

**α Generalized Continuous Mappings in Neutrosophic
Topological Spaces**

**PRISHKA F
(17PMA016)**

**Thesis Submitted to
Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore - 641043**

**In Partial Fulfilment of the Requirements for the Degree of
Master of Science in Mathematics**

April, 2019


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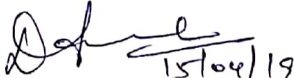
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Signature of the Supervisor

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Introduction

L. A. Zadeh [1965] first introduced the concepts of fuzzy sets and fuzzy logic. Fuzzy sets are sets whose elements have degrees of membership. A fuzzy set in X is a mapping from X into I where X is a non empty set and I is the unit interval $[0, 1]$.

Chang [1968] introduced the concept of fuzzy topological space. Fuzzy topological space is a natural generalization of topological spaces. Several researchers have worked on various basic concepts from general topology using fuzzy sets and developed the theory of fuzzy topological spaces.

Closed sets are fundamental objects in topological spaces. Levine [1970] introduced the concept of generalized closed sets in general topology. K. K. Azad [1981] introduced fuzzy semi-continuous mappings, fuzzy weakly continuous mappings and fuzzy almost continuous mappings.

Fuzzy continuity is one of the main topics in fuzzy topology. Various authors introduced various types of fuzzy continuity. I. M., Hanafy [1999] introduced the concept of fuzzy γ continuity and fuzzy generalized γ continuity in fuzzy topological spaces. Ekici and Etienne kerre [2006] introduced contra continuous mapping in fuzzy topological spaces.

K. Atanassov [1986] introduced the concept of intuitionistic fuzzy sets using the notion of fuzzy sets. Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership.

Coker [1997] introduced intuitionistic fuzzy topology. In fact Coker constructed the fundamental theory on intuitionistic fuzzy topological spaces. I. M., Hanafy [2003] introduced the concept of completely continuous function in intuitionistic fuzzy topological spaces.

Intuitionistic fuzzy contra continuous mapping was introduced by Kresteska and Ekici [2007]. K. Sakthivel [2010] introduced intuitionistic fuzzy α generalized continuous mappings and intuitionistic fuzzy α generalized irresolute mappings.

Florentin Smarandache [2010] introduced neutrosophic set theory. He also developed the concept of single-valued neutrosophic set oriented towards real world scientific and engineering applications. A generalization of the intuitionistic fuzzy set, classical set, fuzzy set, paraconsistent set, and tautological set based on neutrosophic. Florentin Smarandache presents the evaluation of sets from fuzzy set to neutrosophic set and also introduces the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively.

A. A. Salama, Florentin Smarandache and Valeri Kromov [2014] introduced neutrosophic closed sets and neutrosophic continuous functions. Wadei Faris Al-omeri and Florentin Smarandache [2016] introduced new neutrosophic sets via neutrosophic topological spaces.

In this research work, we have analyzed the following topics:

- ❖ Neutrosophic α generalized continuous mappings and Neutrosophic α generalized irresolute mappings
- ❖ Neutrosophic contra α continuous mappings, Neutrosophic contra α generalized continuous mappings and Neutrosophic contra α generalized irresolute mappings
- ❖ Neutrosophic almost α generalized continuous mappings, Neutrosophic almost contra α generalized continuous mappings, Neutrosophic completely α generalized continuous mappings

Chapter 1 deals with the preliminary definitions that are needed for the present study.

In **Chapter 2**, neutrosophic α generalized continuous mappings and neutrosophic α generalized irresolute mappings are introduced. Also we have established the relation between the newly introduced mapping and already existing mappings.

In **Chapter 3**, neutrosophic contra α continuous mappings, neutrosophic contra α generalized continuous mappings and neutrosophic contra α generalized irresolute mappings are introduced which is followed by the relation of them with some of the already existing continuous mappings in neutrosophic topological spaces. Also we have investigated some of their properties.

In **Chapter 4**, neutrosophic almost α generalized continuous mappings, neutrosophic almost contra α generalized continuous mappings and neutrosophic completely α generalized continuous mappings are introduced. Also we have investigated some of their properties.

Review of literature

Research on the field of neutrosophic was developed by Florentin Smarandache [2010]. Many researchers have contributed to the study of generalized forms of neutrosophic sets and their basic properties. We present a review of literature in some of the important article published that are related to this topic.

Levine [1963] introduced the concepts of semi open sets and semi continuity in topological spaces. L. A. Zadeh [1965] introduced fuzzy sets. Chang [1968] introduced the notions of fuzzy topology and basic concepts like fuzzy open set, fuzzy closed set, fuzzy neighborhood, and fuzzy interior of a fuzzy set. He studied and introduced fuzzy continuity which was proved to be fundamental importance in fuzzy topology since then various notions in classical topology have been extended to fuzzy topological spaces by various authors.

Azad [1981] introduced the notions of fuzzy semi open and fuzzy semi closed, fuzzy regular open and fuzzy regular closed sets. Mashour [1983] established α continuous and α open mappings in topological spaces. A. S. Bin Shahana [1991] introduced fuzzy pre open sets and fuzzy pre continuity.

M. K. Singhal and Niti Rajvanshi [1992] introduced the notion of fuzzy α sets and also introduced the concepts of fuzzy α continuous mappings and fuzzy α open mappings. G. Balasubramaniam and P. Sundaram [1997] have introduced fuzzy generalized continuous mappings and $T_{1/2}$ spaces.

M. K. Singhal and A. R. Singhal [1968] introduced almost continuous mapping in topological spaces. Ekici and Etienne kerre [2006] introduced contra continuous mapping in fuzzy topological spaces.

K. Atanassov [1986] introduced the concept of intuitionistic fuzzy sets using the notion of fuzzy sets. Coker [1997] introduced intuitionistic fuzzy topological spaces. Intuitionistic fuzzy contra continuous mapping was introduced by Kresteska and Ekici [2007].

Florentin Smarandache [2010] introduced neutrosophic set theory. He also developed the concept of single-valued neutrosophic set oriented towards real world scientific and engineering applications. As an extension of neutrosophic sets A.A Salama and S.A. Alblowi [2012] introduced neutrosophic set and neutrosophic topological spaces.

A. A. Salama, Florentin Smarandache and Valeri Kromov [2014] introduced neutrosophic closed sets and neutrosophic continuous functions. P. Ishwarya and K. Bageerathi [2016] introduced neutrosophic semi-open sets. Wadei Faris Al-omeri and Florentin Smarandache [2016] introduced new neutrosophic sets via neutrosophic topological spaces. R. Dhavaseelan and S.Jafari [2017] introduced generalized neutrosophic closed sets.

Here we present a brief survey of some of the articles published on neutrosophic closed sets and neutrosophic continuous functions.

1. FUZZY SETS

[Zadeh, L.A., 1965]

In this paper the author has introduced the concept of fuzzy sets. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation and various properties of these notions of fuzzy sets are established.

2. FUZZY TOPOLOGICAL SPACES

[Chang, C. L., 1968]

The author has introduced the basic concepts such as fuzzy open sets, fuzzy closed sets, fuzzy neighborhood, fuzzy interior, fuzzy continuity and fuzzy compactness.

3. GENERALIZED CLOSED SETS IN TOPOLOGY

[Levine., 1970]

In this paper the author has introduced generalized closed sets and generalized open sets in topological spaces. The author also studied and investigated standard properties of these sets.

4. ON FUZZY SEMI CONTINUITY, FUZZY ALMOST CONTINUITY AND FUZZY WEAKLY CONTINUITY

[Azad, K. K., 1981]

In this paper the author has introduced fuzzy semiopen set, fuzzy semiclosed set, fuzzy regular open set, fuzzy regular closed set, fuzzy semi continuous mapping, fuzzy semi open mapping, fuzzy semi closed mapping, fuzzy almost mapping and fuzzy weakly continuous mapping are discussed.

5. α -CONTINUOUS AND α -OPEN MAPPINGS

[Mashour., 1983]

The author has established and studied α -continuous and α -open mappings in topological spaces and also discussed the relations between the concepts of α -continuous mappings and α -open mappings.

6. FUZZY γ OPEN SETS AND FUZZY γ CONTINUITY

[Hanafy, I. M., 1999]

The author has introduced the concept of fuzzy γ open sets which is weaker than each of the concept of fuzzy semi open set or fuzzy pre open set. The author also studied their properties and discussed relationships between these concepts and in fuzzy topological spaces.

7. INTUITIONISTIC FUZZY SETS

[Atanassov, K., 1986]

In this paper the author has introduced the theory of intuitionistic fuzzy set and investigated some of their properties. Intuitionistic fuzzy set is given, the latter being a generalization of the concept of ‘fuzzy set’ and an example is described, various properties are proved, which are connected to the operations and relations over sets, and with modal and topological operators, defined over the set of intuitionistic fuzzy sets.

8. AN INTRODUCTION TO INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Coker, D., 1997]

The author has introduced intuitionistic fuzzy topology and also constructed the fundamental theory on intuitionistic fuzzy topological spaces. The author also introduced the definitions of fuzzy connectedness and fuzzy Hausdorff space, obtain several preservation properties and some characterizations concerning fuzzy compactness and fuzzy connectedness.

9. ON FUZZY CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Gurcay, H., Coker, D., and Haydar, Es. A., 1997]

In this paper, the authors have introduced fuzzy continuity and studied the relations of various continuous mappings and also provided with suitable illustrations.

10. GENERALIZED CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Thakur, S.S., and Rekha Chaturvedi., 2006]

In this article the authors have defined and studied the concept of intuitionistic fuzzy generalized continuous mappings in intuitionistic fuzzy topological spaces. The author also studied the concepts of intuitionistic fuzzy points, intuitionistic fuzzy generalized closed sets and intuitionistic fuzzy generalized open sets.

11. INTUITIONISTIC FUZZY GENERALIZED SEMI CONTINUOUS MAPPINGS

[Santhi, R., Sakthivel, K., 2009]

In this paper the authors have introduced the generalization of intuitionistic fuzzy semi continuous mappings and also establish the relation between the existing continuous mappings with the newly defined continuous mappings.

12. INTUITIONISTIC FUZZY ALPHA GENERALIZED CONTINUOUS MAPPINGS AND INTUITIONISTIC FUZZY ALPHA GENERALIZED IRRESOLUTE MAPPINGS

[Sakthivel, K., 2010]

In this paper the author has introduced intuitionistic fuzzy alpha generalized continuous mappings and intuitionistic fuzzy alpha generalized irresolute mappings and also some of their properties are studied.

13. INTUITIONISTIC FUZZY γ GENERALIZED CONTINUOUS MAPPINGS

[Prema, S., and Jayanthi, D., 2017]

In this paper authors have introduced intuitionistic fuzzy γ generalized continuous mappings and investigated some of their properties. Also they have

provided some characterization of intuitionistic fuzzy γ generalized continuous mappings.

