ANALYSIS OF OPERATING CHARACTERISTIC FUNCTIONS OF SINGLE SAMPLING PLANS

BY

SUDHA, P

(11PM 17)

A DISSERTATION SUBMITTED TO THE
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FOR WOMEN, COIMBATORE – 641 043

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN MATHEMATICS

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CERTIFIED AS A BONAFIDE RESEARCH WORK

Parvathi 9/5/13
SIGNATURE OF THE

HEAD OF THE DEPARTMENT

9/5/13
SIGNATURE OF THE GUIDE

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CONTENTS

INTRODUCTION

In the present scenario of globalization, liberalization and privatization the need for maintaining and improving quality standards is gaining momentum. Due to increasing competitions to market any product to domestic or international level quality manufacturing becomes a prime concern. India too become quality conscious and it is necessary to keep continuous watch over the quality of the goods.

The procedure involved in the above scenario applies statistical principles to specify the requirements of how many units to be inspected and how acceptance or rejection decision shall be made.

A product with a history of consistently good quality requires less inspection than one which has no history or history of erratic quality. Accordingly it is a good practice to include inspection depending on level of quality. The manufacturer knows that when his production process is working correctly the product will usually meet specifications. Producer wants to ensure himself that the manufactured goods are according to the specification and do not contain a large number of defectives and the consumer is anxious not to be paying for substandard merchandise. This brings often the inspection procedure to be agreed to by both manufacturer and consumer so as not to unfairly penalize either. Such procedures are referred to as acceptance sampling.

Acceptance sampling may be performed at many stages as inspection of incoming materials and parts, process inspection at various points in a manufacturing operation and final inspection by manufacturer.

Acceptance sampling plans pioneered by Dodge and Romig are widely used in industries to make a decision whether to accept or reject a lot. With the introduction of modern quality management systems such as ISO 9000, acceptance sampling plans are used in goods inwards, in-process and final inspection stages of the production process. Two major areas of acceptance sampling plan are attribute sampling plan and variable sampling plan. The most popularly used sampling plan for both attribute and variable inspection is the single sampling plan.

Attribute sampling plans represent the most common statistical application used to test the effectiveness and the rate of compliance with established criteria. The result of these plans provide a statistical basis to conclude whether the controls are functioning as intended, reflecting either control compliance or noncompliance. Attribute single sampling plan saves documentation time, because it needs to record the number inspected, the number accepted and the number rejected but do not need to record the actual measurements of each and every unit sampled and inspected.

Acceptance sampling by variable is often used if quality characteristic is measured on a continuous scale following a certain type of stable statistical distribution. The basic theoretical nature of acceptance sampling by variable often assumes the underling distribution of individual measurement to be normal. The measures namely sample mean and sample standard deviation are used as basis in deriving variable sampling plans when the population standard deviation is known as well as unknown.

For selecting single sampling plan one has to choose its parameters relating to the standard quality level with reference to which the plan should operate and the degree of sharpness of inspection around that level.

These circumstances motivated the researcher to study designing methods of attribute and variable single sampling acceptance plans indexed with various quality indices, analysis of their performance with respect to their parameters and comparison of them to evaluate their efficiency.

SYNOPSIS

This dissertation attempts to present a comprehensive idea on single sampling plan .It traces the development of attribute single sampling plan and variable single sampling plan with the comparison between them is also presented.

The content of the dissertation is divided into three chapters.

Basic concepts present important terms definitions and distributions pertaining to the designing of single sampling plan.

Chapter I includes the designing of attributes single sampling plan for two specified points on the OC curve.

In chapter II the designing of variable single sampling plan for two specified points on the OC curve is given

The impact of change in acceptance number, the change in sample size, fixed ratio of sample size and acceptance number, the same AQL, the same RQL, the same IQL are provided for attributes single sampling plan. The effect of change of parameters on the OC function of variable single sampling plan and a comparison of matched ASSP with VSSP is presented in chapter III.

The results and recommendations are presented in summary and Conclusion.

Bibliography is provided at the end

PROFILE OF THE STUDY

In this dissertation the following papers are considered for review.

i. On the design of single sample acceptance sampling plans by Mc Williams et.al (2001)

ii. Acceptance Sampling Plans for percent defective by attributes and by variables of Hamaker (1979)

REVIEW OF LITERATURE

In the field of statistical quality control acceptance sampling plans are classified as acceptance sampling plans by attributes and acceptance sampling plans by variables. A number of articles appeared on acceptance sampling plans. A brief review of pertinent literature on attribute sampling plans and variable sampling plans are presented.

Dodge and Romig (1929) designed single sampling attribute plans based on LTPD with minimum amount of inspection for making decision on submitted lot. They developed sampling plans for known values of LQL with probabilities of acceptance and 0.10 with minimum amount of total inspection.

Campbell (1922) prepared probability curves showing Poisson's exponential summation. Catherine (1941) constructed able of percentage points of χ^2 distribution. Peach and Littauer(1946) presented tables of percentage for designing of single sampling plan when the fraction defective is not greater than 0.10 using Poisson approximation.

Jacobson (1949) constructed the Nomograph for determination of variables inspection plans for fraction defectives. Grubbs (1949) presented designing of single sampling plan using beta approxim'ation to binomial distribution. Cameron (1952) prepared tables for computing the operating characteristic of single sampling plans and for constructing single sampling plans for desired operating ratio.

Dodge - Romig (1959) provided separate tables for the selection of attribute single sampling plans having lot tolerance percent defective protection and also average outgoing quality limit protection for the lots.

Hamaker (1959) has explained the designing of single sampling plan adjusting for finite lot sizes. MIL-STD-105D (1963) presents the sampling procedures and tables for inspection by attributes. Guenther (1969) derived tables of distribution of hypergeometric, binomial and Poisson to obtain sampling plans. Dodge (1969) presented the evolution of acceptance sampling plans.

Gupta and Kapoor (1976) provided basic ideas, methods and the uses of acceptance sampling in quality control. Hald (1981) presented optimal single sampling plans of desired strength and derived theorems related to the designing of single sampling plans with suitable examples to maintain quality of desired standards. Duncan(1986) proposed quality control methods to suit industrial purposes.

Chakraborty (1989) gave the procedure to determine single sampling attribute plans with given indifference quality level and the slope at that point. Chakraborty (1990) gave solution methods for designing single sampling plans by assuming the incoming quality AQL and LQL as random variables.

Govindaraju(1991), Balamurali and Kalyanasundaram (1997) discussed a special type of double sampling plan which has the same OC curve as the fractional acceptance number single sampling plan introduced by Hamaker (1950).

McWilliams et al. (2001) provided a method of finding exact single sample acceptance sampling plan. Montgomery (2004) outlined the basic theory and application of quality control techniques to analyse quality in manufacturing industries. Vijayaraghavan et al. (2005) presented the discussion on choosing the prior distribution for lot fraction nonconforming. Vijayaraghavan et al. (2008) presented a design methodology and tables for the selection of parameters of single sampling plans for specified requirements (strengths) under the conditions of a Poisson sampling distribution.

Bowker and Goode (1952) presented the method of designing variable sampling plan. Resnikoff and Liebermann (1955) developed variable sampling plans for normal population indexed by producer's risk and presented matching plans relating to variance and range.

Extensive tables of variables single sampling plan are given in military standard MIL – STD – 414 (1957). This table includes five levels of inspection along with OC curves.

Owen (1966) introduced one –sided variable sampling plan indexed by AQL and LTPD for the normal population with unknown variance. Owen (1967) developed variable sampling plans with two specification limits. Owen (1969) enlightened on the summary of recent work on variable acceptance sampling plans based on normal distribution.

Guenther (1972) introduced variable sampling plan when the underlying distribution is Poisson. Guenther (1977) Introduced variable sampling plan when the quality characteristic obeys an exponential distribution and gave procedure for finding a variable sampling plan which meets the given specifications $(p, 1 - \alpha)$ and (p_2, β).

Hamaker (1979) presented the methods of adjusting variable and attributes plans having nearly identical OC curves for known sigma plans by using normal approximation. The relative efficiency of variable single sampling plans with respect to attribute single sampling plans are discussed.

Nelson (1981) presented Nomogragh for determining variable sampling plans for the desired degree of discrimination.

Schilling (1982) gave the review of the entire research work on variable sampling plan for proportion non – conforming relating to single and double specification limits for known and unknown sigma cases. Govindaraju (1990) presented procedures and tables for the selection of variable single sampling plans indexed by AQL and AOQL.

BASIC CONCEPTS

Basic concepts section includes the terms and definitions, basic distributions and their approximations which are essential to prepare this dissertation.

INSPECTION

Inspection is the process of measuring, examining, testing or otherwise comparing the unit of product with the stated requirements.

100% INSPECTION

The inspection of every unit of product for the defects listed for an inspection station is 100% inspection.

SAMPLING INSPECTION

The inspection of selected units for the units for the defects concerned from a given lot is sampling inspection.

CONFORMING UNIT

A unit, which meets the acceptance criteria established for the characteristic being considered is a conforming unit.

NONCONFORMING UNIT

A unit which does not meet the acceptance criteria established for the characteristic being considered is a non – conforming unit.

INSPECTION BY ATTRIBUTES

Inspection based on certain characteristics of a unit or product is inspection by attributes. After inspection units are classified as conforming or non-conforming to the specified requirements

INSPECTION BY VARIABLES

Inspection based on measurements of a unit or product is inspection by variables. After inspection measurements of the units of the sample are compared with specification limits.

ACCEPTANCE SAMPLING

A methodology that deals with procedures by which decisions to accept or not to accept are based on the result of the inspection of sample is acceptance sampling .

According to professor Dodge (1969), the major areas of acceptance sampling are

i. lot-by-lot sampling by the method of attributes ,in which each unit in a sample is inspected on a go-non-go basis for one or more characteristics

ii. lot-by-lot sampling by the method of variable, in which each unit in a sample is measured for a single characteristic ,such as weight or strength

iii. continuous sampling of a flow of units in which each unit is inspected by the method of attributes

ACCEPTANCE SAMPLING PLAN

A specified plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria is acceptance sampling plan.

QUALITY INDICES

Quality indices adopted in this dissertation are presented below

ACCEPTABLE QUALITY LEVEL (AQL)

The maximum percentage or proportion of variant units in a lot or batch that for the purpose of acceptance sampling , can be considered satisfactory as a process average.

LIMITING QUALITY LEVEL (LQL)

The percentage or proportion of variant units in a batch or lot for which, for the purpose of acceptance sampling plan consumer wishes the probability of acceptance to be restricted to a specific low value.

In this dissertation, the AQL and LQL are taken as an indices for designing sampling plans at which the probability of acceptance are respectively greater than or equal to (1-α) and less than or equal to β.

