
Introduction

Topological structures on the collection of data are suitable mathematical models for mathematizing not only quantitative data but also qualitative ones. Closedness is the basic concept for the study and investigation in topological spaces. Topology is used in nearly all branches of mathematics in one form or another. It is the mathematical study of the properties that are preserved through deformations, twisting and stretching of objects. Topology is implemented recently to understand diverse topics, such as cell biology, superconductors, robot motion and etc. The first work on Topology came into existence due to Euler, when he published the solution to the Konigsberg bridge problem.

Fuzzy mathematics forms a branch of mathematics related to fuzzy set theory and fuzzy logic. The notion of fuzzy set theory has caused great interest among both pure and applied mathematicians. Classical mathematical methods are not enough to solve the problems of daily life and also are not enough to meet the new requirements. Therefore, some theories such as fuzzy set theory have been developed to solve these problems. Applications of this theory appear in topology and many areas of mathematics. The first publication in fuzzy set theory by Zadeh (1965) and then by Goguen (1967) shows the intention of the authors to generalize the classical set. Through fuzzy sets we can speak only about membership values and it does not give a correct answer for non-membership values.

Atanassov (1986), a Bulgarian mathematician, created an idea about the non-membership value and then he introduced a new set which includes the non-membership value and coined the set as intuitionistic fuzzy set where the degree of membership is denoted by $\mu_A(x) \in [0, 1]$ of each element $x \in X$ to a set A and the degree of non-membership is denoted by $\nu_A(x) \in [0, 1]$.

Intuitionistic fuzzy set is a generalized notion to include both fuzzy sets and vague sets. Intuitionistic fuzzy sets have been found in diverse applied areas of

science and technology and have been applied to logic programming, medical diagnosis, micro electronic fault analysis, decision making problems, image processing, soft computing and many other areas.

Intuitionistic fuzzy sets are extensions of the standard fuzzy sets. The intuitionistic fuzzy set theory is based on

- (i) Extension of corresponding definitions of fuzzy set objects.
- (ii) Definition of new objects and their properties.

After the introduction of intuitionistic fuzzy set, different types of closed sets in intuitionistic fuzzy topological spaces namely intuitionistic fuzzy regular closed set, intuitionistic fuzzy closed set, intuitionistic fuzzy semi closed set, intuitionistic fuzzy preclosed set, intuitionistic fuzzy α -closed set and their generalized closed sets have been introduced and studied by many authors.

In my research work, a new generalization of closed set called β^{**} generalized closed set in intuitionistic fuzzy topological spaces is introduced. Its basic properties, preservation theorems, inter-relations, continuity, separation axioms and theoretical applications are established with necessary counter examples.

Methodology : The analysis has been done by the analytical method of comparing intuitionistic fuzzy β^{**} generalized closed sets with other existing intuitionistic fuzzy closed sets and providing examples wherever necessary to substantiate the results.

Notations : Throughout the thesis, the following notations are used :

- (i) (X, τ) , (Y, σ) and (Z, δ) denote the non-empty intuitionistic fuzzy topological spaces on which no separation axioms are mentioned unless it is stated specifically.
- (ii) The closure and interior of a subset A of a topological space is denoted by $cl(A)$ and $int(A)$ respectively.
- (iii) In all the diagrams, $A \rightarrow B$ represents A implies B but not conversely.

The deliberations in this research work include the following topics :

- ❖ β^{**} generalized closed sets in intuitionistic fuzzy topological spaces.
- ❖ Theoretical applications of β^{**} generalized closed sets in intuitionistic fuzzy topological spaces.
- ❖ β^{**} generalized continuous mappings in intuitionistic fuzzy topological spaces.
- ❖ β^{**} generalized closed mappings in intuitionistic fuzzy topological spaces.
- ❖ β^{**} generalized connectedness in intuitionistic fuzzy topological spaces.

For this, the thesis is organized into six chapters, as given below.

Chapter 1, contains mainly the collection of some basic definitions and results on various already existing closed sets, continuous mappings, closed mappings and connectedness in intuitionistic fuzzy topological spaces which are needed for the present research work.

