size, probability and percent of units which will fail before their shortest life for different parametric models.

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## Section 1.4

### Lifetime Distributions

#### 1.4.1 Introduction

The use of parametric distributions complements non-parametric techniques and provides the following advances:

- Parametric models can be described concisely with just few parameters, instead of having to report an entire curve.
- It is possible to use a parametric model to extrapolate (in time) to the lower or upper tail of a distribution.
- Parametric models provide smooth estimates of failure- time distributions.

#### 1.4.2 Rayleigh distribution

The cumulative distribution function (cdf) of the Rayleigh distribution is given by

$$F(t, \lambda) = 1 - e^{-\frac{1}{2}\left(\frac{t}{\lambda}\right)^2}, \quad t > 0, \lambda > 0 \quad (1.4.1)$$

where  $\lambda$  is the scale parameter

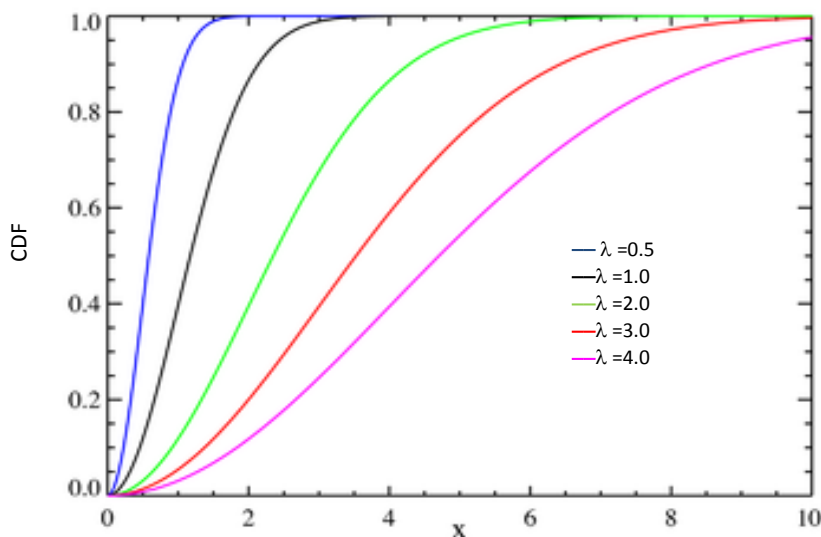


Figure 1.4..1 Cumulative distribution function of Rayleigh distribution

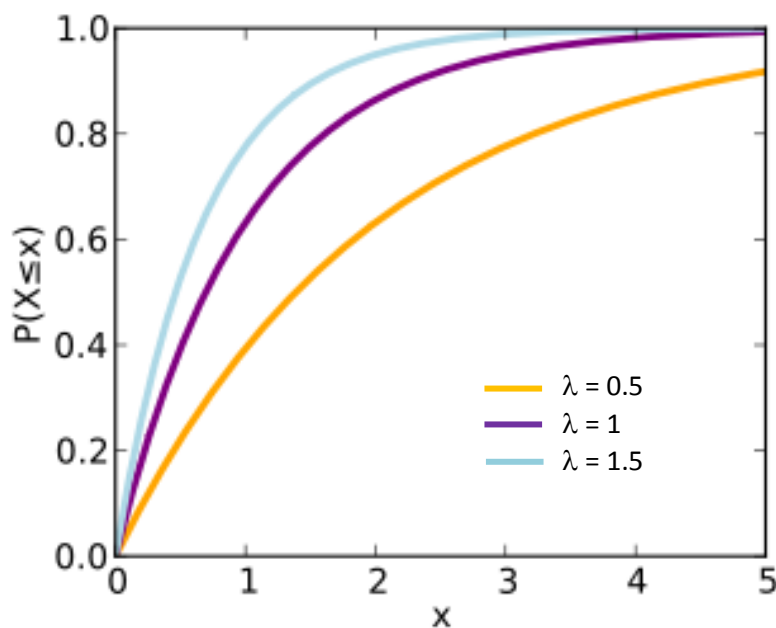
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### 1.4.3 Generalized Exponential distribution