14. INTUITIONISTIC FUZZY γ^* GENERALIZED CONTINUOUS MAPPINGS

[Riya, V.M., and Jayanthi, D., 2017]

In this article the authors have introduced intuitionistic fuzzy γ^* generalized continuous mappings and discussed some of their properties. Also some characterizations of intuitionistic fuzzy γ^* generalized continuous mappings are provided.

15. A GENERALIZATION OF INTUITIONISTIC FUZZY SET

[Florentin Smarandache., 2010]

The author generalizes the intuitionistic fuzzy Para consistent set, and intuitionistic set to the neutrosophic set and also many examples are studied. Many examples are presented and distinctions between neutrosophic set and intuitionistic fuzzy sets are studied.

16. NEUTROSOPHIC SET AND NEUTROSOPHIC TOPOLOGICAL SPACES

[Salama, A.A., and Alblowi, S.A., 2012]

In this paper authors have introduced the neutrosophic set and neutrosophic topological spaces and also basic properties are discussed. The purpose of this paper is to construct a new set theory called the neutrosophic set. Fundamental definitions of neutrosophic set operations, several properties and relation between the neutrosophic sets and others are discussed.

17. NEUTROSOPHIC CLOSED SET AND NEUTROSOPHIC CONTINUOUS FUNCTIONS

[Salama, A.A., Florentin Smarandache and Valeri Kromov., 2014]

In this paper the authors have introduced and studied the concept of neutrosophic closed set and neutrosophic continuous function. The authors are also provided the study on neutrosophic crisp sets.

18. ON NEUTROSOPHIC SEMI-OPEN SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

[Ishwarya, P., and Bageerathi, K., 2016]

In this paper authors have defined the neutrosophic topological spaces and proved some propositions based on the concept of neutrosophic semi-open sets and neutrosophic semi-closed sets and also derived some of their characterization.

19. A STUDY ON NEUTROSOPHIC FRONTIER AND NEUTROSOPHIC SEMI- FRONTIER IN NEUTROSOPHIC TOPOLOGICAL SPACES

[Ishwarya, P., and Bageerathi, K., 2017]

In this paper neutrosophic frontier and neutrosophic semi-frontier in neutrosophic topology are introduced and several properties, characterizations and examples are established.

20. GENERALIZED CLOSED SETS VIA NEUTROSOPHIC TOPOLOGICAL SPACES

[Pushpalatha, A., and Nandhini, T., 2019]

In this paper, the authors have introduced the notion of generalized closed sets in Neutrosophic topological spaces and studied some of their basic properties. The author also studied the propositions with suitable illustrations of neutrosophic generalized closed sets in neutrosophic topological spaces.

CHAPTER 1

Preliminaries

Definition 1.1: [2010]

Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ where $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A .

A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ can be identified as an ordered triple $\langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ in $]0, 1^+[$ on X .

Definition 1.2: [2010]

Let $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ be a neutrosophic set on X , then the complement $C(A)$ may be defined as

1. $C(A) = \{\langle x, 1 - \mu_A(x), 1 - \nu_A(x) \rangle : x \in X\}$
2. $C(A) = \{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X\}$
3. $C(A) = \{\langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X\}$

Note that for any two neutrosophic sets A and B ,

4. $C(A \cup B) = C(A) \cap C(B)$
5. $C(A \cap B) = C(A) \cup C(B)$

Definition 1.3: [2010]

For any two neutrosophic sets $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X\}$ we may have

1. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \forall x \in X$
2. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \forall x \in X$
3. $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x) \rangle$

4. $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \vee \nu_B(x) \rangle$
5. $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x) \rangle$
6. $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \wedge \nu_B(x) \rangle$

Definition 1.4: [2012]

A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

$$(NT_1) \quad 0_N, 1_N \in \tau$$

$$(NT_2) \quad G_1 \cap G_2 \in \tau \text{ for any } G_1, G_2 \in \tau$$

$$(NT_3) \quad \cup G_i \in \tau \forall \{G_i : i \in J\} \subseteq \tau$$

In this case the pair (X, τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (NOS) in X . A neutrosophic set A is a neutrosophic closed set (NCS) if and only if its complement $C(A)$ is a neutrosophic open set in X . Here the empty set (0_N) and the whole set (1_N) may be defined as follows:

$$(0_1) \quad 0_N = \{\langle x, 0, 0, 1 \rangle : x \in X\}$$

$$(0_2) \quad 0_N = \{\langle x, 0, 1, 1 \rangle : x \in X\}$$

$$(0_3) \quad 0_N = \{\langle x, 0, 1, 0 \rangle : x \in X\}$$

$$(0_4) \quad 0_N = \{\langle x, 0, 0, 0 \rangle : x \in X\}$$

$$(1_1) \quad 1_N = \{\langle x, 1, 0, 0 \rangle : x \in X\}$$

$$(1_2) \quad 1_N = \{\langle x, 1, 0, 1 \rangle : x \in X\}$$

$$(1_3) \quad 1_N = \{\langle x, 1, 1, 0 \rangle : x \in X\}$$

$$(1_4) \quad 1_N = \{\langle x, 1, 1, 1 \rangle : x \in X\}$$

Definition 1.5: [2012]

Let (X, τ) be a neutrosophic topological space and let $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ be a neutrosophic set in X . Then the neutrosophic interior and the neutrosophic closure of A are defined by

$$NInt(A) = \cup \{ G : G \text{ is a neutrosophic open set in } X \text{ and } G \subseteq A \}$$

$$NCl(A) = \cap \{ K : K \text{ is a neutrosophic closed set in } X \text{ and } A \subseteq K \}$$

Note that for any neutrosophic set A , $NCl(C(A)) = C(NInt(A))$ and $NInt(C(A)) = C(NCl(A))$.

Definition 1.6: [2016]

A neutrosophic set A of a neutrosophic topological space X is said to be

- (i) a neutrosophic pre-open set if $A \subseteq NInt(NCl(A))$
- (ii) a neutrosophic semi-open set if $A \subseteq NCl(NInt(A))$
- (iii) a neutrosophic α -open set if $A \subseteq NInt(NCl(NInt(A)))$
- (iv) a neutrosophic semi- α -open set if $A \subseteq NCl(\alpha NInt(A))$

Definition 1.7: [2016]

A neutrosophic set A of a neutrosophic topological space X is said to be

- (i) a neutrosophic pre-closed set if $NCl(NInt(A)) \subseteq A$
- (ii) a neutrosophic semi-closed set if $NInt(NCl(A)) \subseteq A$
- (iii) a neutrosophic α -closed set if $NCl(NInt(NCl(A))) \subseteq A$
- (iv) a neutrosophic semi- α -closed set if $NInt(\alpha NCl(A)) \subseteq A$

Definition 1.8: [2018]

A neutrosophic set A in a neutrosophic topological space X is said to be a neutrosophic regular closed set if $NCl(NInt(A)) = A$ and neutrosophic regular open set if $NInt(NCl(A)) = A$.

Definition 1.9: [2018]

Let A be a neutrosophic set of a neutrosophic topological space (X, τ) . Then the neutrosophic α interior and the neutrosophic α closure are defined as

$$N_{\alpha} \text{Int}(A) = \cup \{G: G \text{ is an neutrosophic } \alpha \text{ open set in } X \text{ and } G \subseteq A\}$$

$$N_{\alpha} \text{Cl}(A) = \cap \{K: K \text{ is an neutrosophic } \alpha \text{ closed set in } X \text{ and } A \subseteq K\}$$

Definition 1.10: [2018]

A neutrosophic set A in a neutrosophic topological space X is said to be a neutrosophic α generalized closed set if $N_{\alpha} \text{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is a neutrosophic open set in X .

The complement $C(A)$ of a neutrosophic α generalized closed set A is a neutrosophic α generalized open set in X .

Definition 1.11: [2014]

Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called neutrosophic continuous if the inverse image of every neutrosophic closed set in (Y, σ) is a neutrosophic closed set in (X, τ) .

Definition 1.12: [2016]

Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. Then the mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

- (i) a neutrosophic α continuous if the inverse image of every neutrosophic closed set in (Y, σ) is a neutrosophic α closed set in (X, τ) .
- (ii) a neutrosophic pre continuous if the inverse image of every neutrosophic closed set in (Y, σ) is a neutrosophic pre closed set in (X, τ) .
- (iii) a neutrosophic semi continuous if the inverse image of every neutrosophic closed set in (Y, σ) is a neutrosophic semi closed set in (X, τ) .

Definition 1.13: [2017]

Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called neutrosophic contra continuous if the inverse image of every neutrosophic open set in (Y, σ) is a neutrosophic closed set in (X, τ) .

Definition 1.14: [2017]

Let f be a mapping from a neutrosophic topological space (X, τ) into a neutrosophic topological space (Y, σ) . Then f is said to be a neutrosophic almost continuous if the inverse image of every neutrosophic regular closed set in (Y, σ) is a neutrosophic closed set in (X, τ) .

CHAPTER 2

Section 2.1

Neutrosophic α generalized continuous mappings

In this section we have introduced neutrosophic α generalized continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced mappings and already existing mappings.

Definition 2.1.1:

Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic α generalized continuous mapping if $f^{-1}(B)$ is a neutrosophic α generalized closed set in (X, τ) for every neutrosophic closed set B of (Y, σ) .

Example 2.1.2:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.5, 0.5), (0.2, 0.2), (0.5, 0.5) \rangle$ and $G_2 = \langle y, (0.7, 0.6), (0.1, 0.1), (0.3, 0.3) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2^c = \langle y, (0.3, 0.3), (0.1, 0.1), (0.7, 0.6) \rangle$ is a neutrosophic closed set in Y . Then $f^{-1}(G_2^c) = \langle x, (0.3, 0.3), (0.1, 0.1), (0.7, 0.6) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2^c) \subseteq G_1$ and $N_\alpha Cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \subseteq G_1$, where G_1 is a neutrosophic open set in X . Therefore f is a neutrosophic α generalized continuous mapping.

Proposition 2.1.3:

Every neutrosophic continuous mapping is a neutrosophic α generalized continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic continuous mapping. Let A be a neutrosophic closed set in Y . Then $f^{-1}(A)$ is a neutrosophic closed set in X . Since every neutrosophic closed set is a neutrosophic α generalized closed set in X [2018], $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic α generalized continuous mapping.