RISKS INVOLVED IN ACCEPTANCE SAMPLING

PRODUCER'S RISK(α)

For a given sampling plan ,the probability of not accepting a lot of quality at which it has designated numerical value representing a level which it is generally desired to accept.

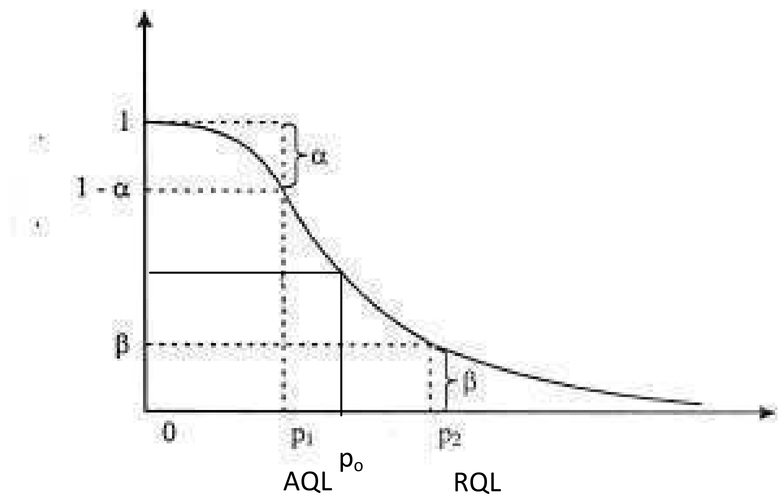
CONSUMER'S RISK(β)

For a given sampling plan, the probability of acceptance of a lot of quality at which it has a designated numerical value representing a level which it is seldom desired to accept.

PERFORMANCE MEASURES

i. OPERATING CHARACTERISTIC (OC) CURVE

Associated with each sampling plan is an Operating Characteristic curve which portrays the performance of the sampling plan against good and poor quality. Operating Characteristic curve shows graphically the interrelationship of risks, associated probabilities and quality of a given sampling plan. The form of Operating Characteristic curve is presented i the following figure . The rectangle with dotted lines represent the ideal OC curve.



Process average

ii. AVERAGE SAMPLE NUMBER (ASN)

The average number of units inspected per lot used for making decisions (acceptance or non–acceptance) is a function of the incoming lot quality p. A plot of ASN against p is called the ASN curve.

ASN = n for Single Sampling plan.

iii. AVERAGE TOTAL INSPECTION (ATI)

The average number of units inspected per lot is based on the sample size for accepted lots and inspected units in not accepted lots. A plot of ATI against p is called the ATI curve .

The expected number of items inspected per lot to arrive at a decision in an acceptance –rectification sampling inspection plan calling for 100% inspection of the rejected lot is called average amount of total inspection (ATI). ATI is also function of the lot quality p.

ATI=ASN + (Average size of inspection of the remainder in the rejected lots)

$$ATI = n P_a(p) + N [1 - P_a(p)]$$

where $P_a(p)$ is probability acceptance of the lot of quality p on the basis of the sampling inspection. It may be also be represented as

$$ATI = n + (N-n) [1 - P_a(p)]$$

iv. **AVERAGE OUTGOING QUALITY (AOQ)**

The expected quality of outgoing product following the use of acceptance sampling plan for a given value of incoming product quality. A plot of AOQ against p is called Average Outgooing Quality curve.

For rectifying inspection single sampling plan calling for 100% inspection of the rejected lots, the AOQ value is given by the formula,

$$AOQ = p (N-n) P_a(p) / N$$

where N is lot size, n is sample size and $P_a(p)$ is probability of acceptance of lot.

PROBABILITY DISTRIBUTION

HYPERGEOMETRIC DISTRIBUTION

A random variable, X is said to have hypergeometric distribution , if its probability mass function is

$$p(x) = \frac{C_x^{Np} \, C_{n-x}^{Nq}}{C_n^{N}}$$

where N - lot size, N > 0

p - proportion defective in the lot , p = 0, 1/N, 2/N, . . . , 1

q - proportion effective in the lot , q = 1-p

n - sample size , n = 1, 2, 3 ,…, N

mean - np

Variance - npq (N-n) / (N-1)

Because of discreteness in the lot,the proportion defective is restricted to one of the values p=0, 1 / N, 2 / N, …, 1. A recursion formula to obtain successive values of the hypergeometric probability is

$$p(x+1)=\frac{(n-x)(Np-x)}{(x+1)(Nq+x-n+1)}p(x)$$

The hypergeometric distribution is fundamental to acceptance sampling. Hypergeometric model is exact for designing the probability of aceptance for isolated lots. It is applicable when sampling an attribute characteristic from a finite lot without replacement.

BIONOMIAL MODEL

A random variable is said to have the binomial distribution if the probability mass function is

$$p(x)=C_x^n\, p^x\, q^{n-x} \quad , \; x = 0 ,1, 2, 3, . . . , n$$

where sample size n ,n > 0

proportion defective p , $0 \le p \le 1$

proportion effective q , q = 1 - p

number of success x , x = 0 , 1 , 2 , … , n

mean = np

variance = npq

The successive probabilities can be calculated recursively using the formula

$$p(x+1) = \frac{(n-x)p}{(x+1)q}\, p(x)$$

Undoubtely the most used distribution in acceptance sampling is the binomial . It compliments the hypergeometric in the characteristic from an infinte lot or from a lot when sampling is replacement . This model is exact for the case of nonconforming units whenever n/N$\leq$ 0 .10,where n and N are respectively the sample and lot size.

POISSON DISTRIBUTION

Arandom variable is said to have Poisson distribution ,if the probability mass function is

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad , \; x = 0, 1, 2, 3, \ldots, \infty$$

Mean number of deffective λ, where $\lambda > 0$

Number of occurance of defectives = x

Mean = λ

Variance = λ

The successive values of the Poisson probabilities can be calculated with the recursion formula

$$p(x+1) = \frac{\lambda}{x+1} p(x)$$

The poisson distribution is used in calculating the characteristics of sampling plans which specify a given number of defects per unit such as the number of defects rivets in a aircraft wing or the number of stones allowed in a piece of glass of a given size.

This model is exact for the case of non-conformties when n/N$\leq$ 0.01, n is large and pis small such that np is finite.

CHI – SQUARE DISTRIBUTION

A random variable is said to have the Chi – Square distribution with degree of freedom n if its probability density function is given by

$$f(x) = \begin{cases} \dfrac{x^{(n/2)-1} e^{\frac{-x}{2}}}{2^{n/2}\Gamma(n/2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

mean = n

variance = 2n

CHAPTER I

DESIGNING OF ATTRIBUTE SINGLE SAMPLING PLAN

This chapter presents the operating procedure of single sampling plan and designing method when two specified points (AQL, 1-α),(LQL, β) on the operating characteristic curve are given . Construction of operating characteristic curve for any given plan, selection of plans to the given specifications and method of construction of tables are also presented.

Probability of acceptance is computed using χ^2 distribution by utilising the fact that the approximation to the partial sums of the Poisson distribution can be computed in terms of chi - square distribution.

OPERATING PROCEDURE

Operating procedure of single sampling plan (N, n, c) has the following steps.

(i) A random sample of size n is randomly selected from the submitted lot of N items

(ii) Each item in the sample is then classified as either defective or non - defective and d be the number of defectives

(iii) Accept the lot if $d \leq c$

(iv) Reject the lot if $d > c$

DESIGNING METHOD

Designing of a sampling plan enables one to find the parameters of sampling plan for the given specification related to quality indices and the corresponding risks of producer and consumer. Designing of a sampling plan means estimation of parameters n and c through specification of quality indices and their risks.

The probability of acceptance based on Poisson distribution for a single sampling plan with parameters n and c is

$$P_a(p) = \sum_{r=0}^{c} e^{-np} \frac{(np)^r}{r\,!}$$, where p is the process average.

The designing of the sampling plan for specified specifications depends on the following two constraints

$$P_a(p_1) = \sum_{r=0}^{c} e^{-np_1} \frac{(np_1)}{r\,!} \geq 1-\alpha \qquad (1.1)$$

$$P_a(p_2) = \sum_{r=0}^{c} e^{-np_2} \frac{(np_2)}{r\,!} \leq \beta \qquad (1.2)$$

for $p_1 -$ AQL , $p_2 -$ LQL, $\alpha -$ producer's risk, $\beta -$ consumer's risk

Using the concept of approximation that the sum of Poisson probabilities can be computed in terms of chi-square distribution one derives

$$np_1 = \frac{1}{2} \chi^2_{1-\alpha,(2c+2)}$$

$$np_2 = \frac{1}{2} \chi^2_{\beta,(2c+2)}$$

For various values of ($p_{1,}$ 1 - α) and $(p_2,\ \beta)$, np_1 and np_2 may be computed.

SELECTION OF PLANS

Method of selection of plan for the specifications of

i. sample size and one point on the operating characteristic curve

and ii. two points on the operating characteristic curve are given

Selection of plan for given n and a point on the OC curve

Table 1.1 canbe used to derive attribute single sampling plan if for some fraction defective p it is desired that the probability acceptance be P_a, the value of np which is either less than or equal to np is located in the column headed by the specified P_a and the corresponding c determines the sampling plan to be used.

The sample size is set at n = 50 and it is desired to accept material with fraction defective p = 0.05 , 90 percent of the time the parameter c is obtained by scanning the column headed for $P_a = 0.90$ to find the value in the table equal to or just less than np = 50 (0.05) = 2.5. From Table 1.1 this turns out to be 2.43252. This is against the value c = 4. Therefore the required attribute single sampling plan is(50 , 4)

Selection of plan for two specified points on the OC curve

To construct sampling plan for given (p_1 , $1-\alpha$) , (p_2 , β) the ratio p_2 / p_1 is to be calculated . The value which is greater than or equal to the desired ratio is located in the appropriate column of Table 1.2 .Corresponding to this ratio the values of np_2 and c can be selected. The sample size is determined by dividing np_2 by p_2 and acceptance number c is read off directly from the table

The sampling plan parameters (n, c) may be obtained which will accept material with fraction defective $p_1 = 0.02$, 90 percent of the time and accept material with fraction defective $p_2 = 0.036$ only 10 percent of the time . Compute $p_2 / p_1 = 1.8$ and enter Table 1.2 for $\alpha = 0.10$ and $\beta = 0.10$ select the value of the ratio p_2 / p_1 in the column for $\alpha = 0.10$, $\beta = 0.10$ equal to or just greater than 1.8 The value is 1.81078 which has associated with it a value of $np_2 = 24.75629$ and value of c = 18. The sample size for the desired plan is taken as

$$n = np_2 / p_2 = 688$$

and the corresponding acceptance number is c = 18.