The cumulative distribution function (cdf) of the Generalized Exponential distribution is given by

$$F(t, \lambda) = \left(1 - e^{-\frac{t}{\lambda}}\right)^\alpha, \quad t > 0, \lambda > 0 \quad (1.4.2)$$

where  $\lambda$  is the scale parameter and  $\alpha$  is the shape parameter and it is fixed as 2



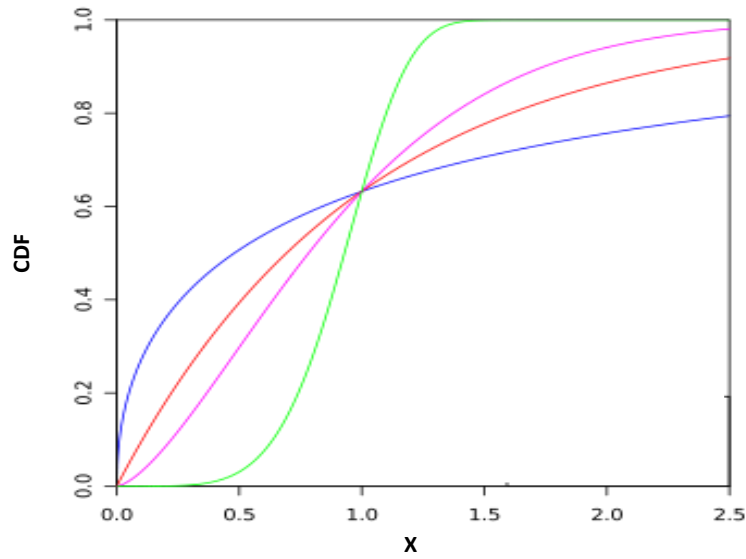
**Figure 1.4.2 Cumulative distribution function of Generalized Exponential distribution**

### 1.4.4 Weibull distribution

The cumulative distribution function (cdf) of the Weibull distribution is given by

$$F(t, \lambda) = 1 - e^{-\left(\frac{t}{\lambda}\right)^m}, \quad t > 0, \lambda > 0 \quad (1.4.3)$$

where  $\lambda$  is the scale parameter and  $m$  is the shape parameter and it is fixed as 2.



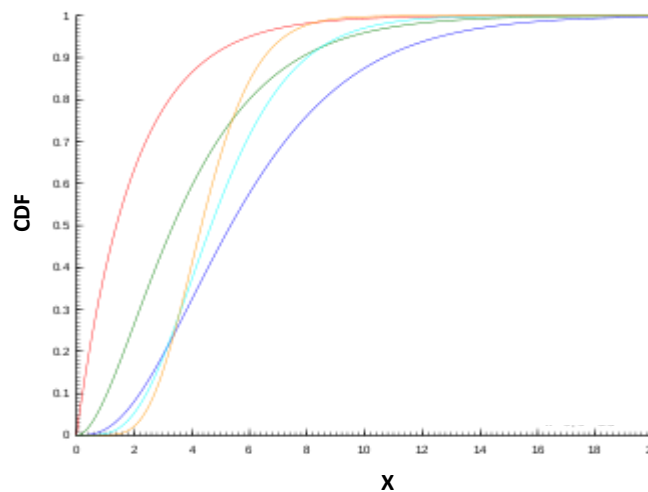
**Figure 1.4.3 Cumulative distribution function of Weibull distribution**

### 1.4.5 Gamma distribution

The cumulative distribution function (cdf) of the Gamma distribution is given by

$$F(t, \lambda) = 1 - e^{-\frac{t}{\lambda}} \sum_{j=0}^{\gamma-1} \left( \frac{t}{\lambda} \right)^j / j! \quad t > 0, \lambda > 0 \quad (1.4.4)$$

where  $\lambda$  is the scale parameter and  $\gamma > 0$  is the shape parameter and it is fixed as 2.