Example 2.1.4:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2, 0.3), (0.1, 0.1), (0.8, 0.7) \rangle$ and $G_2 = \langle y, (0.4, 0.4), (0.2, 0.2), (0.6, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2^c = \langle y, (0.6, 0.6), (0.2, 0.2), (0.4, 0.4) \rangle$ is a neutrosophic closed set in Y . Then $f^{-1}(G_2^c) = \langle x, (0.6, 0.6), (0.2, 0.2), (0.4, 0.4) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2^c) \subseteq 1_N$ and $N_\alpha \text{Cl}(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup \text{NCl}(\text{NInt}(\text{NCl}(f^{-1}(G_2^c)))) = G_1^c \subseteq 1_N$, where G_1^c is a neutrosophic closed set in X . Therefore f is a neutrosophic α generalized continuous mapping but since $f^{-1}(G_2^c)$ is not a neutrosophic closed set in X as $\text{NCl}(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$, f is not a neutrosophic continuous mapping.

Proposition 2.1.5:

Every neutrosophic α continuous mapping is a neutrosophic α generalized continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α continuous mapping. Let A be a neutrosophic closed set in Y . Then $f^{-1}(A)$ is a neutrosophic α closed set in X . Since every neutrosophic α closed set is a neutrosophic α generalized closed set in X [2018], $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic α generalized continuous mapping.

Example 2.1.6:

Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2,0.3), (0.3,0.3), (0.7,0.6) \rangle$ and $G_2 = \langle y, (0.3,0.3), (0.2,0.2), (0.5,0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2^c = \langle y, (0.5,0.6), (0.2,0.2), (0.3,0.3) \rangle$ is a neutrosophic closed set in Y . Then $f^{-1}(G_2^c) = \langle x, (0.5,0.6), (0.2,0.2), (0.3,0.3) \rangle$ is a neutrosophic α generalized in (X, τ) as $f^{-1}(G_2^c) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \subseteq 1_N$, where G_1^c is a neutrosophic closed set in X . Therefore f is a neutrosophic α generalized continuous mapping but since $NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \not\subseteq f^{-1}(G_2^c)$. Hence f is not a neutrosophic α continuous mapping.

Remark 2.1.7:

Every neutrosophic semi continuous mapping and neutrosophic α generalized continuous mapping are independent to each other in general.

Example 2.1.8:

In example 3.2, f is a neutrosophic α generalized continuous mapping but since $NInt(NCl(f^{-1}(G_2^c))) = NInt(G_1^c) = G_1 \not\subseteq f^{-1}(G_2^c) = \langle x, (0.3,0.3), (0.1,0.1), (0.7,0.6) \rangle$, $f^{-1}(G_2^c)$ is not a neutrosophic semi closed set in X . Hence f is not a neutrosophic semi continuous mapping.

Example 2.1.9:

Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.3,0.2), (0.3,0.2), (0.7,0.6) \rangle$ and $G_2 = \langle y, (0.7,0.6), (0.2,0.2), (0.3,0.2) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2^c = \langle y, (0.3,0.2), (0.2,0.2), (0.7,0.6) \rangle$ is a neutrosophic closed set in Y . But $f^{-1}(G_2^c) = \langle x, (0.3,0.2), (0.2,0.2), (0.7,0.6) \rangle$ is not a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2^c) \subseteq G_1$ and $N_\alpha Cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \not\subseteq G_1$, where G_1^c is a neutrosophic closed set in X . Therefore, f is not a neutrosophic α generalized continuous mapping. But since

$NInt(NCl(f^{-1}(G_2^c))) = NInt(G_1^c) = G_1 \subseteq f^{-1}(G_2^c)$ is a neutrosophic semi closed set in X . Hence f is a neutrosophic semi continuous mapping.

Remark 2.1.10:

Every neutrosophic pre continuous mapping and neutrosophic α generalized continuous mapping are independent to each other in general

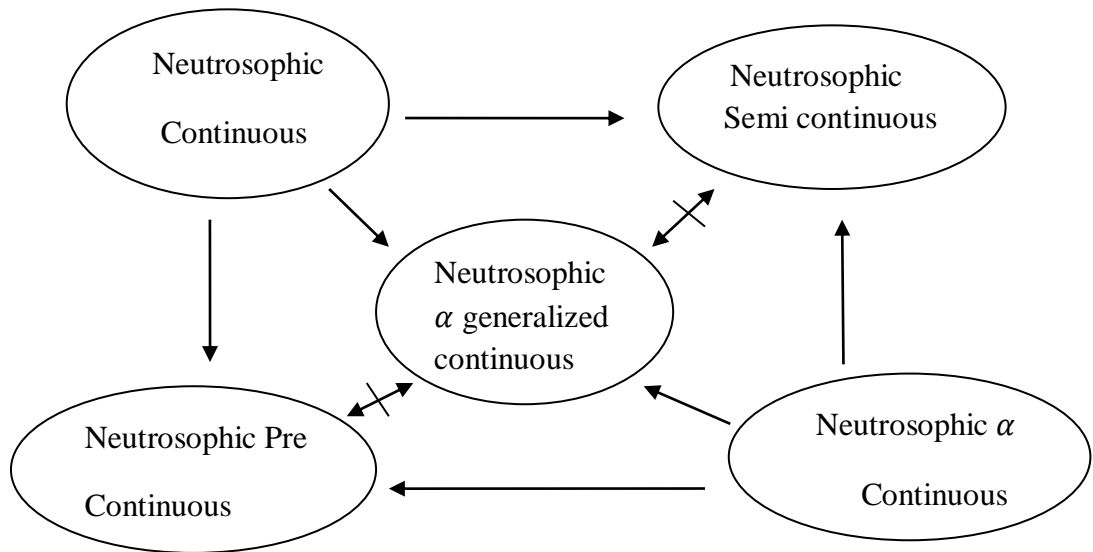
Example 2.1.11:

Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.1,0.1), (0.4,0.4), (0.7,0.3) \rangle$, $G_2 = \langle y, (0.2,0.2), (0.1,0.1), (0.6,0.3) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2^c = \langle y, (0.6,0.3), (0.1,0.1), (0.2,0.2) \rangle$ is a neutrosophic closed set in Y . Then $f^{-1}(G_2^c) = \langle x, (0.6,0.3), (0.1,0.1), (0.2,0.2) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2^c) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \subseteq 1_N$, where G_1^c is a neutrosophic closed set in X . Therefore f is a neutrosophic α generalized continuous mapping. Since $NCl(NInt(f^{-1}(G_2^c))) = G_1^c \not\subseteq f^{-1}(G_2^c)$, $f^{-1}(G_2^c)$ is not a neutrosophic pre closed set in X . Hence f is not a neutrosophic pre continuous mapping.

Example 2.1.12:

Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.5,0.4), (0.3,0.2), (0.5,0.6) \rangle$ and $G_2 = \langle y, (0.8,0.7), (0.3,0.2), (0.2,0.3) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2^c = \langle y, (0.2,0.3), (0.3,0.2), (0.8,0.7) \rangle$ is a neutrosophic closed set in Y . Then $f^{-1}(G_2^c) = \langle x, (0.2,0.3), (0.3,0.2), (0.8,0.7) \rangle$ is not a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2^c) \subseteq G_1$ and $N_\alpha Cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \not\subseteq G_1$, where G_1^c is a neutrosophic closed set in X . Therefore, f is not a neutrosophic α generalized continuous mapping. Since $NCl(NInt(f^{-1}(G_2^c))) = NCl(0_N) = 0_N \subseteq f^{-1}(G_2^c)$, $f^{-1}(G_2^c)$ is a neutrosophic pre closed set in X . Hence f is a neutrosophic pre continuous mapping.

The relation between various types of Neutrosophic continuity is given in the following diagram.



Proposition 2.1.13:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic α generalized continuous if and only if the inverse image of each neutrosophic open set in Y is a neutrosophic α generalized open set in X .

Proof:

Necessity:

Let A be a neutrosophic open set in Y . This implies A^c is neutrosophic closed set in Y . Since f is a neutrosophic α generalized continuous, $f^{-1}(A^c)$ is a neutrosophic α generalized closed set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized open set in X .

Sufficiency:

Let A be a neutrosophic closed set in Y . This implies A^c is neutrosophic open set in Y . By hypothesis, $f^{-1}(A^c)$ is a neutrosophic α generalized open set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic α generalized continuous mapping.

Proposition 2.1.14:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic α generalized continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is a neutrosophic continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic α generalized continuous mapping.

Proof:

Let A be a neutrosophic closed set in Z . Then $g^{-1}(A)$ is a neutrosophic closed set in Y , by hypothesis. Since f is a neutrosophic α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . Hence $g \circ f$ is a neutrosophic α generalized continuous mapping.

Definition 2.1.15:

Let (X, τ) be a neutrosophic topological space. The Neutrosophic alpha generalized closure ($N_{\alpha g}Cl(A)$) for any neutrosophic set A is defined as follows:

$$N_{\alpha g}Cl(A) = \cap \{ K : K \text{ is a neutrosophic } \alpha \text{ generalized closed set in } X \text{ and } A \subseteq K \}.$$

If A is a neutrosophic α generalized closed set, then $N_{\alpha g}Cl(A) = A$.

Proposition 2.1.16:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized continuous mapping. Then the following conditions are hold:

- (i) $f(N_{\alpha g}Cl(A)) \subseteq NCl(f(A))$, for every neutrosophic set A in X .
- (ii) $N_{\alpha g}Cl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$, for every neutrosophic set B in Y .

Proof:

(i) Since $NCl(f(A))$ is a neutrosophic closed set in Y and f is a neutrosophic α generalized continuous mapping, then $f^{-1}(NCl(f(A)))$ is a neutrosophic α generalized closed set in X . That is $N_{\alpha g}Cl(f^{-1}(NCl(f(A)))) = f^{-1}(NCl(f(A)))$. Now, $f(N_{\alpha g}Cl(f^{-1}(NCl(f(A)))) = ff^{-1}(NCl(f(A))) \subseteq NCl(f(A))$. Then $f(N_{\alpha g}Cl(A)) \subseteq f(N_{\alpha g}Cl(f^{-1}(f(A)))) \subseteq f(N_{\alpha g}Cl(f^{-1}(NCl(f(A)))) \subseteq NCl(f(A))$. Therefore $f(N_{\alpha g}Cl(A)) \subseteq NCl(f(A))$, for every neutrosophic set A in X .

(ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(N_{\alpha g}Cl(f^{-1}(B))) \subseteq NCl(f(f^{-1}(B))) \subseteq NCl(B)$. Hence $N_{\alpha g}Cl(f^{-1}(B)) \subseteq f^{-1}(f(N_{\alpha g}Cl(f^{-1}(B)))) \subseteq f^{-1}(NCl(B))$, for every neutrosophic set B in Y .

Definition 2.1.17:

A neutrosophic topological space (X, τ) is said to be a neutrosophic $_{\alpha a}T_{1/2}$ ($N_{\alpha a}T_{1/2}$) space if every neutrosophic α generalized closed set in X is a neutrosophic closed set in X .