Thus the required single sampling plan is (688 , 18)

Construction of OC curves of Single Sampling plans

Table 1.3 gives the OC values of two attribute single sampling plans (100, 1) and (100, 3) which are obtained from Table 1.1

Table 1.3 OC values of Attribute Single Sampling plans

(100, 1)		(100, 3)	
p	P_a	p	P_a
0.001	0.995	0.0067	0.995
0.0015	0.990	0.0082	0.990
0.0036	0.950	0.0137	0.950
0.0053	0.900	0.0174	0.900
0.0168	0.500	0.0367	0.500
0.0387	0.100	0.0668	0.100
0.0474	0.050	0.0775	0.050
0.0664	0.010	0.1005	0.010
0.0743	0.005	0.1098	0.005
0.9233	0.001	0.1306	0.001

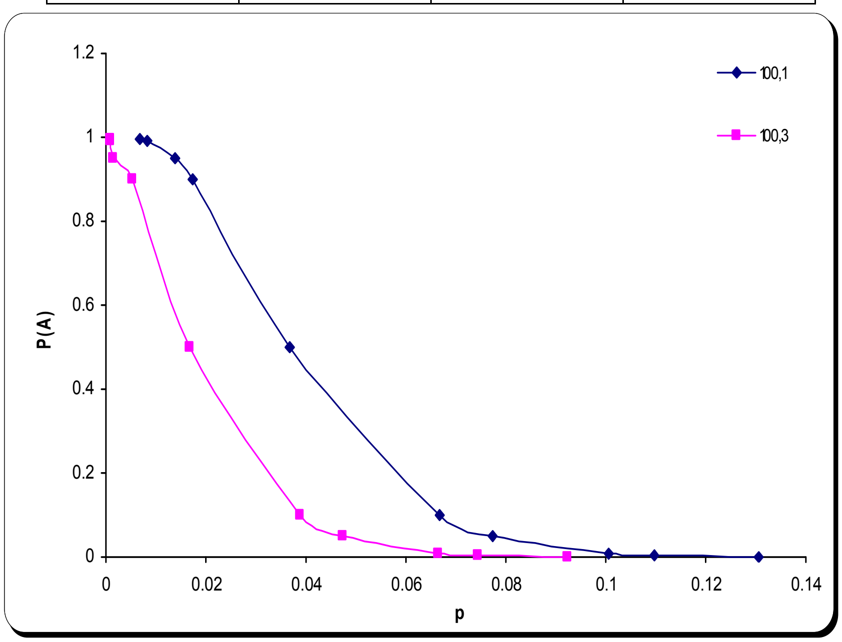


Fig 1.3 OC values of Attribute Single Sampling Plans (100.1) and (100,3)

CONCLUSION

In case of single sampling plan (n, c) one may see that

i increases in c probability of acceptance increases.

ii Increase in n probability of acceptance decreases.

CONSTRUCTION OF TABLES

By using expressions (1.1) and (1.2) the values of np are computed for various values of α and β and the acceptance number c.The Table 1.1 gives np for probability acceptance corresponding to 0.995, 0.99, 0.95, 0.90, 0.05, 0.10, 0.05 , 0 .005, 0 .001 for various acceptance number c = 0, 1, 2, 3, …, 30.

Table 1.2 is constructed to derive sampling plans of desired points on operating characteristic curve. The two points through which the operating characteristic curve requires to pass are the fraction defective p_1 for which the probability of acceptance is greater than or equal to $1-\alpha$ and p_2 for which the probability of acceptance is less than or equal to β. Table 1.2 gives values of the operating ratio p_2 / p_1 and np_2 corresponding to the various combinations of α [0.005 , 0.001 , 0.05 , 0.10] and β [0.001 , 0.10] for various values of c .

Table 1.3 presents the probability of acceptance corresponding to various incoming quality levels of desired plans (100, 1) and (100, 3). This table is derived from Table 1.1 by dividing each entry in the row by the number n for various probabilities of acceptance.

TABLE 1.1 VALUES OF np

c	probabilities of acceptance									
	0.995	0.99	0.95	0.9	0.5	**0.1**	**0.05**	**0.01**	**0.005**	**0.001**
0	0.005	0.010	0.051	0.105	0.693	2.303	2.996	4.605	5.298	6.908
1	0.103	0.149	0.355	0.532	1.678	3.890	4.744	6.638	7.430	9.233
2	0.338	0.436	0.818	1.102	2.674	5.322	6.296	8.406	9.274	11.229
3	0.672	0.823	1.366	1.745	3.672	6.681	7.754	10.045	10.977	13.062
4	1.078	1.279	1.970	2.433	4.671	7.994	9.154	11.605	12.594	14.794
5	1.537	1.785	2.613	3.152	5.670	9.275	10.513	13.108	14.150	16.455
6	2.037	2.330	3.285	3.895	6.670	10.532	11.842	14.571	15.660	18.062
7	2.571	2.906	3.981	4.656	7.669	11.771	13.148	16.000	17.134	19.626
8	3.132	3.507	4.695	5.432	8.669	12.995	14.435	17.403	18.578	21.156
9	3.717	4.130	5.425	6.221	9.669	14.206	15.705	18.783	19.998	22.657
10	4.321	4.771	6.169	7.021	10.669	15.407	16.962	20.145	21.398	24.134
11	4.943	5.428	6.924	7.829	11.668	16.598	18.208	21.490	22.779	22.779
12	5.580	6.099	7.690	8.646	12.668	17.782	19.443	22.821	24.145	24.145
13	6.231	6.782	8.464	9.470	13.668	18.958	20.669	24.139	25.497	28.446
14	6.893	7.477	9.246	10.300	14.668	20.128	21.886	25.446	26.836	29.852
15	7.567	8.181	10.036	11.135	15.668	21.292	23.097	26.743	28.164	31.244

CONTINUATION OF TABLE 1.1

C	Probability of acceptance									
	0.995	**0.99**	**0.95**	**0.9**	**0.5**	**0.1**	**0.05**	**0.01**	**0.005**	**0.001**
16	8.251	8.895	10.832	11.976	16.668	22.452	24.301	28.030	29.482	32.624
17	8.943	9.616	11.634	12.822	17.668	23.606	25.499	29.310	30.791	33.993
18	9.644	10.346	12.442	13.671	18.668	24.756	26.692	30.581	32.091	35.351
19	10.353	11.082	13.255	14.525	19.668	25.903	27.879	31.845	33.383	36.701
20	11.069	11.825	14.072	15.383	20.668	27.045	29.062	33.103	34.668	38.042
21	11.792	12.574	14.894	16.244	21.668	28.184	30.240	34.355	35.946	39.375
22	12.521	13.329	15.720	17.108	22.668	29.320	31.415	35.601	37.218	40.700
23	13.255	14.089	16.549	17.975	23.668	30.453	32.585	36.841	38.484	42.019
24	13.995	14.853	17.382	18.844	24.667	31.584	33.752	38.077	39.745	43.330
25	14.741	15.623	18.219	19.717	25.667	32.711	34.916	39.308	41.000	44.636
26	15.491	16.397	19.058	20.592	26.667	33.836	36.077	40.534	42.251	45.936
27	16.245	17.175	19.901	21.469	27.667	34.959	37.234	41.757	43.497	43.497
28	17.004	17.957	20.746	22.348	28.667	36.080	38.389	42.975	44.738	48.519
29	17.767	18.742	21.594	23.229	29.667	37.199	39.541	44.190	45.976	49.804
30	18.534	19.532	22.445	24.113	30.667	38.315	40.691	45.401	47.209	51.083

TABLE 1.2 VALUES OF p_2 / p_1

c	Values of p_2/p_1 for				np_2 for β=0.1	c	Values of p_2/p_1				np_2 for β=.001
	α=.005	α=.01	α=.05	α=.10			α=.005	α=.01	α=.05	α=.10	
	β=0.1	β=0.1	β=0.1	β=0.1			β=.001	β=.001	β=.001	β=.001	
0	459.365	229.105	44.891	21.854	2.303	0	1378.094	687.316	134.672	65.563	6.908
1	37.584	26.184	10.946	7.314	3.890	1	89.216	62.155	25.983	17.362	9.233
2	15.753	12.206	6.509	4.829	5.322	2	33.235	25.752	13.732	10.189	11.229
3	9.939	8.115	4.890	3.829	6.681	3	19.432	15.867	9.560	7.487	13.062
4	7.416	6.249	4.057	3.286	7.994	4	13.725	11.566	7.509	6.082	14.794
5	6.035	5.195	3.549	2.943	9.275	5	10.706	9.217	6.297	5.221	16.455
6	5.170	4.520	3.206	2.704	10.532	6	8.865	7.751	5.498	4.637	18.062
7	4.578	4.050	2.957	2.528	11.771	7	7.633	6.753	4.930	4.215	19.626
8	4.148	3.705	2.768	2.392	12.995	8	6.754	6.032	4.506	3.894	21.156
9	3.822	3.440	2.618	2.283	14.206	9	6.096	5.486	4.176	3.642	22.657
10	3.565	3.229	2.497	2.194	15.407	10	5.585	5.058	3.912	3.438	24.134
11	3.358	3.058	2.397	2.120	16.598	11	4.608	4.196	3.290	2.909	22.779
12	3.187	2.915	2.312	2.057	17.782	12	4.327	3.959	3.140	2.793	24.145
13	3.043	2.795	2.240	2.002	18.958	13	4.566	4.194	3.361	3.004	28.446
14	2.920	2.692	2.177	1.954	20.128	14	4.330	3.993	3.228	2.898	29.852
15	2.814	2.603	2.122	1.912	21.292	15	4.129	3.819	3.113	2.806	31.244

CONTINUATION OF TABLE 1.2

c	Values of p_2/p_1 for				np_2 for β=0.1	c	Values of p_2/p_1				np_2 for β=.001
	α=.005	α=.01	α=.05	α=.10			α=.005	α=.01	α=.05	α=.10	
	β=0.1	β=0.1	β=0.1	β=0.1			β=.001	β=.001	β=.001	β=.001	
16	2.721	2.524	2.073	1.875	22.452	16	3.954	3.668	3.012	2.724	32.624
17	2.640	2.455	2.029	1.841	23.606	17	3.801	3.535	2.922	2.651	33.993
18	2.567	2.393	1.990	1.811	24.756	18	3.665	3.417	2.841	2.586	35.351
19	2.502	2.337	1.954	1.783	25.903	19	3.545	3.312	2.769	2.527	36.701
20	2.443	2.287	1.922	1.758	27.045	20	3.437	3.217	2.703	2.473	38.042
21	2.390	2.241	1.892	1.735	28.184	21	3.339	3.131	2.644	2.424	39.375
22	2.342	2.200	1.865	1.714	29.320	22	3.251	3.054	2.589	2.379	40.700
23	2.297	2.162	1.840	1.694	30.453	23	3.170	2.982	2.539	2.338	42.019
24	2.257	2.126	1.817	1.676	31.584	24	3.096	2.917	2.493	2.299	43.330
25	2.219	2.094	1.795	1.659	32.711	25	3.028	2.857	2.450	2.264	44.636
26	2.184	2.064	1.775	1.643	33.836	26	2.965	2.802	2.410	2.231	45.936
27	2.152	2.036	1.757	1.628	34.959	27	2.678	2.533	2.186	2.026	43.497
28	2.122	2.009	1.739	1.614	36.080	28	2.853	2.702	2.339	2.171	48.519
29	2.094	1.985	1.723	1.601	37.199	29	2.803	2.657	2.306	2.144	49.804
30	2.067	1.962	1.707	1.589	38.315	30	2.756	2.615	2.276	2.119	51.083

CHAPTER II

DESIGNING OF VARIABLE SINGLE SAMPLING PLAN

This chapter presents the assumption, procedure and the designing of variable sampling plan for desired points on the OC curve when the standard deviation of the underlying normal population is known as well as unknown.