**Figure 1.4.4 Cumulative distribution function of Gamma distribution**

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### 1.4.6 Marshall- Olkin Extended Lomax distribution

The cumulative distribution function (cdf) of the Marshall-Olkin extended Lomax distribution is given by

$$F(t, \lambda) = \frac{(1+t/\lambda)^\theta - 1}{(1+t/\lambda)^\theta - \bar{\gamma}}, \quad t > 0, \lambda > 0 \quad (1.4.5)$$

where  $\bar{\gamma} = 1 - \gamma$  and  $\gamma$  is the shape parameter and let us assume  $\gamma = 2$ ,  $\lambda$  is scale parameter and  $\theta$  is shape parameter.

In Real life application the experimenter wants to establish the lifetime of the electrical devices these distributions are more significant than other distribution.

These distributions simultaneously satisfy both the Producers risk and Consumer risk.

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## Section 1.5

### Notations and Symbols

$n$	-	Sample size
$n_1$	-	Size of the first sample
$n_2$	-	Size of the second sample
$g$	-	Number of groups
$r$	-	Number of items in a group
$c_1$	-	Acceptance number of the first sample
$c_2$	-	Acceptance number of the second sample
$d$	-	Number of defectives
$m, k, \gamma$	-	Shape parameter
$\lambda$	-	Scale parameter, Mean life
$t_0$	-	Termination time
$a$	-	Test termination time multiplier
$\alpha$	-	Producer's risk
$\beta$	-	Consumer's risk
$p$	-	Failure probability
$L(p)$	-	Probability of acceptance
$\lambda_0$	-	Specified life
$t/\lambda_0$	-	Time termination ratio

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## **Section 1.6**

### **Review of Literature**

The objective of reliability acceptance sampling plans are developed to obtain information concerning failures in order to quantify reliability and to improve product reliability. As required by the principles of statistical inference it is necessary to specify the probability distribution of the variable quality characteristic under consideration. In the absence of such specifications, it is taken as the well known normal distribution. However, if the normal distribution is not a good fit to the data under consideration, the decision process constructed on this basis would be misleading. At the same time, an appeal to central limit theorem as a justification to normality assumption is not always valid as the sample size in quality control data is not large enough to adopt normality. In this backdrop, acceptance sampling plans based on truncated life tests for a variety of distributions was discussed by many authors.

#### **Attributes sampling plan**

In (1952), Cameron constructed and computed tables for the operating characteristics of attributes Single sampling plans. Dodge and Romig presented sampling inspection tables for the Single and Double sampling plans in (1959), Evolution of acceptance sampling plans was discussed by Dodge in the year (1969). Schilling (1980), compiled a book on acceptance sampling in quality control. Schilling along with Johnson (1982), presented tables for the construction of matched Single, Double and Multiple sampling plans with application to MIL-STD-105E in the year (1989), Vedaldi (1984), designed a new criterion for the construction of Single sampling plans by attributes. Paul Mathews, et.al. (2008), introduced the design of Single sampling plans for defectives.

Guenther (1969), studied the use of the Binomial, Hypergeometric and poisson tables to obtain sampling plans. Also Guenther (1970), find a procedure for

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finding Double sampling Plans for attributes. Hald (1981), has proposed Double sampling plan with acceptance number 0 and 1 and rejection number 2. Govindaraju presented some interesting observations in acceptance sampling plans in the year (1987).

Dodge and Stephens (1966), studied the Chain sampling plans and in (1974), they gave comparison of Chain sampling plan with other sampling plans. Stephens, (2001a), compiled a book on applied acceptance sampling which comprises of different plans, procedures and principles of different acceptance sampling plans. In (2002), Radhakrishnan contributed a study on selection of certain acceptance sampling plans. Radhakrishnan and Alagirisamy (2011), constructed Group acceptance sampling plan using weighted binomial distribution.

#### **Sampling plans for truncated life test :**

Even though the acceptance sampling plans based on truncated life test has gradually gained its importance in the later part of the twentieth century, it is gaining much more attention in the twenty first century. Single sampling plan based on truncated life tests have been proposed for a variety of life distribution by many authors. In (1953), Epstein and Sobel introduced life testing and Epstein in (1954), presented truncated life tests in the exponential case. Sobel and Tischendorf (1959), discussed acceptance sampling plans with new life test objectives. In (1961), Goode and Kao presented sampling plans based on the Weibull distribution. Gamma distribution in acceptance sampling based on life tests was discussed by Gupta and Groll in the same year. Kantam et.al. (1998), studied the half Logistic distribution in acceptance sampling based on life tests.