Definition 2.1.18:

A neutrosophic topological space (X, τ) is said to be a neutrosophic $_{\alpha b}T_{1/2}$ ($N_{\alpha b}T_{1/2}$) space if every neutrosophic α generalized closed set in X is a neutrosophic generalized closed set in X .

Proposition 2.1.19:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized continuous mapping, then f is a neutrosophic continuous mapping, if X is a $N_{\alpha a}T_{1/2}$ space.

Proof:

Let A be a neutrosophic closed set in Y . Then $f^{-1}(A)$ is a neutrosophic α generalized closed set in X , by hypothesis. Since X is a $N_{\alpha a}T_{1/2}$ space, $f^{-1}(A)$ is a neutrosophic closed in X . Hence f is a neutrosophic continuous mapping.

Proposition 2.1.20:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized continuous mapping, then f is a neutrosophic generalized continuous mapping, if X is a $N_{\alpha b}T_{1/2}$ space.

Proof:

Let A be a neutrosophic closed set in Y . Then $f^{-1}(A)$ is a neutrosophic α generalized closed set in X , by hypothesis. Since X is a $N_{\alpha b}T_{1/2}$ space, $f^{-1}(A)$ is a neutrosophic generalized closed set in X . Hence f is a neutrosophic generalized continuous mapping.

Proposition 2.1.21:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from a neutrosophic topological space X into a neutrosophic topological space Y . Then the following conditions are equivalent if X is a $N_{\alpha\alpha}T_{1/2}$ space:

- (i) f is a neutrosophic α generalized continuous mapping
- (ii) If B is a neutrosophic open set in Y , then $f^{-1}(B)$ is a neutrosophic α generalized closed set in X
- (iii) $f^{-1}(\text{NInt}(B)) \subseteq \text{NInt}(\text{NCl}(\text{NInt}(f^{-1}(B))))$ for every neutrosophic set B in Y

Proof:

(i) \Rightarrow (ii) is obviously true.

(ii) \Rightarrow (iii) Let B be any neutrosophic set in Y . Then $\text{NInt}(B)$ is a neutrosophic open set in Y . Then $f^{-1}(\text{NInt}(B))$ is neutrosophic α generalized open set in X . Since X is a $N_{\alpha\alpha}T_{1/2}$ space, $f^{-1}(\text{NInt}(B))$ is a neutrosophic open set in X . Therefore, $f^{-1}(\text{NInt}(B)) = \text{NInt}(f^{-1}(\text{NInt}(B))) \subseteq \text{NInt}(\text{NCl}(\text{NInt}(f^{-1}(B))))$.

(iii) \Rightarrow (i) Let B be a neutrosophic closed set in Y . Then its complement B^c is a neutrosophic open set in Y . By hypothesis, $f^{-1}(\text{NInt}(B^c)) \subseteq \text{NInt}(\text{NCl}(\text{NInt}(f^{-1}(B^c))))$. This implies $f^{-1}(B^c) \subseteq \text{NInt}(\text{NCl}(\text{NInt}(f^{-1}(B^c))))$. Hence $f^{-1}(B^c)$ is a neutrosophic α open set in X . Since every neutrosophic α open set is a neutrosophic α generalized open set, $f^{-1}(B^c)$ is a neutrosophic α generalized open set in X . Therefore, $f^{-1}(B)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic α generalized continuous mapping.

Proposition 2.1.22:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from a neutrosophic topological space X into a neutrosophic topological space Y . Then the following conditions are equivalent if X is a $N_{\alpha\alpha}T_{1/2}$ space:

- (i) f is a neutrosophic α generalized continuous mapping

- (ii) If $f^{-1}(B)$ is a neutrosophic α generalized closed set in X , for every neutrosophic closed set B in Y
- (iii) $NCl(NInt(NCl(f^{-1}(A)))) \subseteq f^{-1}(NCl(A))$ for every neutrosophic set B in Y

Proof:

(i) \Rightarrow (ii) is obviously true.

(ii) \Rightarrow (iii) Let A be any neutrosophic set in Y . Then $NCl(A)$ is a neutrosophic closed set in Y . By hypothesis, $f^{-1}(NCl(A))$ is a neutrosophic α generalized closed set in X . since X is a $N_{\alpha\alpha}T_{1/2}$ space, $f^{-1}(NCl(A))$ is a neutrosophic closed set in X . Therefore $NCl(f^{-1}(NCl(A))) = f^{-1}(NCl(A))$. Now $NCl(NInt(NCl(f^{-1}(A)))) \subseteq NCl(NInt(NCl(f^{-1}(NCl(A)))) \subseteq f^{-1}(NCl(A))$.

(iii) \Rightarrow (i) Let A be a neutrosophic closed set in Y . Then by hypothesis $NCl(NInt(NCl(f^{-1}(NCl(A)))) \subseteq f^{-1}(NCl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is a neutrosophic α closed set in X and hence it is a neutrosophic α generalized closed set in X . Therefore f is a neutrosophic α generalized continuous mapping.

Section 2.2

Neutrosophic α generalized irresolute mappings

In this section we have introduced neutrosophic α generalized irresolute mappings and studied some of their properties.

Definition 2.2.1:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic α generalized irresolute mapping if $f^{-1}(A)$ is a neutrosophic α generalized closed set in (X, τ) for every neutrosophic α generalized closed set A of (Y, σ) .

Proposition 2.2.2:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized irresolute, then f is neutrosophic α generalized continuous mapping but not conversely.

Proof:

Let f be a neutrosophic α generalized irresolute mapping. Let A be any neutrosophic closed set in Y . Since every neutrosophic closed set is a neutrosophic α generalized closed set, A is a neutrosophic α generalized closed set in Y . By hypothesis $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic α generalized continuous mapping.

Example 2.2.3:

Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.1, 0.3), (0.3, 0.3), (0.6, 0.3) \rangle$ and $G_2 = \langle y, (0.3, 0.1), (0.1, 0.1), (0.5, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic α generalized continuous mapping but not a neutrosophic α generalized irresolute mapping. Since the neutrosophic set $A = \langle y, (0.1, 0.3), (0.2, 0.2), (0.6, 0.4) \rangle$ is a neutrosophic α generalized closed set in Y but $f^{-1}(A)$ is not a neutrosophic α generalized closed set in X as $f^{-1}(A) = \langle x, (0.1, 0.3), (0.2, 0.2), (0.6, 0.4) \rangle \subseteq G_1$ but $N_\alpha \text{Cl}(f^{-1}(A)) = f^{-1}(A) \cup \text{NCl}(\text{NInt}(\text{NCl}(f^{-1}(A)))) = G_1^c \not\subseteq G_1$. Hence f is not a neutrosophic α generalized irresolute mapping.

Proposition 2.2.4:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be any two neutrosophic α generalized irresolute mappings, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic α generalized irresolute mapping.

Proof:

Let A be a neutrosophic α generalized closed set in Z . Then $g^{-1}(A)$ is a neutrosophic α generalized closed set in Y . Since f is a neutrosophic α generalized irresolute mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . Hence $g \circ f$ is a neutrosophic α generalized irresolute mapping.

Proposition 2.2.5:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized irresolute mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be a neutrosophic α generalized continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic α generalized continuous mapping.

Proof:

Let A be a neutrosophic closed set in Z . Then $g^{-1}(A)$ is a neutrosophic α generalized closed set in Y , by hypothesis. Since f is a neutrosophic α generalized irresolute mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . Hence $g \circ f$ is a neutrosophic α generalized continuous mapping.

Proposition 2.2.6:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized irresolute mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be a neutrosophic continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic α generalized continuous mapping.

Proof:

Let A be a neutrosophic closed set in Z . Then $g^{-1}(A)$ is a neutrosophic closed set in Y . Since every neutrosophic closed set is a neutrosophic α generalized closed set [2018], $g^{-1}(A)$ is a neutrosophic α generalized closed set in Y . Therefore, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X , by hypothesis. Hence $g \circ f$ is a neutrosophic α generalized continuous mapping.

Proposition 2.2.7:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic α generalized irresolute mapping if and only if the inverse image of each neutrosophic α generalized open set in Y is a neutrosophic α generalized open set in X .

Proof:

Necessity:

Let A be a neutrosophic α generalized open set in Y . Then A^c is a neutrosophic α generalized closed set in Y . Since f is neutrosophic α generalized irresolute, $f^{-1}(A^c)$ is neutrosophic α generalized closed set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized open set in X .

Sufficiency:

Let A be a neutrosophic α generalized closed set in Y . This implies A^c is neutrosophic α generalized open set in Y . By hypothesis, $f^{-1}(A^c)$ is a neutrosophic α generalized open set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic α generalized irresolute mapping.

Proposition 2.2.8:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from a neutrosophic topological space X into a neutrosophic topological space Y . Then the following conditions are equivalent if X and Y are $N_{\alpha\alpha}T_{1/2}$ spaces:

- (i) f is a neutrosophic α generalized irresolute mapping
- (ii) $f^{-1}(B)$ is a neutrosophic α generalized open set in X for each neutrosophic α generalized open set in Y
- (iii) $NCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for each neutrosophic set B of Y

Proof:

(i) \Rightarrow (ii) is obviously true from the Proposition 2.2.7.

(ii) \Rightarrow (iii) Let B be any neutrosophic set in Y and $B \subseteq NCl(B)$. Then $f^{-1}(B) \subseteq f^{-1}(NCl(B))$. Since $NCl(B)$ is a NCS in Y , $f^{-1}(NCl(B))$ is a neutrosophic α generalized closed set in X , by hypothesis. Since X is a $N_{\alpha\alpha}T_{1/2}$ space, $f^{-1}(NCl(B))$ is a neutrosophic closed set in X . Hence $NCl(f^{-1}(B)) \subseteq NCl(f^{-1}(NCl(B))) = f^{-1}(NCl(B))$.

(iii) \implies (i) Let B be a neutrosophic α generalized closed set in Y . Since Y is a $N_{\alpha\alpha}T_{1/2}$ space, B is a neutrosophic closed set in Y and $NCl(B) = B$. Hence $f^{-1}(B) = f^{-1}(NCl(B)) \supseteq NCl(f^{-1}(B))$. But $f^{-1}(B) \subseteq NCl(f^{-1}(B))$. Therefore, $NCl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is a neutrosophic closed set and hence it is a neutrosophic α generalized closed set in X . Thus f is a neutrosophic α generalized irresolute mapping.

CHAPTER 3

Section 3.1

Neutrosophic contra α continuous mappings

In this section we have introduced neutrosophic contra α continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced mapping and already existing mappings.

Definition 3.1.1:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic contra α continuous mapping if $f^{-1}(B)$ is a neutrosophic α closed set in (X, τ) for every neutrosophic open set B in (Y, σ) .