In Chapter 2, section 2.1 gives a short introduction to intuitionistic fuzzy β closed set and its history. In section 2.2 we have introduced a new class of intuitionistic fuzzy generalized closed sets called intuitionistic fuzzy β^{**} generalized closed sets in intuitionistic fuzzy topological spaces. A comparison of β^{**} generalized closed sets with other existing intuitionistic fuzzy closed sets, properties of intuitionistic fuzzy β^{**} generalized closed sets and their characterizations are studied. Also it is shown that the converses are not true in general and they are proved with necessary counter examples. In section 2.3, a similar work is carried out for intuitionistic fuzzy β^{**} generalized open sets which are the complements of intuitionistic fuzzy β^{**} generalized closed sets. In section 2.4, as an theoretical application of β^{**} generalized closed sets, two new spaces namely, $IF\beta^{**}gT_{1/2}$ space, $IF\beta^{**}pT_{1/2}$ space are introduced and we have proved some interesting propositions on these spaces.

In Chapter 3, a new type of intuitionistic fuzzy continuous mapping called intuitionistic fuzzy β^{**} generalized continuous mapping has been introduced. This chapter is divided into six sections. In section 3.1, we give a short introduction to intuitionistic fuzzy generalized continuous mappings. In section 3.2, we have introduced intuitionistic fuzzy β^{**} generalized continuous mappings and analyzed the interrelations between intuitionistic fuzzy β^{**} generalized continuous mapping with other already existing continuous mappings. Also the composition of two intuitionistic fuzzy β^{**} generalized continuous mappings and their respective properties, preservation theorems are discussed with necessary counter examples. Some fascinating theorems concerning intuitionistic fuzzy β^{**} generalized continuous mappings are provided and discussed. In section 3.3, intuitionistic fuzzy contra β^{**} generalized continuous mapping has been introduced and some of its properties are studied. In section 3.4, intuitionistic fuzzy almost β^{**} generalized continuous mappings are introduced and some of their properties are analyzed. In section 3.5, intuitionistic fuzzy almost contra β^{**} generalized continuous mappings has been introduced and some interesting propositions and results are studied and investigated. In section 3.6, intuitionistic fuzzy β^{**} generalized irresolute mappings have been introduced and some of their properties are proved and they are compared with intuitionistic fuzzy β^{**} generalized continuous mappings.

In Chapter 4, the section 4.1 presents a short introduction to intuitionistic fuzzy continuous mappings. In section 4.2, the concept of intuitionistic fuzzy completely β^{**} generalized continuous mappings has been introduced and some of their properties are discussed. In section 4.3, the concept of intuitionistic fuzzy perfectly β^{**} generalized continuous mappings is established and their characterizations are discussed. In section 4.4, the concept of intuitionistic fuzzy quasi β^{**} generalized continuous mappings is brought into limelight.

In Chapter 5, section 5.1 presents a short introduction to intuitionistic fuzzy closed mappings and intuitionistic fuzzy open mappings. In section 5.2, intuitionistic

fuzzy β^{**} generalized closed mappings and intuitionistic fuzzy β^{**} generalized open mappings have been introduced and some of their properties are discussed. In this section, some characterizations of intuitionistic fuzzy β^{**} generalized closed mappings and intuitionistic fuzzy β^{**} generalized open mappings are proved. Also interrelation between intuitionistic fuzzy β^{**} generalized open mapping with other open mappings in intuitionistic fuzzy topological spaces are established with counter examples. In section 5.3, intuitionistic fuzzy almost β^{**} generalized closed mappings and some of their properties and characterizations are introduced and analyzed. In section 5.4, intuitionistic fuzzy contra β^{**} generalized open mappings are introduced and some interesting characterizations are acquired.

In Chapter 6, section 6.1 deals with the introduction to connectedness in intuitionistic fuzzy topological spaces. In section 6.2, we have introduced the notion of intuitionistic fuzzy β^{**} generalized connected space and intuitionistic fuzzy β^{**} generalized super connected space. We have investigated some of their properties also we have characterized the intuitionistic fuzzy β^{**} generalized super connected space.