Balasoorya Saw and Gadag (2000), presented a progressively censored reliability sampling plans for the Weibull distributions. Kantam, et.al. (2001a,b), proposed an economic reliability test plan for log-logistic distribution, and Birnbaum-Saunders distribution for percentiles. Baklizi (2003), presented an acceptance sampling plan based on truncated life tests based on Pareto distribution of the second kind. In (2004), Baklizi and El Masri proposed an acceptance sampling plan based on truncated life tests based on the Birnbaum Saunders model. In

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(2005), Rosaiah and Kantam presented an acceptance sampling plan based on the inverse Rayleigh distribution.

Tsai, et.al. (2006), proposed an acceptance sampling plan based on truncated life tests for Generalized Rayleigh distribution. Kantam, et.al and Rosaiah (2006a), discussed half logistic distribution in acceptance sampling based on life tests. Kantam, et.al (2006b), proposed a reliability test plan for exponentiated log-logistic distribution. Balakrishnan, et.al.(2007), proposed an acceptance sampling plan for truncated life tests based on the generalized Birnbaum-Saunders distribution. In (2007a), Aslam and Shabaz proposed an economic reliability test plans using the Generalized Exponential distribution. Kantam, et.al. (2007a,b), proposed an economic reliability test plan with Exponentiated log-logistic distribution and Inverse Rayleigh variate.

Srinivasa Rao, et.al. (2008), presented an acceptance sampling plan for truncated life test based on Marshall-Olkin extended Lomax distribution. Tsai et.al. (2008), presented reliability sampling plans under truncated life tests with inverse gaussian data and Aslam and Kantam (2008), proposed an economic reliability acceptance sampling based on truncated life tests based on the Birnbaum-Saunders distribution. Srinivasa Rao, et.al. (2009b), proposed an acceptance sampling plan for truncated life tests based on Marshall- Olkin extended Lomax distribution. Srinivasa Rao et.al. (2009c) presented an economic reliability test plan for Marshall-Olkin extended Exponential distribution. Tsai, et.al. (2010) proposed an acceptance sampling plans for truncated life tests based on Burr type XII percentiles. Aslam, et.al. (2010a), presented a time truncated acceptance sampling plans for Generalized Exponential distribution. Mughal, et.al.(2011), proposed an economic reliability acceptance sampling plans from truncated life tests based on the Burr Type XII percentiles. Srinivasa Rao, et.al. (2011d), proposed an acceptance sampling plan based on life tests for a log-logistic model. Srinivasa Rao, et.al. (2012b), proposed a reliability test plans for type – II exponentiated log – logistic distribution.

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Muhammad Aslam (2007b), proposed a Double acceptance sampling plan based on truncated life tests based on the Rayleigh distribution. In (2012a), Srinivasa Rao proposed a Double acceptance sampling plans based on truncated life tests for the Marshall-Olkin extended exponential distribution. In (2012 a,b), Sudamani Ramaswamy and Priya Anburajan presented Double acceptance sampling plans based on truncated life for Generalized Exponential and Marshall-Olkin extended lomax distribution.

In Group sampling plan a multiple number of items as a group can be tested simultaneously in a tester. Aslam and Jun (2009a), have designed a Group acceptance sampling plan for a truncated life test when the lifetime of an item follows either inverse Rayleigh or a log-logistic distribution. Srinivasa Rao (2009a), have designed a Group acceptance sampling plans for lifetimes following a Generalized Exponential distribution. Aslam et.al.(2009b), have studied a Group sampling plan based on truncated life test for Gamma distributions.

In (2010b), Aslam, et.al. proposed an economic reliability Group acceptance sampling plans for lifetimes following a Marshall-Olkin extended distribution. Aslam, et.al. (2010c,d), proposed an economic reliability Group acceptance sampling plans for lifetimes following a Marshall-Olkin extended distribution.