Definition 3.1.2:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic contra pre continuous mapping if $f^{-1}(B)$ is a neutrosophic pre closed set in (X, τ) for every neutrosophic open set B in (Y, σ) .

Proposition 3.1.3:

Every neutrosophic contra continuous mapping is a neutrosophic contra α continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic contra continuous mapping. Let A be a neutrosophic open set in Y . By hypothesis, $f^{-1}(A)$ is a neutrosophic closed set in X . Since every neutrosophic closed set is a neutrosophic α -closed set in X , f is a neutrosophic contra α continuous mapping.

Example 3.1.4:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where

$G_1 = \langle x, (0.4, 0.6), (0.1, 0.1), (0.3, 0.3) \rangle$ and $G_2 = \langle y, (0.5, 0.6), (0.2, 0.2), (0.1, 0.1) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2 = \langle y, (0.5, 0.6), (0.2, 0.2), (0.1, 0.1) \rangle$ is a neutrosophic open set in Y . Then $f^{-1}(G_2) = \langle x, (0.5, 0.6), (0.2, 0.2), (0.1, 0.1) \rangle$ is a neutrosophic α -closed set in (X, τ) , as $NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq f^{-1}(G_2)$. Therefore f is a neutrosophic contra α continuous mapping but since $f^{-1}(G_2)$ is not a neutrosophic closed set in X , f is not a neutrosophic contra continuous mapping.

Proposition 3.1.5:

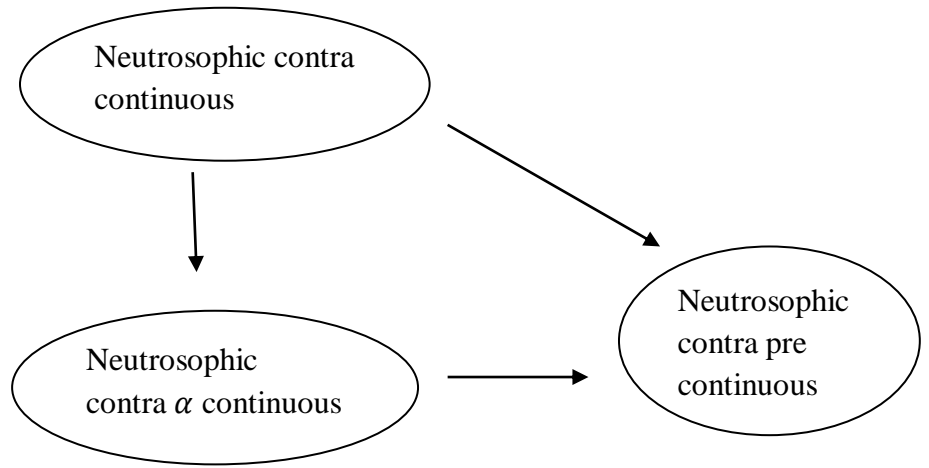
Every neutrosophic contra α -continuous mapping is a neutrosophic contra pre-continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic contra α -continuous mapping. Let A be a neutrosophic open set in Y . By hypothesis, $f^{-1}(A)$ is a neutrosophic α -closed set in X . Every neutrosophic α -closed set is a neutrosophic pre-closed set in X , f is a neutrosophic contra pre-continuous mapping.

Example 3.1.6:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.4, 0.6), (0.3, 0.3), (0.3, 0.3) \rangle$ and $G_2 = \langle y, (0.5, 0.6), (0.2, 0.2), (0.1, 0.1) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2 = \langle y, (0.5, 0.6), (0.2, 0.2), (0.1, 0.1) \rangle$ is a neutrosophic open set in Y . Then $f^{-1}(G_2) = \langle x, (0.5, 0.6), (0.2, 0.2), (0.1, 0.1) \rangle$ is a neutrosophic pre-closed in (X, τ) as $NCl(NInt(f^{-1}(G_2))) = G_1^c \subseteq f^{-1}(G_2)$. Therefore f is a neutrosophic pre-continuous mapping but since $f^{-1}(G_2)$ is not a neutrosophic α closed set in X , f is not a neutrosophic contra α -continuous mapping.



Section 3.2

Neutrosophic contra α generalized continuous mappings

In this section we have introduced neutrosophic contra α generalized mapping and studied some of its properties.

Definition 3.2.1:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic contra generalized continuous mapping if the inverse image of every neutrosophic open set in (Y, σ) is a neutrosophic generalized closed set in (X, τ) .

Definition 3.2.2:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic contra α generalized continuous mapping if $f^{-1}(B)$ is a neutrosophic α generalized closed set in (X, τ) for every neutrosophic open set B of (Y, σ) .

Example 3.2.3:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2, 0.3), (0.1, 0.1), (0.8, 0.7) \rangle$ and $G_2 = \langle y, (0.4, 0.4), (0.2, 0.2), (0.6, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2 = \langle y, (0.4, 0.4), (0.2, 0.2), (0.6, 0.6) \rangle$ is a neutrosophic open set in Y . Then $f^{-1}(G_2) = \langle x, (0.4, 0.4), (0.2, 0.2), (0.6, 0.6) \rangle$ is a neutrosophic α generalized closed set

in (X, τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha \text{Cl}(f^{-1}(G_2)) = f^{-1}(G_2) \cup \text{NCl}(\text{NInt}(\text{NCl}(f^{-1}(G_2))))$
 $= G_1^c \subseteq 1_N$. Therefore f is a neutrosophic contra α generalized continuous mapping.

Proposition 3.2.4:

Every neutrosophic contra continuous mapping is a neutrosophic contra α generalized continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic contra continuous mapping. Let A be a neutrosophic open set in Y . By hypothesis, $f^{-1}(A)$ is a neutrosophic closed set in X . Since every neutrosophic closed set is a neutrosophic α generalized closed set in X , f is a neutrosophic contra α generalized continuous mapping.

Example 3.2.5:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.1, 0.2), (0.1, 0.1), (0.7, 0.6) \rangle$ and $G_2 = \langle y, (0.3, 0.3), (0.1, 0.1), (0.5, 0.5) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2 = \langle y, (0.3, 0.3), (0.1, 0.1), (0.5, 0.5) \rangle$ is a neutrosophic open set in Y . Then $f^{-1}(G_2) = \langle x, (0.3, 0.3), (0.1, 0.1), (0.5, 0.5) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha \text{Cl}(f^{-1}(G_2)) = f^{-1}(G_2) \cup \text{NCl}(\text{NInt}(\text{NCl}(f^{-1}(G_2))))$
 $= G_1^c \subseteq 1_N$. Therefore f is a neutrosophic contra α generalized continuous mapping but since $f^{-1}(G_2)$ is not a neutrosophic closed set in X as $\text{NCl}(f^{-1}(G_2)) = G_1^c \neq f^{-1}(G_2)$, f is not a neutrosophic contra continuous mapping.

Proposition 3.2.6:

Every neutrosophic contra α continuous mapping is a neutrosophic contra α generalized continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic contra α continuous mapping. Let A be a neutrosophic open set in Y . Then by hypothesis, $f^{-1}(A)$ is a neutrosophic α

closed set in X . Since every neutrosophic α closed set is a neutrosophic α generalized closed set, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic contra α generalized continuous mapping.

Example 3.2.7:

Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2,0.3), (0.3,0.3), (0.7,0.6) \rangle$ and $G_2 = \langle y, (0.3,0.3), (0.2,0.2), (0.5,0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the neutrosophic set $G_2 = \langle y, (0.3,0.3), (0.2,0.2), (0.5,0.6) \rangle$ is a neutrosophic open set in Y . Then $f^{-1}(G_2) = \langle x, (0.3,0.3), (0.2,0.2), (0.5,0.6) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$. Therefore f is a neutrosophic contra α generalized continuous mapping but since $NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \not\subseteq f^{-1}(G_2)$ is not a neutrosophic α closed set in X , f is not a neutrosophic contra α continuous mapping.

Remark 3.2.8:

Every neutrosophic contra pre-continuous mapping and neutrosophic contra α generalized continuous mapping are independent to each other in general.

Example 3.2.9:

Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.1,0.1), (0.4,0.4), (0.7,0.3) \rangle$, $G_2 = \langle y, (0.2,0.2), (0.1,0.1), (0.6,0.3) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=u$ and $f(b) =v$. Here the neutrosophic set $G_2 = \langle y, (0.2,0.2), (0.1,0.1), (0.6,0.3) \rangle$ is a neutrosophic open set in Y . Then $f^{-1}(G_2) = \langle x, (0.2,0.2), (0.1,0.1), (0.6,0.3) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$. Therefore f is a neutrosophic contra α generalized continuous mapping. Since $NCl(NInt(f^{-1}(G_2))) = G_1^c \not\subseteq f^{-1}(G_2)$, $f^{-1}(G_2)$ is not a neutrosophic pre-closed set in X . Hence f is not a neutrosophic contra pre-continuous mapping.

Example 3.2.10:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.3, 0.4), (0.3, 0.2), (0.7, 0.6) \rangle$ and $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.7, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic contra pre continuous mapping. Since for neutrosophic open set $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.7, 0.6) \rangle$, $f^{-1}(G_2)$ is a neutrosophic pre-closed set in X as $NCl(NInt(f^{-1}(G_2))) = NCl(0_N) = 0_N \subseteq f^{-1}(G_2)$. But f is not a neutrosophic contra α generalized continuous mapping, since for a neutrosophic open set $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.7, 0.6) \rangle$ in Y , $f^{-1}(G_2)$ is not a neutrosophic α generalized closed set in X as $f^{-1}(G_2) \subseteq G_1$ where as $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \not\subseteq G_1$.

Proposition 3.2.11:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic contra α generalized continuous mapping if and only if the inverse image of each neutrosophic closed set in Y is a neutrosophic α generalized open set in X .

Proof:**Necessity:**

Let A be a neutrosophic closed set in Y . This implies A^c is a neutrosophic open set in Y . Since f is a neutrosophic contra α generalized continuous mapping, $f^{-1}(A^c)$ is a neutrosophic α generalized closed set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized open set in X .

Sufficiency:

Let A be a neutrosophic open set in Y . This implies A^c is a neutrosophic closed set in Y . By hypothesis, $f^{-1}(A^c)$ is neutrosophic α generalized open set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic contra α generalized continuous mapping.

Proposition 3.2.12:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic contra α generalized continuous mapping, then f is a neutrosophic contra continuous mapping if X is a $N_{\alpha\alpha}T_{1/2}$ space.

Proof:

Let A be a neutrosophic open set in Y . Then by hypothesis, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Since X is a $N_{\alpha\alpha}T_{1/2}$ space, $f^{-1}(A)$ is a neutrosophic closed set in X . Hence f is a neutrosophic contra continuous mapping.

Proposition 3.2.13:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic contra α generalized continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is a neutrosophic contra continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic α generalized continuous mapping.