Tables are prepared to enable the user in selecting plans for desired discrimination.

ASSUMPTIONS

The assumptions of variable sampling plan are

i. only single quality characteristic measurable on continuous scale is taken into consideration
ii. X , the measurements of the items in the lots are distributed according to normal with mean μ and the standard deviation σ
iii. the purpose of inspection is to control the fraction defective, p in the submitted lot

Under these assumptions the fraction defective in a lot is

$$p = \int_{k_p}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

where K_p is a quantity determined from normal probability tables.

SPECIFICATION LIMIT

Specification limits can be of two types . A single specification limit implies only one boundary value for acceptability. A product is confirming if the measurement, $X \leq U$ for an upper specification limit , or $X \geq L$ for an lower specification limit. A product confirm to the specification if the measurement, X such that $L \leq X \leq U$ for a double specification limit .

OPERATING PROCEDURE OF VARIABLE SINGLE SAMPLING PLAN

Step 1: Take a random sample of size n and obtain the measurements of the quality characteristics from the submitted lot ,

Step 2: Compute $\bar{x}$, the sample mean and s, the sample standard deviation

Step 3: Acceptance criteria

If upper specification limit U is specified, then accept the unit for $X \leq U$

$\bar{x} + k\sigma \leq U$, accept the lot when σ is known.

$\bar{x} + ks \leq U$, accept the lot when σ is unknown.

If lower specification limit L is specified , then accept the unit for $X \geq L$

$\bar{x} - k\sigma \geq L$, accept the lot when σ is known.

$\bar{x} - ks \geq L$, accept the lot when σ is unknown.

If double specification limits U and L are specified, then accept the unit for $L \leq X \leq U$

$L + k\sigma \leq \bar{x} \leq U - k\sigma$, accept the lot when σ is known.

$L + ks \leq \bar{x} \leq U - ks$, accept the lot when σ is unknown.

Only the designing of plans with upper specification limit is provided.

OC FUNCTION OF KNOWN σ VARIABLE SINGLE SAMPLING PLAN (VSSP)

It is assumed that the individual measurements of the submitted lot follow normal distribution with mean μ and standard deviation σ . Products which are defectives, have the quality characteristic $X > U$.The proportional area to the right of the value U of the normal curve is equal to p which is shown in following fig 2.1.

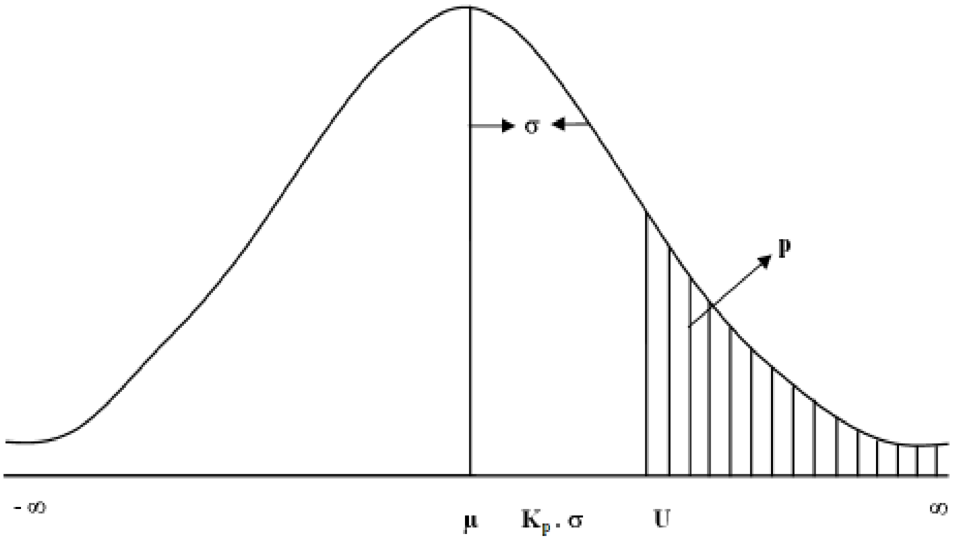


Fig. 2.1 Distribution of X

From this fig 2.1

$$\frac{U-\mu}{\sigma} = K_p \qquad (2.1)$$

The mean, $\bar{x}$ is distributed normally with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$, where n is the sample size . This implies that follows normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$. The distribution is shown in Fig. 2.2.

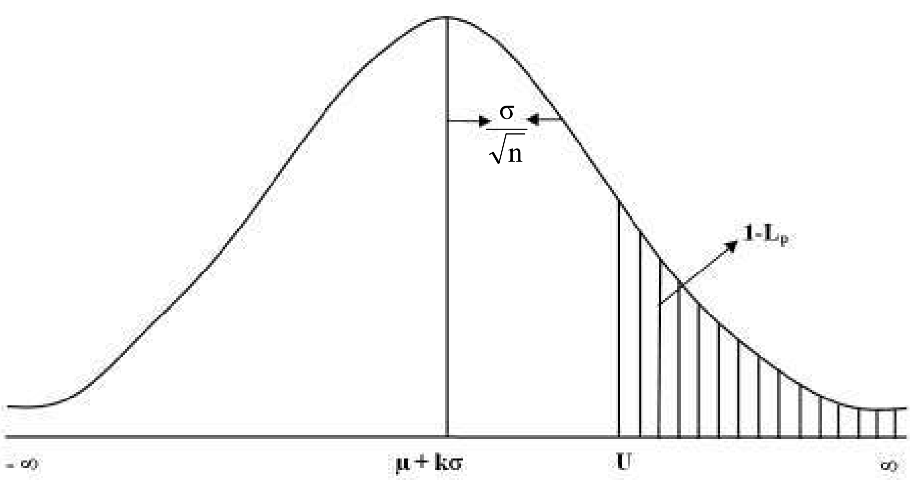


Fig .2.2 Sampling Distribution of $\bar{x} + k\sigma$

From the sampling distribution of $\bar{x} + k\sigma$, one can obtain the probability of acceptance, L_p From Fig.2.2

$$L_p = P\left(\bar{x} + k\sigma \le U\right)$$

therefore $1 - L_p = P\left(\bar{x} + k\sigma > U\right)$

Also $K_{-L_p} \dfrac{\sigma}{\sqrt{n}} = U - \left(\mu - k\sigma\right)$

$$K_{-L_p} = -K_{L_p}$$

Thus $-K_{L_p} \dfrac{\sigma}{\sqrt{n}} = U - \mu + k\sigma$

Simplifying further one obtains,

$$K_p = k - \frac{1}{\sqrt{n}} K_{L_p} \qquad (2.2)$$

Equation (2.2) provides L_p for any given p and vice- versa.
Using this equation one can

i. obtain the plan for desired points on Operating Characteristic curve

and ii. compute various points on the OC curve of the desired sampling plan

DETERMINATION OF n AND k FOR GIVEN AQL and LQL

One can obtain the plan parameters n and k of a known variable sampling plan for two points on the Operating Characteristic curve $(p_1, 1-\alpha)$ and (p_2, β) by using the relation (2.2)

$$K_1 = k - \frac{1}{\sqrt{n}} K_{1-\alpha} \qquad (2.3)$$

$$K_2 = k - \frac{1}{\sqrt{n}} K_{\beta} \qquad (2.4)$$

where K_1 is the value of K_p when p = p_1

therefore $$p_1 = \int_{K_1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}u^2} du \qquad (2.5)$$

K_2 is the value of K_p when p = p_2

therefore $$p_2 = \int_{K_2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}u^2} du \quad (2.6)$$

Equations (2.3) and (2.4) can be written as ,

$$K_1 = k + \frac{1}{\sqrt{n}} K_\alpha \quad (2.7)$$

$$K_2 = k - \frac{1}{\sqrt{n}} K_\beta \quad (2.8)$$

where K_α and K_β are given by

$$\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}u^2} du \quad (2.9)$$

$$\beta = \int_{K_\beta}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}u^2} du \quad (2.10)$$

Multiplying (2.5) by K_β , (2.6) by K_α and adding one get

$$k = \frac{K_1 K_\beta + K_2 K_\alpha}{K_\alpha + K_\beta} \quad (2.11)$$

subtracting (2.6) from (2.5), one obtains

$$n = \frac{(K_\alpha + K_\beta)^2}{(K_1 + K_2)^2} \quad (2.12)$$

SIGNIFICANCE OF THE VALUES OF k and n

The choice of the two parameters k and n determine the sampling plan completely.

In the known sigma variable sampling plan one can observe from (2.11) and (2.12) that

i. increase in $(K_\alpha + K_\beta)$ increases n,
ii. decrease in $(K_1 + K_2)$ increases n when p_1 and p_2 are closer,

iii. given the indifference quality $p = p_0$, the relation (2.2) becomes,

$$K_0 = k - \frac{1}{\sqrt{n}} K_{0.5} \text{ ,when } \sigma \text{ is known}$$

But $K_{0.5} = 0$ from normal tables. Thus in this case $k = K_0$ where K_0 is the value of K_p at $p = p_0$. This shows that k is the independent of the sample size n at indifference quality .

Wallis approximation is used in obtaining sample size , n_s for unknown sigma plans by multiplying corresponding n_σ with the factor $\left(1+\frac{k^2}{2}\right)$

In general , n fixes the steepness of the operating characteristic curve and k fixes the location of operating characteristic curve.

SELECTION OF VARIABLE SAMPLING PLAN FOR GIVEN AQL and LQL

Tables 2.1 and 2.2 provide the sampling plan parameters namely acceptance criteria, k sample size, n_σ for known standard deviation plan for $\alpha = 0.05$, $\beta = 0.05$; $\alpha = 0.1$, $\beta = 0.05$; $\alpha = 0.05$, $\beta = 0.1$; and $\alpha = 0.1$,$\beta = 0.1$ for various values of p_1 and p_2 such that $p_1 < p_2$.

These tables can be utilized to select plans for parameters for the desired discrimination.