In (2011a), Aslam, et.al. presented a Group acceptance sampling plan for the generalized Rayleigh distribution. Srinivasa Rao, (2011a), has studied the Group sampling plan based on truncated life tests for the Marshall-olkin extended Lomax distribution, and also he (2011b), introduced a hybrid Group acceptance sampling plans for lifetimes based on Generalized Exponential distribution and log – logistic distributions. Aslam, et.al. (2011c), presented economic reliability Group acceptance sampling plans for lifetimes following a Generalized Exponential distribution. Aslam, et.al. (2011b), proposed an improved Group sampling plans based on time – truncated life tests based on Weibull distribution,

In (2012c), Sudamani Ramaswamy and Priya Anburajan presented the Group acceptance sampling plans using weighted binomial on truncated life

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test for Marshall – Olkin Extended Distributions and (2012d), they presented the Group acceptance sampling plans using weighted binomial on truncated life test for inverse Rayleigh and log-logistic distributions. Sudamani Ramaswamy and Priyah Anburajan (2012e), had studied the hybrid Group acceptance sampling plans for lifetimes based on Weibull distribution. In (2012f,g), Sudamani Ramaswamy and Priya Anburajan presented hybrid Group sampling plan for truncated life tests based on inverse Rayleigh, Marshall-Olkin extended Lomax distribution and Weibull distribution.

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## Section 1.7

### Thesis Outline and Contributions Overview

This thesis is devoted to the application of the minimum angle method to design some acceptance sampling plans for truncated life tests based on different life time distributions. The distributions considered in this thesis are the Rayleigh distribution, Generalized Exponential distribution, Weibull distribution, Gamma distribution and Marshall-Olkin Extended Lomax distribution.

This thesis consists of seven chapters :

Chapter 1 comprises of sections about the notations, definitions, review of literature, thesis outline and contribution overview and methodologies used.

In Chapter 2 Single acceptance sampling plans for truncated life tests are considered.

The following is the procedure for a Single acceptance sampling plan based on truncated life test. The Single sampling plan is completely specified by three numbers  $N$ ,  $n$  and  $c$ ,

Where  $N$  is the lot size,

$n$  is the sample size, and

$c$  is the acceptance number,

1. Select a random sample of size  $n$  from lot of size  $N$ , and put them on test for time  $t_0$ .
2. Inspect all the articles included in the sample. Let  $d$  be the number of defectives in the sample.
3. (i) If  $d \leq c$ , accept the lot.

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(ii) If  $d > c$ , reject the lot.

A new approach for designing Single sampling plans for truncated life tests using minimum angle method, is proposed when the life time of the items follow Rayleigh distributions, Generalized Exponential distribution, Weibull distribution and Gamma distribution. The test termination time and mean ratio time are specified. The acceptance number is also specified. The design parameter is obtained such that it satisfies both producer's risk and consumer's risk simultaneously and at the same time it minimizes the sum of the risks. The tables of design parameter are provided for easy selection of the plan parameter. The results are analysed with the help of tables and examples. Among all the distributions used, when Weibull distribution is followed, the sample size is very much less than the sample size of all other distributions.

Chapter 3 deals with Double acceptance sampling plans based on truncated life tests. The following is the procedure for the Double acceptance sampling plan based on truncated life test.

1. Draw the first sample of size  $n_1$  and put them on test for time  $t_0$ .
2. Accept the lot if not more than  $c_1$  failures occur. Reject the lot and terminate the test if there are more than  $c_2$  failures.
3. If the number of failures is between  $c_1$  and  $c_2$ , then draw the second sample of size  $n_2$  and put them on test for time  $t_0$ .
4. Accept the lot if the total number of failures from the first and second samples is not more than  $c_2$ . Otherwise, reject the lot and terminate the test.

A new approach is proposed for designing Double sampling plans for truncated life test using minimum angle method, when the life time of the items follows Rayleigh distribution, Generalized Exponential distribution, Weibull distribution, Gamma distribution and Marshall-Olkin Extended Lomax distribution. The test termination time and mean ratio time are specified. The acceptance numbers are specified as  $c_1 = 0$  and  $c_2 = 2$ . The design parameter is obtained such that it satisfies both the producer's risk and consumer's risk simultaneously and at

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the same time it minimizes the sum of the risks. The tables of design parameter are provided for easy selection of the plan parameter. The results are analysed with the help of tables and examples. Among all the distributions used, when the Weibull distribution is followed, the sample size is very much less than the sample size of all other distributions.