Proof:

Let A be a neutrosophic open set in Z . Then by hypothesis, $g^{-1}(A)$ is a neutrosophic closed set in Y . Since f is a neutrosophic contra α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized open set in X . That is $(g \circ f)^{-1}(A)$ is a neutrosophic α generalized open set in X . Hence $g \circ f$ is a neutrosophic α generalized continuous mapping.

Proposition 3.2.14:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is a neutrosophic contra continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic contra α generalized continuous mapping.

Proof:

Let A be a neutrosophic open set in Z . Then $g^{-1}(A)$ is a neutrosophic closed set in Y , by hypothesis. Since f is a neutrosophic α generalized continuous

mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . That is $(g \circ f)^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence $g \circ f$ is a neutrosophic contra α generalized continuous mapping.

Proposition 3.2.15:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be two mappings. Then the following conditions are equivalent if X is a $N_{\alpha\alpha}T_{1/2}$ space:

- (i) $g \circ f$ is a neutrosophic contra α generalized continuous mapping
- (ii) $NCl(NInt(NCl((g \circ f)^{-1}(B)))) \subseteq (g \circ f)^{-1}(B)$ for every neutrosophic open set B in Z

Proof:

(i) \Rightarrow (ii) Let B be any neutrosophic open set in Z . Then $(g \circ f)^{-1}(B)$ is a neutrosophic α generalized closed set in X , by hypothesis. Since X is a $N_{\alpha\alpha}T_{1/2}$ space, $(g \circ f)^{-1}(B)$ is a neutrosophic closed set in X . Therefore, $NCl((g \circ f)^{-1}(B)) = (g \circ f)^{-1}(B)$. Now $NCl(NInt(NCl((g \circ f)^{-1}(B)))) = NCl(NInt((g \circ f)^{-1}(B))) \subseteq NCl((g \circ f)^{-1}(B)) = (g \circ f)^{-1}(B)$.

(ii) \Rightarrow (i) Let B be a neutrosophic closed set in Z . Then its complement B^c is a neutrosophic open set in Z . By hypothesis, $NCl(NInt(NCl((g \circ f)^{-1}(B^c)))) \subseteq (g \circ f)^{-1}(B^c)$. Hence $(g \circ f)^{-1}(B^c)$ is a neutrosophic α closed set in X . Since every neutrosophic α closed set is a neutrosophic α generalized closed set, $(g \circ f)^{-1}(B^c)$ is a neutrosophic α generalized closed set in X and hence $(g \circ f)^{-1}(B)$ is a neutrosophic α generalized open set in X as $(g \circ f)^{-1}(B) = ((g \circ f)^{-1}(B^c))^c$. Thus $(g \circ f)$ is a neutrosophic contra α generalized continuous mapping.

Proposition 3.2.16:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic contra α generalized continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is a neutrosophic continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic contra α generalized continuous mapping.

Proof:

Let A be a neutrosophic open set in Z . Then by hypothesis, $g^{-1}(A)$ is a neutrosophic open set in Y . Since f is a neutrosophic contra α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . That is $(g \circ f)^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence $g \circ f$ is a neutrosophic contra α generalized continuous mapping.

Proposition 3.2.17:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be any two mappings. Then the following conditions are equivalent if X is a $N_{\alpha\alpha}T_{1/2}$ space.

- (i) $g \circ f$ is a neutrosophic contra α generalized continuous mapping
- (ii) $(g \circ f)^{-1}(B) \subseteq \text{NInt}(\text{NCl}(\text{NInt}((g \circ f)^{-1}(B)))$ for each neutrosophic closed set B of Z .

Proof:

(i) \Rightarrow (ii) Let B be any neutrosophic closed set in Z . By hypothesis, $(g \circ f)^{-1}(B)$ is a neutrosophic α generalized open set in X . Since X is a $N_{\alpha\alpha}T_{1/2}$ space, $(g \circ f)^{-1}(B)$ is a neutrosophic open set in X . Therefore, $(g \circ f)^{-1}(B) = \text{NInt}((g \circ f)^{-1}(B))$. But $\text{NInt}((g \circ f)^{-1}(B)) \subseteq \text{NInt}(\text{NCl}(\text{NInt}((g \circ f)^{-1}(B))))$. This implies $(g \circ f)^{-1}(B) \subseteq \text{NInt}(\text{NCl}(\text{NInt}((g \circ f)^{-1}(B)))$ for every neutrosophic closed set B in Z .

(ii) \Rightarrow (i) Let B be any neutrosophic closed set in Z . By hypothesis, $(g \circ f)^{-1}(B) \subseteq \text{NInt}(\text{NCl}(\text{NInt}((g \circ f)^{-1}(B))))$. This implies $(g \circ f)^{-1}(B)$ is a neutrosophic α open set in X and hence $(g \circ f)^{-1}(B)$ is a neutrosophic α generalized open set in X . Therefore f is a neutrosophic contra α generalized continuous mapping.

Section 3.3

Neutrosophic contra α generalized irresolute mappings

In this section we have introduced neutrosophic contra α generalized irresolute mapping and discussed some of its properties.

Definition 3.3.1:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic contra α generalized irresolute mapping if $f^{-1}(A)$ is a neutrosophic α generalized closed set in (X, τ) for every neutrosophic α generalized open set A of (Y, σ) .

Proposition 3.3.2:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic contra α generalized irresolute mapping, then f is a neutrosophic contra α generalized continuous mapping but not conversely in general.

Proof:

Let f be a neutrosophic contra α generalized irresolute mapping. Let A be any neutrosophic open set in Y . Since every neutrosophic open set is a neutrosophic α generalized open set, A is a neutrosophic α generalized open set in Y . By hypothesis, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic contra α generalized continuous mapping.

Example 3.3.3:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2, 0.3), (0.1, 0.1), (0.5, 0.4) \rangle$ and $G_2 = \langle y, (0.6, 0.2), (0.2, 0.2), (0.1, 0.5) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic contra α generalized continuous mapping but not a neutrosophic contra α generalized irresolute mapping. Since the neutrosophic set $A = \langle y, (0.1, 0.3), (0.2, 0.2), (0.6, 0.4) \rangle$ is a neutrosophic α generalized open set in Y but $f^{-1}(A)$ is not a neutrosophic α generalized closed set in X as

$f^{-1}(A) = \langle x, (0.1, 0.3), (0.2, 0.2), (0.6, 0.4) \rangle \subseteq G_1$ but $N_{\alpha}Cl(f^{-1}(A)) = f^{-1}(A) \cup NCl(NInt(NCl(f^{-1}(A)))) = G_1^c \not\subseteq G_1$.

Proposition 3.3.4:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \delta)$ are neutrosophic contra α generalized irresolute mapping then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic α generalized irresolute mapping.

Proof:

Let A be a neutrosophic α generalized open set in Z . Then $g^{-1}(A)$ is a neutrosophic α generalized closed set in Y . Since f is a neutrosophic contra α generalized irresolute mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized open set in X . That is $(g \circ f)^{-1}(A)$ is a neutrosophic α generalized open set in X . Hence $g \circ f$ is a neutrosophic α generalized irresolute mapping.

Proposition 3.3.5:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic contra α generalized irresolute mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is a neutrosophic contra α generalized continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic α generalized continuous mapping.

Proof:

Let A be a neutrosophic open set in Z . Then by hypothesis, $g^{-1}(A)$ is a neutrosophic α generalized closed set in Y . Since f is a neutrosophic contra α generalized irresolute mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized open set in X . That is $(g \circ f)^{-1}(A)$ is a neutrosophic α generalized open set in X . Hence $g \circ f$ is a neutrosophic α generalized continuous mapping.

CHAPTER 4

Section 4.1

Neutrosophic almost α generalized continuous mappings

In this section we have introduced neutrosophic almost α generalized continuous mappings and investigated some of their properties.

Definition 4.1.1:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be a neutrosophic almost α generalized continuous mapping if $f^{-1}(A)$ is a neutrosophic α generalized closed set in (X, τ) for every neutrosophic regular closed set A in (Y, σ) .

Example 4.1.2:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2, 0.3), (0.1, 0.1), (0.8, 0.7) \rangle$ and $G_2 = \langle y, (0.4, 0.4), (0.2, 0.2), (0.6, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic almost α generalized continuous mapping. For, consider the neutrosophic set $G_2^c = \langle y, (0.6, 0.6), (0.2, 0.2), (0.4, 0.4) \rangle$ which is a neutrosophic regular closed set in Y , since $NCl(NInt(G_2^c)) = NCl(G_2) = G_2^c$. We have $f^{-1}(G_2^c) = \langle x, (0.6, 0.6), (0.2, 0.2), (0.4, 0.4) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2^c) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \subseteq 1_N$.

Proposition 4.1.3:

Every neutrosophic continuous mapping is a neutrosophic almost α generalized continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic continuous mapping. Let A be a neutrosophic regular closed set in Y . Since every neutrosophic regular closed set is

a neutrosophic closed set, A is a neutrosophic closed set in Y . Then $f^{-1}(A)$ is a neutrosophic closed set in X , by hypothesis. Since every neutrosophic closed set is a neutrosophic α generalized closed set, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic almost α generalized continuous mapping.

Example 4.1.4:

Let $X=\{a,b\}$ and $Y=\{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.1, 0.2), (0.1, 0.1), (0.7, 0.6) \rangle$ and $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.5, 0.5) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic almost α generalized continuous mapping but not a neutrosophic continuous mapping. For, consider the neutrosophic set $G_2^c = \langle y, (0.5, 0.5), (0.2, 0.2), (0.3, 0.3) \rangle$ which is a neutrosophic regular closed set in Y as $NCl(NInt(G_2^c)) = NCl(G_2) = G_2^c$. We have $f^{-1}(G_2^c) = \langle x, (0.5, 0.5), (0.2, 0.2), (0.3, 0.3) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2^c) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \subseteq 1_N$. Therefore f is a neutrosophic almost α generalized continuous mapping. Now, consider the neutrosophic closed set $G_2^c = \langle y, (0.5, 0.5), (0.2, 0.2), (0.3, 0.3) \rangle$ in Y , where $f^{-1}(G_2^c)$ is not a neutrosophic closed set in X as $NCl(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$. Therefore, f is not a neutrosophic continuous mapping.

Proposition 4.1.5:

Every neutrosophic α continuous mapping is a neutrosophic almost α generalized continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α continuous mapping. Let A be a neutrosophic regular closed set in Y . Since every neutrosophic regular closed set is a neutrosophic closed set, A is a neutrosophic closed set in Y . Then $f^{-1}(A)$ is a neutrosophic α closed set in X , by hypothesis. Since every neutrosophic α closed

set is a neutrosophic α generalized closed set, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic almost α generalized continuous mapping.