For example, consider the designing of a plan for desired values $p_1 = 0.01$, $p_2 = 0.05$, $\alpha = 0.10$, $\beta = 0.05$ From table 2 .1 one obtains the plan with $n_\sigma = 14.40$, and k = 1.99 for known sigma variable sampling plan

To determine variable sampling plan to the specified values $p_1 = 0.02$, $p_2 = 0.06$ $\alpha = 0.10$, $\beta = 0.05$, From Table 2.2 one gets

$n_\sigma = 34.57$, and $k = 1.14$

From the numerical values obtained for various p_1, p_2 with certain α and β presented in Tables 2.1and 2.2 the following features could be observed

i. for a fixed p_1 increase in p_2 decreases k, and n_σ
ii. for a fixed p_2 increase in p_1 decreases the value of k and increases the value of n_σ

iii. for a fixed α increase in β decreases k and n_σ

and iv. a fixed β increase in α increases k and decreases n_σ

COMPUTATION OF POINTS ON OC CURVE

The operating characteristic curve of variable sampling plan with known sigma may be obtained from the relation (2.2) .The points on the operating characteristic curve may be obtained by either computing L_p for given p or by computing p for given L_p .

For illustration, consider the determination of points on the operating characteristic curve for known sigma variable single sampling plan

i. for various values of n when k is fixed
ii. for various values of k when n is fixed

Operating characteristic values for fixed k with n = 5, 10, 12 are computed and presented in Table 2.3 .Operating characteristic values for k = 1, 1.44, 1.70 are calculated and given in Table 2.4.

The OC curves for variable single sampling plans with fixed n and fixed k are shown in fig.2.3 and fig.2.4. The properties observed are

i. OC values decrease for increase in p
ii. OC values decrease for increase in k
iii. OC values decrease for increases in n for inferior quality
iv. OC values increase for increase in n for superior quality

METHOD OF CONSTRUCTION OF TABLES

Tables 2.1 and 2.2 are constructed to obtain the acceptance constant k, sample sizes for standard deviation known, n_σ for single sampling variable plan . These tables are indexed by the combination of producer's quality level with α values (0.05, 0.10) and the consumer's quality level with β values (0.05 , 0.10) .Plans are derived using

$$k = \frac{K_1 K_\beta + K_2 K_\alpha}{K_\alpha + K_\beta}$$

$$n_{\sigma} = \frac{(K_{\alpha} + K_{\beta})^2}{(K_1 + K_2)^2}$$

Wallis approximation is used in obtaining sample size , n_s for unknown sigma plans by multiplying corresponding n_{σ} with the factor $\left(1 + \frac{k^2}{2}\right)$

where K_1 - area of p_1 in upper tail of normal curve

K_2 - area of p_2 in upper tail of normal curve

K_{α} - area of α in upper tail of normal curve

K_{β} - area of β in upper tail of normal curve

TABLE 2.1 VARIABLE SINGLE SAMPLING PLAN PARAMETERS INDEXED BY AQL and LQL

p_1	p_2	α = 0.05, β = 0.05		α = 0.05, β = 0.10	
		k	n_σ	k	n_σ
0.01	0.015	2.25	425.39	2.24	337.64
0.01	0.02	2.20	149.38	2.18	118.57
0.01	0.03	2.11	56.25	2.08	44.65
0.01	0.04	2.05	33.52	2.01	26.60
0.01	0.05	1.99	23.55	1.95	18.69
0.02	0.025	2.02	1344.44	2.01	1067.11
0.02	0.03	1.98	376.82	1.96	299.09
0.02	0.04	1.91	121.00	1.89	96.04
0.02	0.05	1.86	64.78	1.83	51.42
0.02	0.06	1.81	43.56	1.78	34.57
0.05	0.055	1.63	4356.00	1.62	3457.44
0.05	0.06	1.61	1344.44	1.60	1067.11
0.05	0.07	1.57	376.82	1.55	299.09
0.05	0.08	1.53	189.06	1.52	150.06
0.05	0.09	1.50	121.00	1.48	96.04
0.07	0.075	1.46	6806.25	1.46	5402.25
0.07	0.08	1.45	2222.45	1.44	1764.00
0.07	0.09	1.42	644.38	1.41	511.46
0.07	0.1	1.39	301.66	1.37	239.43
0.08	0.0991	1.27	2074.60	1.34	138.30

TABLE 2.2 VARIABLE SINGLE SAMPLING PLAN PARAMETERS INDEXED BY AQL and LQL

p_1	p_2	α = 0.10, β = 0.05		α = 0.10, β = 0.10	
		K	n_σ	k	n_σ
0.01	0.015	2.26	337.64	2.25	260.02
0.01	0.02	2.21	118.57	2.20	91.31
0.01	0.03	2.14	44.65	2.11	34.38
0.01	0.04	2.08	26.60	2.05	20.49
0.01	0.05	2.03	18.69	1.99	14.40
0.02	0.025	2.02	1067.11	2.02	821.78
0.02	0.03	1.99	299.09	1.98	230.33
0.02	0.04	1.93	96.04	1.91	73.96
0.02	0.05	1.88	51.42	1.86	39.60
0.02	0.06	1.84	34.57	1.81	26.63
0.05	0.055	1.63	3457.44	1.63	2662.56
0.05	0.06	1.61	1067.11	1.61	821.78
0.05	0.07	1.58	299.09	1.57	230.33
0.05	0.08	1.54	150.06	1.53	115.56
0.05	0.09	1.52	96.04	1.50	73.96
0.07	0.075	1.46	5402.25	1.46	4160.25
0.07	0.08	1.45	1764.00	1.45	1358.45
0.07	0.09	1.42	511.46	1.42	393.87
0.07	0.1	1.40	239.43	1.39	184.39
0.07	0.11	1.37	138.30	1.36	106.50

TABLE 2.3 OC VALUES OF VSS PLANS WITH KNOWN SIGMA and FIXED k = 1.55

p	OC VALUES		
	n=5	n=10	n=12
0.0075	0.97615	0.99836	0.99886
0.018	0.89065	0.97062	0.97193
0.027	0.80234	0.91309	0.90658
0.048	0.59871	0.69497	0.64809
0.083	0.36317	0.36693	0.29116
0.129	0.17619	0.12502	0.07493
0.181	0.07636	0.03074	0.01122
0.216	0.04272	0.0116	0.00379
0.289	0.01355	0.00149	0.0003
0.313	0.0089	0.00071	0.00012
0.345	0.00508	0.00025	3E-05

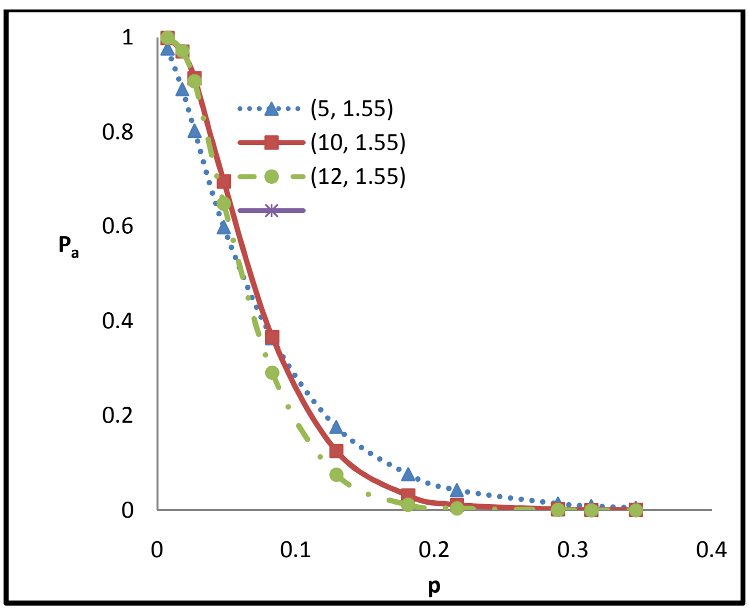


Fig.2.3 OC curves of VSS Plans with fixed k = 1.55

TABLE 2.4 OC VALUES OF VSS PLANS WITH KNOWN SIGMA and FIXED n =7

p	OC VALUES		
	k=1	k=1.44	k=1.70
0.0075	0.99992	0.996	0.9732
0.018	0.99891	0.9599	0.85543
0.027	0.99305	0.9032	0.72907
0.048	0.95728	0.719	0.4562
0.083	0.8485	0.4483	0.20611
0.129	0.64431	0.2061	0.06811
0.181	0.40517	0.0808	0.01831
0.216	0.28096	0.0401	0.00755
0.289	0.12302	0.0099	0.00131
0.313	0.08851	0.00604	0.00069
0.345	0.05592	0.00298	0.00029

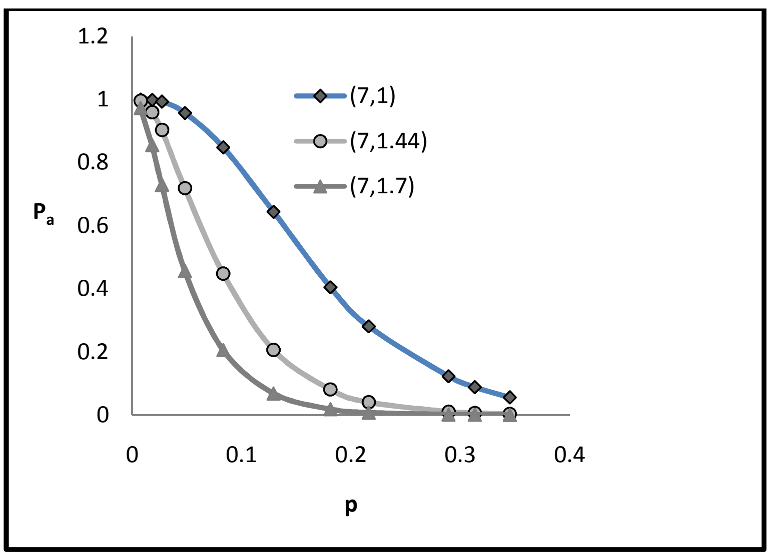


Fig.2.4 OC curves of VSS Plans with fixed n

CHAPTER III

ANALYSIS OF OPERATING CHACTERISTIC FUNCTION

This chapter presents the properties of operating characteristic function with reference to its parameters and quality indices.

For attribute single sampling plans, with regard to parameters - the effect of acceptance number, sample size, fixed ratio of sample size to acceptance number on operating characteristic values are discussed. With reference to quality indices - the trend of operating characteristic values of single sampling plans having identical AQL, RQL and IQL values are examined.

For variable single sampling plans the effect of parameters on the OC function are analysed by considering fixed n with varying k and fixed k with varying n. A comparison is provided to study the efficiency of matched attribute single sampling plan and variable single sampling plan.

Operating characteristic function plays a major role in the theory of sampling inspection. The two main aspects for evaluation of the effect of sample size and acceptance number on operating characteristic functions are to estimate

i. the dependence of operating characteristic function on the sample size
ii. the dependence of operating characteristic function on the acceptance number

Operating characteristic function for attribute single sampling plan may be derived using Hypergeometric distribution, Binomial distribution and Poisson distribution.