Chapter 4 deals with Special purpose Double sampling plan of type DSP (0,1) based on truncated life tests. The following is the procedure for the Special purpose Double sampling plan of type DSP (0,1) based on truncated life test. The plan is implemented in the following way

- (i) From a lot, select a sample size  $n_1$ , and observe the number of defectives  $d_1$ , during the time  $t_0$ .
- (ii) If  $d_1 = 0$ , accept the lot.  
If  $d_1 > 1$ , reject the lot.
- (iii) If  $d_1 = 1$ , select a second sample of size  $n_2$  and observe  $d_2$  during the time  $t_0$ .  
If  $d_2 = 0$ , accept the lot. otherwise reject the lot.

A new approach for designing of Special purpose Double Sampling Plan DSP (0,1) for truncated life test using minimum angle method, is proposed when the life time of the items follows Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution. The test termination time and mean ratio time are specified. The design parameter is obtained such that it satisfies both the producer's risk and consumer's risk simultaneously and at the same time it minimizes the sum of the risks. The tables of design parameter are provided for easy selection of the plan parameter. The results are analysed with the help of tables and examples. Comparing all distributions we see that when Weibull distribution is followed, the sample size is very much less than the sample size of all other distributions.

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Chapter 5 deals with Chain sampling plan based on truncated life tests. The following is the procedure for the Chain sampling plan based on truncated life test. The plan is implemented in the following way

- 1) For each lot, select a sample of  $n$  units and test each unit for conformance to the specified requirements during the time  $t_0$ .
- 2) Accept the lot if  $d$  (the observed number of defectives) is zero in the sample of  $n$  unit, and reject if  $d > 1$ .
- 3) Accept the lot if  $d$  is equal to 1 and if no defectives are found in the immediately preceding  $i$  samples of size  $n$ .

A new approach of designing Chain sampling plans for truncated life test using minimum angle method, is proposed when the life time of the items follow Rayleigh distributions, Generalized Exponential distribution, Weibull distribution and Gamma distribution. The test termination time, mean ratio time and number of preceding sample  $i = 2$  are specified. The design parameter is obtained such that it satisfies both the producer's risk and consumer's risk simultaneously and at the same time it minimizes the sum of the risks. The tables of design parameter are provided for easy selection of the plan parameter. The results are analysed with the help of tables and examples. We see that when Weibull distribution is followed, the sample size is much less than the sample size of all other distribution.

Chapter 6 deals with Group acceptance sampling plans based on truncated life tests. The following is the operating procedure of the Group acceptance sampling plan based on the truncated life test.

- 1) Select the number of groups  $g$  and allocate predefined  $r$  items to each group so that the sample size for a lot will be  $n = gr$ .
- 2) Select the acceptance number  $c$  for a group and specify the experiment time  $t_0$ .
- 3) Perform the experiment for  $g$  groups simultaneously and record the number of failures for each group.

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- 4) Accept the lot if most  $c$  failures occurs in each of all groups by the experiment time.
  - 5) Terminate the experiment as soon as more  $c$  failures occur in any group and reject the lot.

A new approach of designing a Group acceptance sampling plan for a truncated life test when the life time of the test items follow Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution. Minimum angle method is applied to determine the design parameter group size  $g$  by satisfying both the risks at the specified quality levels simultaneously and at the same time minimizing the sum of risks. Tables of design parameters are provided. The results are explained with some examples and comparisons are made among the distributions considered. Among all the distributions used, when the Generalized Exponential distribution is followed, the sample size is very much less than the sample size of all other distribution.

Chapter 7 deals with Group acceptance sampling plans with weighted binomial distribution based on truncated life tests. The following is the operating procedure of the Group sampling plans for truncated life tests.

A new approach for Group acceptance sampling plan with weighted binomial distribution for a truncated life test when the life time of the test items follow Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution is proposed. Minimum angle method is applied to determine the design parameter group size  $g$  by satisfying both the risks at the specified quality levels simultaneously and at the same time minimizing the sum of risks. Tables of design parameters are provided. The results are explained with some examples and comparisons are made among the distributions considered. Among all the distributions used, when the Generalized Exponential and Weibull distribution is followed, the sample size is very much less than the sample sizes of all other distributions.