Example 4.1.6:

Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2,0.3), (0.3,0.3), (0.7,0.6) \rangle$ and $G_2 = \langle y, (0.3,0.3), (0.2,0.2), (0.5,0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic almost α generalized continuous mapping but not a neutrosophic α continuous mapping. For, consider the neutrosophic set $G_2^c = \langle y, (0.5,0.6), (0.2,0.2), (0.3,0.3) \rangle$ which is a neutrosophic regular closed set in Y , as $NCl(NInt(G_2^c)) = NCl(G_2) = G_2^c$. We have $f^{-1}(G_2^c) = \langle x, (0.5,0.6), (0.2,0.2), (0.3,0.3) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2^c) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \subseteq 1_N$. Therefore, f is a neutrosophic almost α generalized continuous mapping. Now, consider the neutrosophic closed set $G_2^c = \langle y, (0.5,0.6), (0.2,0.2), (0.3,0.3) \rangle$ in Y , where $f^{-1}(G_2^c)$ is not a neutrosophic α closed set in X as $NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \not\subseteq f^{-1}(G_2^c)$. Therefore, f is not a neutrosophic α continuous mapping.

Proposition 4.1.7:

Every neutrosophic almost continuous mapping is a neutrosophic almost α generalized continuous mapping but not conversely in general.

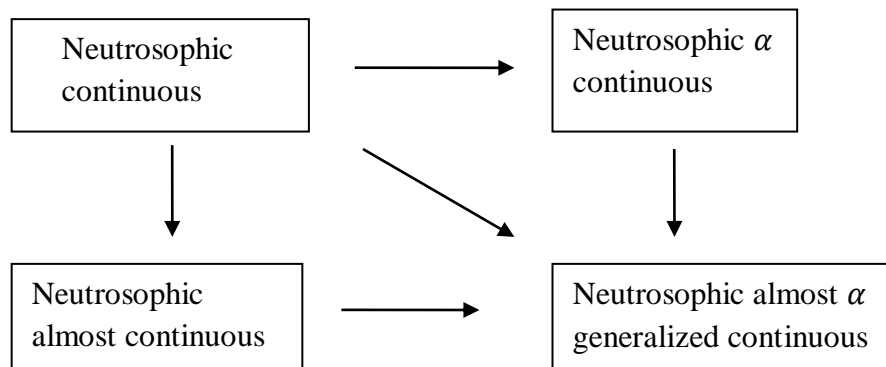
Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic almost continuous mapping. Let A be a neutrosophic regular closed set in Y . Since f is a neutrosophic almost continuous mapping, $f^{-1}(A)$ is a neutrosophic closed set in X . Since every neutrosophic closed set is a neutrosophic α generalized closed set, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic almost α generalized continuous mapping.

Example 4.1.8:

Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.3,0.2), (0.3,0.3), (0.6,0.7) \rangle$ and $G_2 = \langle y, (0.4,0.2), (0.2,0.2), (0.5,0.4) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic almost α generalized continuous mapping but not a neutrosophic almost continuous mapping. For, consider the neutrosophic set $G_2^c = \langle y, (0.5,0.4), (0.2,0.2), (0.4,0.2) \rangle$ which is a neutrosophic regular closed set in Y , as $NCl(NInt(G_2^c)) = NCl(G_2) = G_2^c$. We have $f^{-1}(G_2^c) = \langle x, (0.5,0.4), (0.2,0.2), (0.4,0.2) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2^c) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \cup NCl(NInt(NCl(f^{-1}(G_2^c)))) = G_1^c \subseteq 1_N$. Therefore, f is a neutrosophic almost α generalized continuous mapping. Now, consider the neutrosophic closed set $G_2^c = \langle y, (0.5,0.4), (0.2,0.2), (0.4,0.2) \rangle$ in Y , where $f^{-1}(G_2^c)$ is not a neutrosophic closed set in X as $NCl(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$. Therefore, f is not a neutrosophic almost continuous mapping.

The relation between various types of neutrosophic continuity is given in the following diagram.



Proposition 4.1.9:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized irresolute mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be a neutrosophic almost α generalized continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic almost α generalized continuous mapping.

Proof:

Let A be a neutrosophic regular closed set in Z . Since g is a neutrosophic almost α generalized continuous mapping, $g^{-1}(A)$ is a neutrosophic α generalized closed set in Y . Since f is a neutrosophic α generalized irresolute mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . Since $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$, $g \circ f$ is a neutrosophic almost α generalized continuous mapping.

Proposition 4.1.10:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be a neutrosophic almost continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic almost α generalized continuous mapping.

Proof:

Let A be a neutrosophic regular closed set in Z . Since g is a neutrosophic almost continuous mapping, $g^{-1}(A)$ is a neutrosophic closed set in Y . Since f is a neutrosophic α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . Since $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$, $g \circ f$ is a neutrosophic almost α generalized continuous mapping.

Proposition 4.1.11:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be a neutrosophic almost continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic almost α generalized continuous mapping.

Proof:

Let A be a neutrosophic regular closed set in Z . Since g is a neutrosophic almost continuous mapping, $g^{-1}(A)$ is a neutrosophic closed set in Y . Since f is a neutrosophic continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic closed set in X . Since every neutrosophic closed set is a neutrosophic α generalized closed set, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . Since $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$, $g \circ f$ is a neutrosophic almost α generalized continuous mapping.

Proposition 4.1.12:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be a neutrosophic almost continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic almost α generalized continuous mapping.

Proof:

Let A be a neutrosophic regular closed set in Z . Since g is a neutrosophic almost continuous mapping, $g^{-1}(A)$ is a neutrosophic closed set in Y . Since f is a neutrosophic α continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α closed set in X , $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . Since $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$, $g \circ f$ is a neutrosophic almost α generalized continuous mapping.

Proposition 4.1.13:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic almost α generalized continuous mapping if and only if the inverse image of each neutrosophic regular open set in Y is a neutrosophic α generalized open set in X .

Proof:**Necessity:**

Let A be a neutrosophic regular open set in Y . This implies A^c is a neutrosophic regular closed set in Y . Since f is a neutrosophic almost α generalized continuous, $f^{-1}(A^c)$ is a neutrosophic α generalized closed set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized open set in X .

Sufficiency:

Let A be a neutrosophic regular closed set in Y . This implies A^c is a neutrosophic regular open set in Y . By hypothesis, $f^{-1}(A^c)$ is a neutrosophic α generalized open set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic almost α generalized continuous mapping.

Proposition 4.1.14:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from a neutrosophic topological space X into a neutrosophic topological space Y . Then the following conditions are equivalent if X is a $N_{\alpha\alpha}T_{1/2}$ space:

- (i) f is a neutrosophic almost α generalized continuous mapping
- (ii) $f^{-1}(B)$ is a neutrosophic α generalized open set in X for every neutrosophic regular open set B in Y
- (iii) $f^{-1}(NInt(B)) \subseteq NInt(NCl(NInt(f^{-1}(B))))$ for every neutrosophic regular open set B in Y

Proof:

(i) \Rightarrow (ii) is obviously true.

(ii) \Rightarrow (iii) Let B be any neutrosophic regular open set in Y . Then $NInt(B)$ is a neutrosophic open set in Y . By hypothesis, $f^{-1}(NInt(B))$ is a neutrosophic α generalized open set in X . Since X is a $N_{\alpha\alpha}T_{1/2}$ space, $f^{-1}(NInt(B))$ is a neutrosophic open set in X . Therefore, $f^{-1}(NInt(B)) = NInt(f^{-1}(NInt(B))) \subseteq NInt(NCl(NInt(f^{-1}(B))))$.

(iii) \Rightarrow (i) Let B be a neutrosophic regular closed set in Y . Then its complement B^c is a neutrosophic regular open set in Y . By hypothesis, $f^{-1}(NInt(B^c)) \subseteq NInt(NCl(NInt(f^{-1}(B^c))))$. This implies $f^{-1}(B^c) = f^{-1}(NInt(B^c)) \subseteq NInt(NCl(NInt(f^{-1}(B^c))))$. Hence $f^{-1}(B^c)$ is a neutrosophic α open set in X . Since every neutrosophic α open set is a neutrosophic α generalized open set, $f^{-1}(B^c)$ is a neutrosophic α generalized open set in X . Therefore, $f^{-1}(B)$ is a neutrosophic α

generalized closed set in X as $f^{-1}(B^c) = (f^{-1}(B))^c$. Hence f is a neutrosophic almost α generalized continuous mapping.

Proposition 4.1.15:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from a neutrosophic topological space X into a neutrosophic topological space Y . Then the following conditions are equivalent if X is a $N_{\alpha\alpha}T_{1/2}$ space:

- (i) f is a neutrosophic almost α generalized continuous mapping
- (ii) $f^{-1}(B)$ is a neutrosophic α generalized closed set in X , for every neutrosophic closed set B in Y
- (iii) $NCl(NInt(NCl(f^{-1}(A)))) \subseteq f^{-1}(NCl(A))$ for every neutrosophic regular closed set B in Y

Proof:

(i) \Rightarrow (ii) is obviously true.

(ii) \Rightarrow (iii) Let A be any neutrosophic regular closed set in Y . Then $NCl(A)$ is a neutrosophic closed set in Y . By hypothesis, $f^{-1}(NCl(A))$ is a neutrosophic α generalized closed set in X . since X is a $N_{\alpha\alpha}T_{1/2}$ space, $f^{-1}(NCl(A))$ is a neutrosophic closed set in X . Therefore $NCl(f^{-1}(NCl(A))) = f^{-1}(NCl(A))$. Now $NCl(NInt(NCl(f^{-1}(A)))) \subseteq NCl(NCl(f^{-1}(NCl(A)))) = NCl(f^{-1}(NCl(A))) \subseteq f^{-1}(NCl(A))$.

(iii) \Rightarrow (i) Let A be a neutrosophic regular closed set in Y . Then by hypothesis $NCl(NInt(NCl(f^{-1}(A)))) \subseteq f^{-1}(NCl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is a neutrosophic α closed set in X and hence it is a neutrosophic α generalized closed set in X . Therefore f is a neutrosophic almost α generalized continuous mapping.

Proposition 4.1.16:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping . If $f^{-1}(N_{\alpha}\text{Int}(B)) \subseteq N_{\alpha}\text{Int}(f^{-1}(B))$ for every neutrosophic set B in Y , then f is a neutrosophic almost α generalized continuous mapping.