The hypergeometric distribution is fundamental to acceptance sampling. It is applicable when sampling an attribute characteristic from a finite lot without replacement. This model is exact for the case of nonconforming units for isolated lots of smaller size.

Undoubtedly the most used distribution in acceptance sampling is the binomial. It compliments the hypergeometric in the characteristic from an infinte lot or from a lot when sampling is done with replacement .

This model is exact for the case of nonconforming units whenever

$n / N \leq 0.10$, where n and N are respectively the sample and lot size.

Poisson model is exact for the case of non-conformties when $n / N \leq 0.01$, sample size is large and p is small such that np is finite.

The simplest of three functions is based on Poisson distribution given. The probability of acceptance depends only on c and m where m = np. It does not matter whether the expected number of defects per sample is the product of a small sample size and a large proportion of occurrence of defects or a large sample size and a small proportion of occurrence of defects.

The OC function under Poisson model is defined by

$$G(c,m) = \sum_{x=0}^{c} g(x,m) \qquad (3.1)$$

where $g(x,m) = \frac{e^{-m} m^{x}}{x!}$, $x = 0 , 1 , 2 , . . . , \infty$

and $m = np,\ n \geq 0 , p \geq 0$

The following aspects are considered for study of operating characteristic values.

i. Effect of acceptance number with fixed sample size

ii. Effect of sample size with fixed acceptance number

iii. Effect of fixed ratio of acceptance number to sample size

iv. Effect of fixed AQL

v. Effect of fixed RQL

and vi. Effect of fixed IQL

Effect of Acceptance Number with Fixed Sample Size

The acceptance probability depends only on c and m where m = np. Table 3.1 presents the probability of acceptance obtained by using expression (3.1) with excel worksheet relating to single sampling plan (100, c) for acceptance number c = 1, 3, 4, and 5 as a function of m. The operating characteristic curves corresponding to these single sampling plans are shown in fig.3.1.

The following salient features may be observed

i. probability of acceptance is an increasing function of c

and ii. probability of acceptance is a decreasing function of m

Effect of Sample Size with Fixed Acceptance Number

The effect of sample size on operating Characteristic function is studied with various n values. Probability of acceptance for a single sampling plan may be considered as a function of m for a given value of acceptance number. Table 3.2 presents the probability of acceptance corresponding to single sampling plan (n, 2) for n = 50, 150, 200, 250, 500 for various fraction defective. Probability acceptance values are computed through excel worksheet for the specified parameters with equation (3.1). The operating characteristic curves for single sampling plans (n , 2) for n = 50, 150, 200, 250, 500 are shown in fig. 3.2. The following salient features may be seen from the trend of Operating Characteristic curves.

i. Operating Characteristic is a decreasing function of n for a fixed c

ii. Operating Characteristic is a decreasing function of p for any specified n and c

and iii. the point of inflexion may be seen at $p = \frac{c}{n}$

Effect of Fixed Ratio of Acceptance Number and Sample Size

The effect of fixed ratio of sample size and acceptance number, $h = c / n$ on OC values are studied in this part.

The probability of acceptance for the single sampling plans having the fixed ratio of acceptance number to the sample size as h = 0.05 are computed and presented in Table 3.3. The operating characteristic curves of these sampling plans are given in fig.3.3.

For fixed h, fig. 3.3 shows that

i. the operating characteristic values increase in a complicated manner without any uniform trend

and ii. the operating characteristic values increase with decrease in p

Effect of same AQL on Operating Characteristic Function

Table 3.4 presents the probabilities of acceptance for certain selected plans having the same AQL which means that the probability of acceptance corresponding to this proportion defective, p is 0.95 for all single sampling plans considered.

Fig.3.4 shows that the operating characteristic curves of single sampling plans with same AQL. These curves indicate that

$$n = m(p^{-1} - 1/2) + (c/2)$$

Effect of same RQL on Operating Characteristic Function

Table 3.5 presents the probability of acceptance for certain selected plan having the same RQL which means that the probability of acceptance corresponding to this proportion defective p, is 0.10 for all single sampling plans considered.

Figure 3.5 shows the operating characteristic curves of single sampling plans with same RQL which implies that probability of acceptance of single sampling plans at RQL is 0.10 for all the plans selected.

These curves indicate that

$$n = m(p^{-1} - (1/2)) + (c/2)$$

Effect of same IQL on Operating Characteristic Function

Table 3.6 presents the probabilities of acceptance for certain plans which are having the same IQL which means that the probability of acceptance corresponding to the proportion defective p is 0.5 for all single sampling plans considered.

Fig. 3.6 shows the Operating Characteristic curves of Single Sampling Plans having the same IQL. These curves

i. indicate the relation between c and n as

$(n+(1/3))\ IQL \approx c + (2/3)$

and ii. show the probability of acceptance decreases for the proportion

defective < IQL and increases for the proportion defectives > IQL for any n and c

TABLE 3.1 OC VALUES OF SINGLE SAMPLING PLAN (100, c)

m	C			
	1	3	4	5
0.5	0.985612	0.998248	0.999828	0.999986
1	0.919699	0.981012	0.99634	0.999406
2	0.676676	0.857123	0.947347	0.983436
3	0.42319	0.647232	0.815263	0.916082
4	0.238103	0.43347	0.628837	0.78513
5	0.124652	0.265026	0.440493	0.615961
6	0.061969	0.151204	0.285057	0.44568
7	0.029636	0.081765	0.172992	0.300708
8	0.013754	0.04238	0.099632	0.191236
9	0.006232	0.021226	0.054964	0.115691

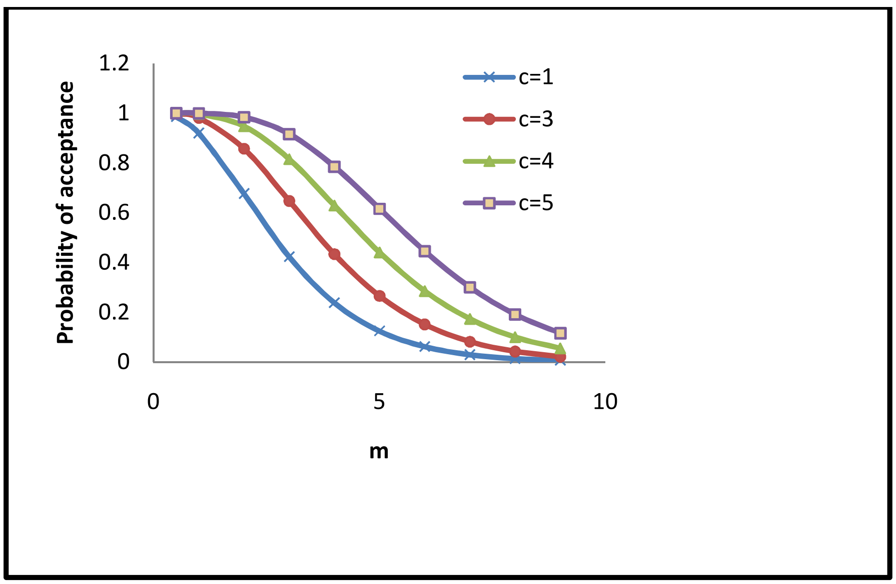


Fig.3.1 OC curves of Single Sampling Plans with fixed n =100

TABLES 3.2 OC VALUES OF SINGLE SAMPLING PLAN (n , 2)

p	n				
	50	100	200	250	500
0.001	1.0000	0.9995	0.9989	0.9978	1.0018
0.002	0.9998	0.9964	0.9921	0.9856	0.8034
0.003	0.9995	0.9891	0.9769	0.9595	0.5952
0.004	0.9989	0.9769	0.9526	0.9197	0.4243
0.005	0.9978	0.9595	0.9197	0.8685	0.2957
0.01	0.9856	0.8088	0.6767	0.5438	0.0405
0.02	0.9197	0.4232	0.2381	0.1247	0.0005
0.03	0.8088	0.1736	0.0620	0.0203	5E-06
0.04	0.6767	0.0620	0.0138	0.0028	4E-08
0.05	0.5438	0.0203	0.0028	0.0003	4E-10
0.06	0.4232	0.0062	0.0005	4E-05	3E-12
0.07	0.3208	0.0018	9.4E-05	4E-06	2E-14
0.08	0.2381	0.0005	1.6E-05	5E-07	2E-16
0.09	0.1736	0.0001	2.8E-06	5E-08	1E-18

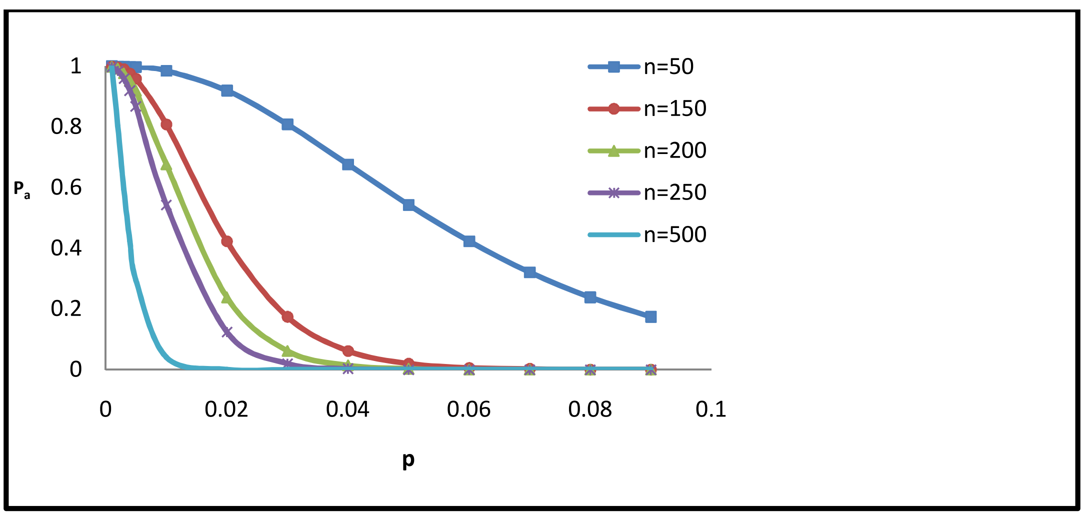


Fig. 3.2 OC curves of Single Sampling Plans with fixed c = 2

TABLE 3.3 OC VALUES OF SS PLANS WITH FIXED RATIO of n and c

p	n=25,c=1	n=50,c=2	n=75,c=3	n=100,c=4
0.005	0.992809	0.997839	0.9993884	0.999827884
0.01	0.973501	0.985612	0.9927078	0.996340153
0.02	0.909796	0.919699	0.9343575	0.947346983
0.03	0.826641	0.808847	0.8094331	0.815263245
0.04	0.735759	0.676676	0.6472319	0.628836935
0.05	0.644636	0.543813	0.4837674	0.440493285
0.06	0.557825	0.42319	0.342296	0.2850565
0.07	0.477878	0.320847	0.2316697	0.172991608
0.08	0.406006	0.238103	0.1512039	0.0996324
0.09	0.342547	0.173578	0.0957651	0.054963641