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## Section 1.8

### Methodology

Acceptance sampling plans are statistical tools for rectifying quality assurance. The sampling plan provides the vendor and buyer the decision rules for product acceptance to meet the present quality requirements. Several types of decision rules have been proposed for the acceptance sampling problem but work on designing the parameters to control quality of the received lots based on Minimum Angle Method is scarce. According to the author's knowledge in acceptance sampling plan based on truncated life test, no one has applied minimum angle method to design the parameter. In acceptance sampling plan based on truncated life test, the design parameters are usually determined for the specified experiment time and for the acceptance number which satisfies consumer's risk only. This method used the one-point approach on operating characteristic curve, so it may not always satisfy the producer's risk. In this thesis a new approach for designing parameter for acceptance sampling plans based on truncated life test is introduced. An attempt is made to design the parameter such that it satisfies both the conditions producer's risk as well as consumer's risk and at the same time it minimizes the sum of the risks. Using minimum angle method the design parameter is selected such that  $L(p_1) \geq 1-\alpha$  and  $L(p_2) \leq \beta$  and at the same time sum of risk is minimum.

#### 1.8.1 Minimum Angle Method

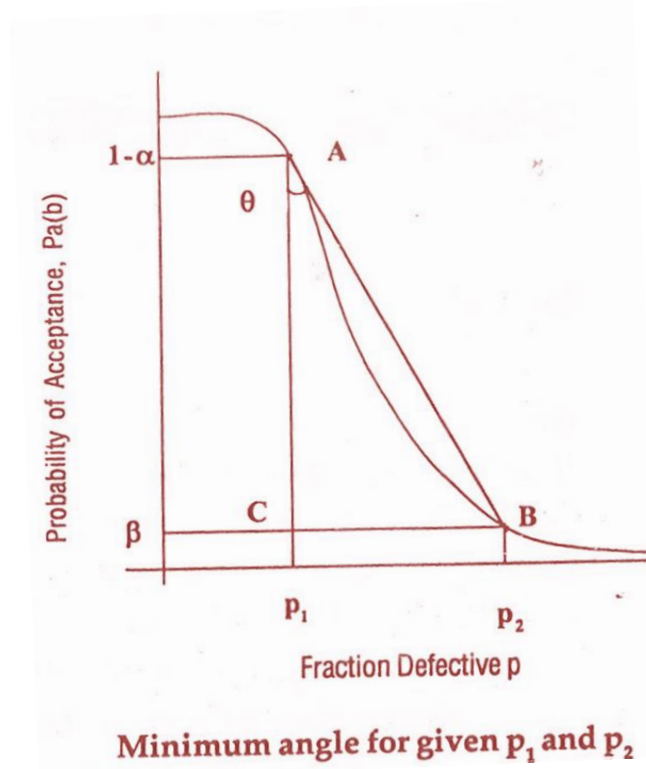
The practical performance of a sampling plan is revealed by its operating characteristic curve. Norman Bush et. al. (1953), have used different techniques involving comparison of some portion of the OC curve to that of the ideal curve. The approach of minimum angle method by considering the tangent of the angle between the lines joining the points  $(AQL, 1-\alpha)$ ,  $(LQL, \beta)$  is shown in Figure 1.8.1 where  $p_1 = AQL$ ,  $p_2 = LQL$ . By employing this method one can get a better discriminating plan with the minimum angle. Tangent of angle made by lines AB and AC is

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$$\tan\theta = BC/AC$$

$$\tan\theta = (p_2 - p_1) / (Pa(p_1) - Pa(p_2)) \quad (1.4.6)$$

The smaller the value of this  $\tan\theta$ , closer is the angle  $\theta$  approaching zero and the chord AB approaching AC, the ideal condition through (AQL,  $1-\alpha$ ). This criterion minimizes simultaneously the consumer's and producer's risks. Thus both the producer and consumer favour the plans evolved by the criterion.



**Figure 1.8.1 Minimum angle for given  $p_1$  and  $p_2$**