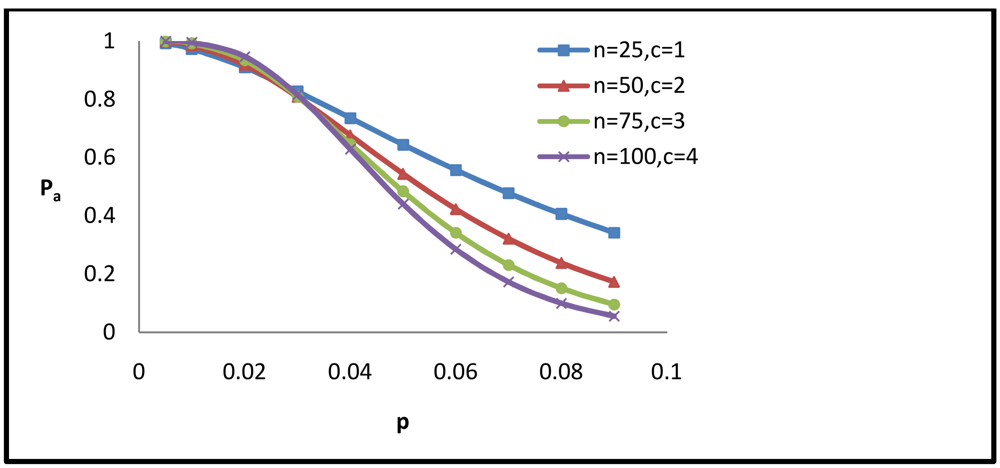


Fig. 3.3 OC values of Single Sampling Plans with fixed ratio of n and c.

TABLE 3.4 OC VALUES OF SSPLANS WITH SAME AQL

p	n=18,c=1	n=69,c=3	n=201,c=7	n=504,c=15
0.001	0.99984	0.999999	1	1
0.002	0.999367	0.999986	1	1
0.003	0.998593	0.999935	0.9999997	1
0.004	0.997529	0.999806	0.9999979	0.999999999
0.005	0.996185	0.999551	0.9999894	0.999999988
0.01	0.985619	0.994526	0.9988685	0.999924445
0.02	0.94884	0.948504	0.9476666	0.94842631
0.03	0.897432	0.844276	0.7398392	0.55579923
0.04	0.837214	0.700823	0.4473914	0.14841369
0.05	0.772482	0.547459	0.2157504	0.020392185
0.06	0.706359	0.406608	0.0869161	0.001715296
0.07	0.641058	0.289705	0.0304233	0.000100334
0.08	0.578104	0.199443	0.0095309	4.4524E-06
0.09	0.518495	0.133425	0.0027315	1.59201E-07

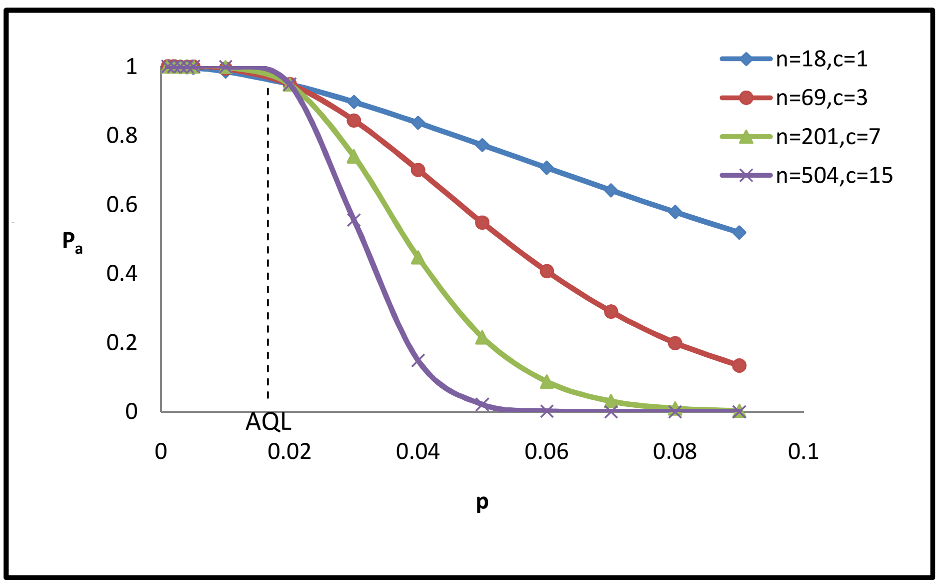


Fig.3.4 OC curves of Single Sampling plans with same AQL

TABLE 3.5 OC VALUES OF SSPLANS WITH SAME RQL

p	n=18 c=1	n=69 c=3	n=115 c=7	n=210 c=15
0.001	0.999332	0.999999	1	1
0.002	0.997393	0.999989	1	1
0.003	0.994277	0.999948	1	1
0.004	0.990071	0.999845	1	1
0.005	0.984859	0.999641	1	1
0.01	0.946306	0.995552	0.999973	1
0.02	0.830178	0.956905	0.997411	0.999708
0.03	0.695369	0.866031	0.975141	0.992412
0.04	0.564541	0.736002	0.904949	0.952714
0.05	0.448126	0.591408	0.777623	0.854236
0.06	0.349721	0.453247	0.613611	0.696401
0.07	0.269322	0.33393	0.446004	0.506632
0.08	0.205203	0.238065	0.301	0.325753
0.09	0.154984	0.165099	0.190333	0.185111

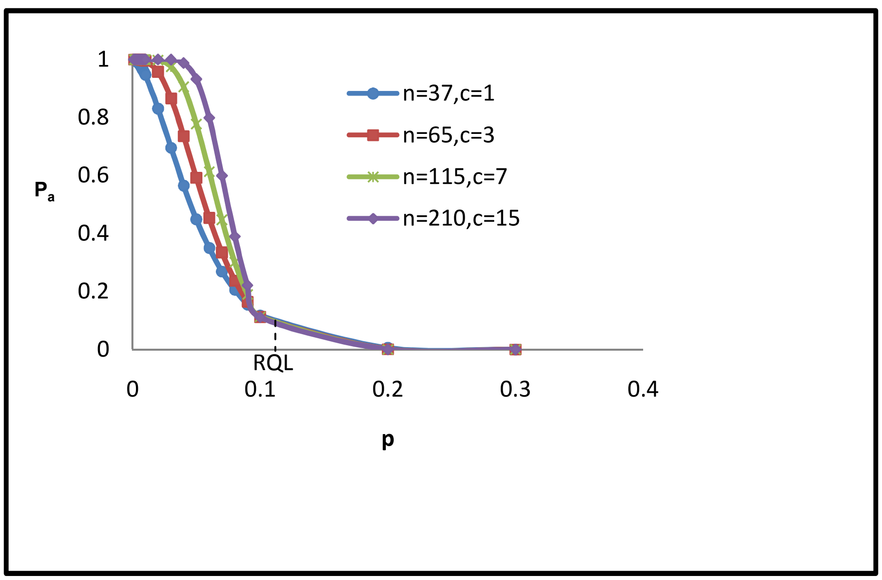


Fig.3.5 OC curves of Single Sampling Plans with same RQL

TABLE 3.6 OC VALUES OF SSPLANS WITH SAME IQL

p	n=33 c=1	n=73 c=3	n=153 c=7	n=313 c=15
0.001	0.999467	0.999999	1	1
0.002	0.997916	0.999983	1	1
0.003	0.995411	0.99992	1	1
0.004	0.992018	0.99976	1	1
0.005	0.987796	0.999447	0.999999	1
0.01	0.956169	0.993352	0.999806	1
0.02	0.857973	0.939277	0.986747	0.999209
0.03	0.739438	0.821315	0.905816	0.969352
0.04	0.619754	0.665149	0.7273	0.804406
0.05	0.508932	0.504638	0.502786	0.501811
0.06	0.411446	0.362945	0.303242	0.229435
0.07	0.328555	0.249923	0.162925	0.079557
0.08	0.259755	0.166063	0.079532	0.021901
0.09	0.203674	0.107123	0.035857	0.004985

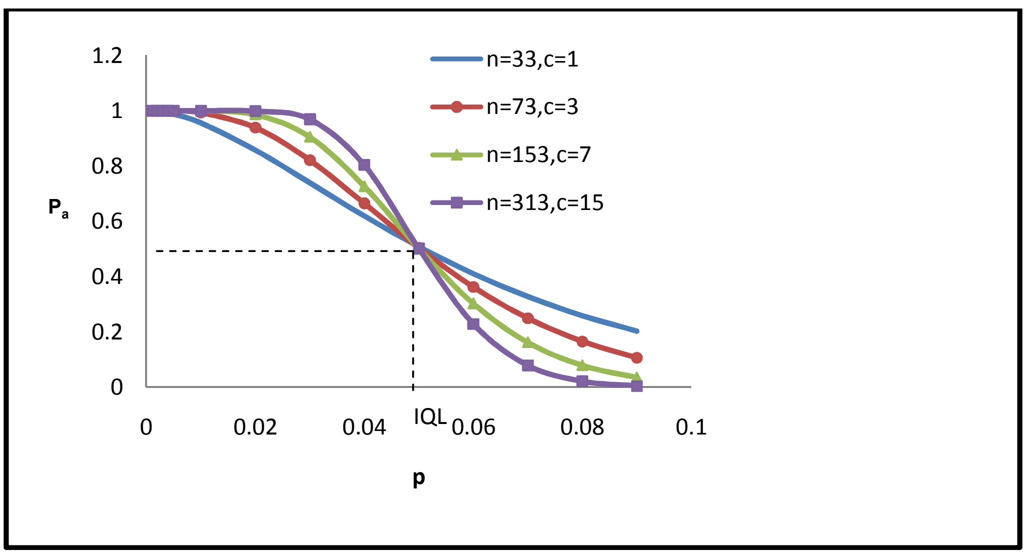


Fig. 3.6 OC curves of Single Sampling Plans with same IQL

Operating Characteristic function for Variable Single Sampling Plan is derived using Normal Distribution.

A random variable is said to have normal distribution, if its probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty$$

where mean = μ

variance = σ^2

It is denoted as random variable, $x \sim N(\mu,\sigma^2)$

It is assumed that the individual measurements of the submitted lot follow normal distribution with mean μ and standard deviation σ. Products which are defectives, have the quality characteristic X > U. The proportional area to the right of the value U of the normal curve is equal to p with given,

$$\frac{U-\mu}{\sigma} = K_p \qquad (3.\ 1)$$

The mean $\bar{x}$ is distributed normally with mean μ and standard deviation $\sigma/\sqrt{n}$, where n is the sample size .

This implies that, $\bar{x} \sim N\left(\mu,\ \sigma^2/n\right)$

From the sampling distribution of $\bar{x} + k\sigma$, one can obtain the probability of acceptance , P_a.

$$P_a = P\left(\bar{x} + k\sigma \le U\right)$$

therefore $1 - P_a = P\left(\bar{x} + k\sigma > U\right)$

Also $K_{-P_a}\frac{\sigma}{\sqrt{n}} = U - \left(\mu - k\sigma\right)$

But $K_{-P_a} = -K_{P_a}$ in normal distribution

Therefore $-K_{P_a}\frac{\sigma}{\sqrt{n}} = U - \mu + k\sigma$

Simplifying further with (3.1) one obtains,

$$K_p = k - \frac{1}{\sqrt{n}} K_{L_p} \qquad (3.2)$$

Equation (3.2) provides P_a for any given p and vice- versa. Using this equation one can

i. design the plan for desired points on operating characteristic curve

and ii. compute various points on the OC curve of the desired sampling plan . Using the normal distribution calculation of OC curves for variable sampling plans may be performed, when the standard deviation is known.

Computation of OC Values with Upper Specification Limit

For any given value of p, the probability of acceptance can be determined as follows for an upper specification limit

i. Determine k_p from p

ii. Obtain $K_{P_a} = k_p - k$

iii. Convert K_{P_a} to the distribution of sample means as $P_a = \sqrt{n} K_{P_a}$

iv. The probability of a normal variate exceeding P_a is the probability of rejection. Its complement P_a, is the probability of acceptance.

Computation of OC Values with Lower Specification Limit

i. Determine k_p

ii. Obtain $K_{P_a} = k_p + k$

iii. Convert to the distribution means as $P_a = \sqrt{n} K_{P_a}$

iv. the probability of a normal variate equal to or exceeding P_a is the probability of acceptance . Its complement P_a, is the probability of rejection .

The relation (3.2) is used to compute the probability of acceptance for various proportion defective. The probability of acceptance values obtained through excel work sheet by using normal probability values corresponding to the given p.

Effect of Sample Size with Fixed Acceptance Constant

OC values corresponding to various fraction defectives are computed for variable single sampling plans (n, 1.55) for n =5, 10, 12 are given in Table 3.7. The OC curves for Variable Single Sampling Plans (n, 1.55) for n = 5, 10, 12 are shown in fig.3.7.

OC curves of variable single sampling plans reveal the following facts

i. OC values decrease for increase in p

ii. OC values decrease for increase in n for inferior quality

and iii. OC values increase for increase in n for superior quality

Effect of Acceptance Constant with Fixed Sample Size

Table 3.8 presents the probability of acceptance obtained for Variable Single Sampling Plans with fixed n and acceptance constants k = 1, 1.44, 1.70. The OC curves corresponding to these Variable Single Sampling Plans are shown in fig 3.8.

From OC values of Variable Single Sampling Plans one observes

i. OC values decrease for increase in k

and ii. OC values decrease for increase in p

TABLE3.7 OC VALUES OF VSSP WITH KNOWN SIGMA AND FIXED K=1.55

p	OC values		
	n=5	n=10	n=12
0.0075	0.97615	0.99836	0.99886
0.018	0.89065	0.97062	0.97193
0.027	0.80234	0.91309	0.90658
0.048	0.59871	0.69497	0.64809
0.083	0.36317	0.36693	0.29116
0.129	0.17619	0.12502	0.07493
0.181	0.07636	0.03074	0.01122
0.216	0.04272	0.0116	0.00379
0.289	0.01355	0.00149	0.0003
0.313	0.0089	0.00071	0.00012
0.345	0.00508	0.00025	3E-05

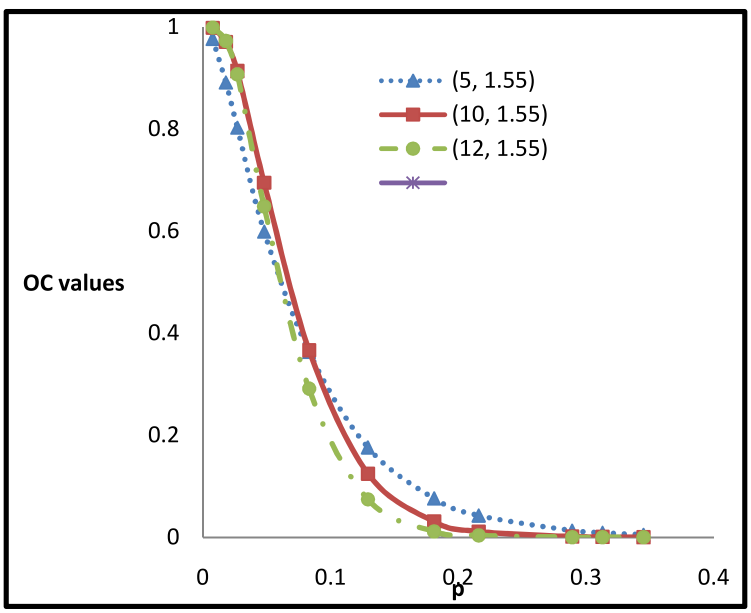


Fig.3.7 OC curves of VSS Plans with fixed k = 1.44

TABLE 3.8 OC VALUES OF VSSP WITH KNOWN SIGMA AND FIXED n =7

p	OC values		
	k=1	k=1.44	k=1.70
0.0075	0.99992	0.996	0.9732
0.018	0.99891	0.9599	0.85543
0.027	0.99305	0.9032	0.72907
0.048	0.95728	0.719	0.4562
0.083	0.8485	0.4483	0.20611
0.129	0.64431	0.2061	0.06811
0.181	0.40517	0.0808	0.01831
0.216	0.28096	0.0401	0.00755
0.289	0.12302	0.0099	0.00131
0.313	0.08851	0.00604	0.00069
0.345	0.05592	0.00298	0.00029

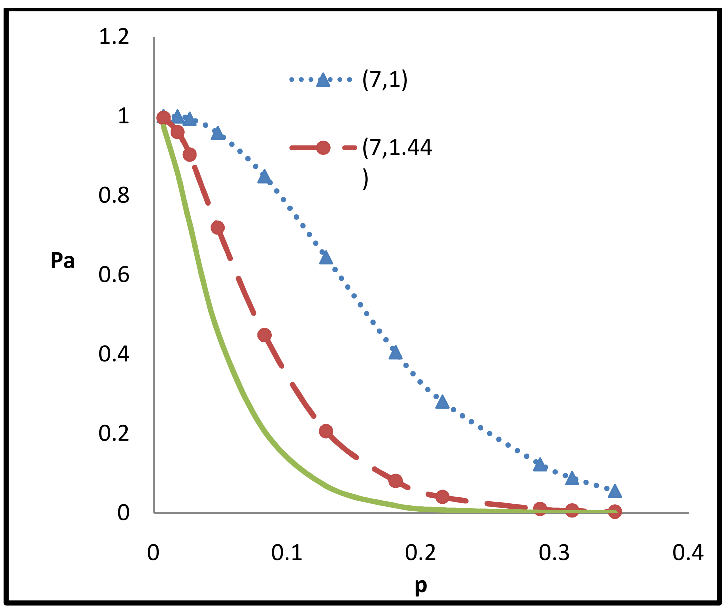


Fig. 3.8 OC curves of VSS Plans with fixed n

COMPARISION OF ATTRIBUTE AND VARIABLE SINGLE SAMPLING PLANS

Comparison of any two sampling plans is based on the OC function. Plans with same OC points are assumed to be matched plans. All the points of OC curve are not considered for matching. Only the points on the OC curve corresponding to AQL and RQL are considered for matching

The Attribute Single Sampling Plan n = 20, c = 1 matches with Variable Single Sampling Plan n = 7, k = 1.44. The probability of acceptance corresponding to these two plans are shown in Table 3.7.

The probability of acceptance values show that Variable Single Sampling Plan is more efficient than Attribute Single Sampling Plan. The advantages of Variable Single Sampling Plan over attribute single sampling plan are

i. same protection with smaller sample size

ii. feedback of data on process

and iii. extent of conformity of each unit

However the above advantages depend on the correctness of the assumption of shape of the underlying distribution of the measurements.

The principal advantage of variable plan over attribute plan is reduction in sample size.

TABLE 3.9 OC VALUES OF MATCHED ASSP AND VSSP

p	ASSP	VSSP
	n=20,c=1	n=7,k=1.44
0.0075	0.9956	0.9898
0.018	0.9599	0.9488
0.027	0.9032	0.8974
0.048	0.719	0.7505
0.083	0.4483	0.5058
0.129	0.2061	0.2713
0.181	0.0808	0.1237
0.216	0.0401	0.0708
0.289	0.0099	0.0209

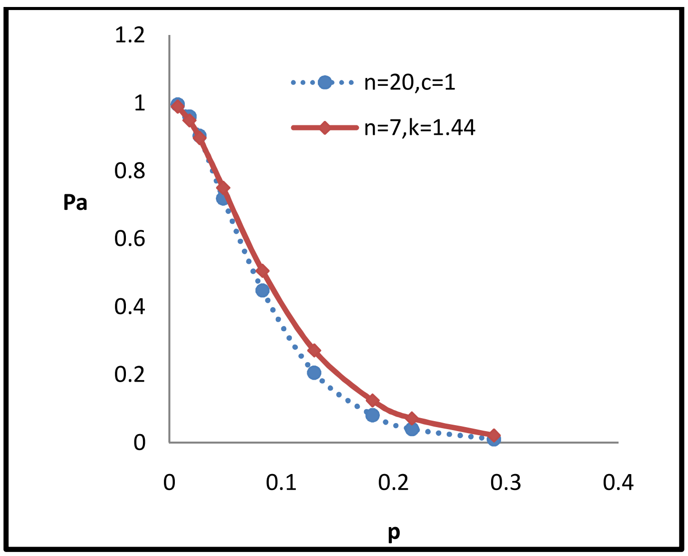


Fig.3.9 OC curves of ASSP(20, 1) and VSSP (7, 1.44)

SUMMARY AND CONCLUSION

In this dissertation attribute single sampling plan and variable single sampling plan are analysed.

Important definitions are presented in Basic Concepts.

In chapter I designing of attribute single sampling plan is explained in detail.

In chapter II designing variable single sampling plan is discussed in detail. Properties of OC function with respect to parameter and quality indices along with comparison are presented in chapter III. At the end a bibliography is added.

Recommendations for further study

(i) Cost model using quadratic loss function may be designed

(ii) Attribute single sampling plan and variable single sampling plan may be designed to accommodate the process with non-constant proportion of defective.

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