Proof:

Let B be a neutrosophic regular open set in Y . By hypothesis, $f^{-1}(N_{\alpha}\text{Int}(B)) \subseteq N_{\alpha}\text{Int}(f^{-1}(B))$. Since B is a neutrosophic regular open set, it is a neutrosophic α open set in Y . Therefore, $N_{\alpha}\text{Int}(B) = B$. Hence $f^{-1}(B) = f^{-1}(N_{\alpha}\text{Int}(B)) \subseteq N_{\alpha}\text{Int}(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B) = N_{\alpha}\text{Int}(f^{-1}(B))$. This implies $f^{-1}(B)$ is a neutrosophic α open set in X and hence $f^{-1}(B)$ is a neutrosophic α generalized open set in X . Thus f is a neutrosophic almost α generalized continuous mapping.

Proposition 4.1.17:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping . If $N_{\alpha}\text{Cl}(f^{-1}(B)) \subseteq f^{-1}(N_{\alpha}\text{Cl}(B))$ for every neutrosophic set B in Y , then f is a neutrosophic almost α generalized continuous mapping.

Proof:

Let B be a neutrosophic regular closed set in Y . By hypothesis, $N_{\alpha}\text{Cl}(f^{-1}(B)) \subseteq f^{-1}(N_{\alpha}\text{Cl}(B))$. Since B is a neutrosophic regular closed set, it is a neutrosophic α closed set in Y . Therefore, $N_{\alpha}\text{Cl}(B) = B$. Hence $f^{-1}(B) = f^{-1}(N_{\alpha}\text{Cl}(B)) \supseteq N_{\alpha}\text{Cl}(f^{-1}(B)) \supseteq f^{-1}(B)$. Therefore $f^{-1}(B) = N_{\alpha}\text{Cl}(f^{-1}(B))$. This implies $f^{-1}(B)$ is a neutrosophic α closed set in X and hence $f^{-1}(B)$ is a neutrosophic α generalized closed set in X . Thus f is a neutrosophic almost α generalized continuous mapping.

Section 4.2

Neutrosophic almost contra α generalized continuous mappings

In this section we have introduced almost contra α generalized continuous mappings and studied some of their properties.

Definition 4.2.1:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic almost contra continuous mapping if $f^{-1}(A)$ is a neutrosophic closed set in (X, τ) for every neutrosophic regular open set A of (Y, σ) .

Definition 4.2.2:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic almost contra generalized continuous mapping if $f^{-1}(A)$ is a neutrosophic generalized closed set in (X, τ) for every neutrosophic regular open set A of (Y, σ) .

Definition 4.2.3:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic almost contra α generalized continuous mapping if $f^{-1}(A)$ is a neutrosophic α generalized closed set in (X, τ) for every neutrosophic regular open set A of (Y, σ) .

Example 4.2.4:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2, 0.3), (0.1, 0.1), (0.8, 0.7) \rangle$ and $G_2 = \langle y, (0.4, 0.4), (0.2, 0.2), (0.6, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic almost contra α generalized continuous mapping. For, consider the neutrosophic set $G_2 = \langle y, (0.4, 0.4), (0.2, 0.2), (0.6, 0.6) \rangle$ is a neutrosophic regular open set in Y , since $NInt(NCl(G_2)) = NInt(G_2^c) = G_2$. We have, $f^{-1}(G_2) = \langle x, (0.4, 0.4), (0.2, 0.2), (0.6, 0.6) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$.

Proposition 4.2.5:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic almost contra α generalized continuous if and only if the inverse image of each neutrosophic regular closed set in Y is a neutrosophic α generalized open set in X .

Proof:**Necessary:**

Let A be a neutrosophic regular closed set in Y . This implies A^c is a neutrosophic regular open set in Y . Since f is a neutrosophic almost contra α generalized continuous mapping, $f^{-1}(A^c)$ is a neutrosophic α generalized closed set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized open set in X .

Sufficiency:

Let A be a neutrosophic regular open set in Y . This implies A^c is a neutrosophic regular closed set in Y . By hypothesis, $f^{-1}(A^c)$ is neutrosophic α generalized open set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic almost contra α generalized continuous mapping.

Proposition 4.2.6:

Every neutrosophic contra continuous mapping is a neutrosophic almost contra α generalized continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic contra continuous mapping. Let A be a neutrosophic regular open set in Y . This implies A is a neutrosophic open set in Y . Since f is a neutrosophic contra continuous mapping, $f^{-1}(A)$ is a neutrosophic closed set in X . Since every neutrosophic closed set is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic almost contra α generalized continuous mapping.

Example 4.2.7:

Let $X=\{a,b\}$ and $Y=\{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.1, 0.2), (0.1, 0.1), (0.7, 0.6) \rangle$ and $G_2 = \langle y, (0.3, 0.3), (0.1, 0.1), (0.5, 0.5) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic almost contra α generalized continuous mapping but not a neutrosophic contra continuous mapping. For, consider the neutrosophic set $G_2 = \langle y, (0.3, 0.3), (0.1, 0.1), (0.5, 0.5) \rangle$ is a neutrosophic regular open set in Y as $NInt(NCl(G_2)) = NInt(G_2^c) = G_2$. We have $f^{-1}(G_2) = \langle x, (0.3, 0.3), (0.1, 0.1), (0.5, 0.5) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$. Therefore, f is a neutrosophic almost contra α generalized continuous mapping. Now, consider the neutrosophic open set $G_2 = \langle y, (0.3, 0.3), (0.1, 0.1), (0.5, 0.5) \rangle$ in Y , where $f^{-1}(G_2)$ is not a neutrosophic closed set in X as $NCl(f^{-1}(G_2)) = G_1^c \neq f^{-1}(G_2)$. Therefore, f is not a neutrosophic contra continuous mapping.

Proposition 4.2.8:

Every neutrosophic contra α continuous mapping is a neutrosophic almost contra α generalized continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic contra α continuous mapping. Let A be a neutrosophic regular open set in Y . This implies neutrosophic open set in Y . Then by hypothesis, $f^{-1}(A)$ is a neutrosophic α closed set in X . Since every neutrosophic α closed set is a neutrosophic α generalized closed set, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic almost contra α generalized continuous mapping.

Example 4.2.9:

Let $X=\{a,b\}$ and $Y=\{u,v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2, 0.3), (0.3, 0.3), (0.7, 0.6) \rangle$ and $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.5, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic almost contra α generalized continuous mapping but not a neutrosophic contra α continuous mapping. For, consider the neutrosophic set $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.5, 0.6) \rangle$ is a neutrosophic regular open set in Y as $NInt(NCl(G_2)) = NInt(G_2^c) = G_2$. We have $f^{-1}(G_2) = \langle x, (0.3, 0.3), (0.2, 0.2), (0.5, 0.6) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$. Therefore f is a neutrosophic almost contra α generalized continuous mapping. Now, consider the neutrosophic open set $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.5, 0.6) \rangle$ in Y , $f^{-1}(G_2)$ is not a neutrosophic α closed set in X as $NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \not\subseteq f^{-1}(G_2)$. Therefore f is not a neutrosophic contra α continuous mapping.

Remark 4.2.10:

Every neutrosophic contra pre continuous mapping and neutrosophic almost contra α generalized continuous mapping are independent to each other in general.

Example 4.2.11:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.1, 0.1), (0.4, 0.4), (0.7, 0.3) \rangle$ and $G_2 = \langle y, (0.2, 0.2), (0.1, 0.1), (0.6, 0.3) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic almost contra α generalized continuous mapping but not a neutrosophic contra pre continuous mapping. For, consider the neutrosophic set $G_2 = \langle y, (0.2, 0.2), (0.1, 0.1), (0.6, 0.3) \rangle$ is a neutrosophic regular open set in Y as $NInt(NCl(G_2)) = NInt(G_2^c) = G_2$. We have $f^{-1}(G_2) = \langle x, (0.2, 0.2), (0.1, 0.1), (0.6, 0.3) \rangle$ is a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$. Therefore f is a

neutrosophic almost contra α generalized continuous mapping. Now, consider the neutrosophic open set $G_2 = \langle y, (0.2, 0.2), (0.1, 0.1), (0.6, 0.3) \rangle$ in Y , $f^{-1}(G_2)$ is not a neutrosophic pre closed set in X as $NCl(NInt(f^{-1}(G_2))) = G_1^c \not\subseteq f^{-1}(G_2)$. Therefore f is not a neutrosophic contra pre continuous mapping.

Example 4.2.12:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.3, 0.4), (0.3, 0.2), (0.7, 0.6) \rangle$ and $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.7, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic contra pre continuous mapping but not a neutrosophic almost contra α generalized continuous mapping. For, consider the neutrosophic set $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.7, 0.6) \rangle$ is a neutrosophic regular open set in Y as $NInt(NCl(G_2)) = NInt(G_2^c) = G_2$. We have $f^{-1}(G_2) = \langle x, (0.3, 0.3), (0.2, 0.2), (0.7, 0.6) \rangle$ is not a neutrosophic α generalized closed set in (X, τ) as $f^{-1}(G_2) \subseteq G_1$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \not\subseteq G_1$. Therefore f is not a neutrosophic almost contra α generalized continuous mapping. Now, consider the neutrosophic open set $G_2 = \langle y, (0.3, 0.3), (0.2, 0.2), (0.7, 0.6) \rangle$ in Y , where $f^{-1}(G_2)$ is a neutrosophic pre closed set in X as $NCl(NInt(f^{-1}(G_2))) = NCl(0_N) = 0_N \subseteq f^{-1}(G_2^c)$. Therefore, f is a neutrosophic contra pre continuous mapping.

Proposition 4.2.13:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic almost contra α generalized continuous mapping, then f is a neutrosophic almost contra generalized continuous mapping if X is a $N_{\alpha b}T_{1/2}$ space.

Proof:

Let A be a neutrosophic regular open set in Y . Then $f^{-1}(A)$ is a neutrosophic α generalized closed set in X , by hypothesis. Since X is a $N_{\alpha b}T_{1/2}$ space, $f^{-1}(A)$ is a neutrosophic generalized closed set in X . Hence f is a neutrosophic almost contra generalized continuous mapping.

Proposition 4.2.14:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic almost contra α generalized continuous mapping, then f is a neutrosophic almost contra continuous mapping if X is a $N_{\alpha\alpha}T_{1/2}$ space.

Proof:

Let A be a neutrosophic regular open set in Y . Then $f^{-1}(A)$ is a neutrosophic α generalized closed set in X , by hypothesis. Since X is a $N_{\alpha\alpha}T_{1/2}$ space, $f^{-1}(A)$ is a neutrosophic closed in X . Hence f is a neutrosophic almost contra continuous mapping.

Proposition 4.2.15:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic α generalized continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be a neutrosophic almost contra continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic almost contra α generalized continuous mapping.

Proof:

Let A be a neutrosophic regular open set in Z . Then $g^{-1}(A)$ is a neutrosophic closed set in Y , by hypothesis. Since f is a neutrosophic α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X . Hence $g \circ f$ is a neutrosophic almost contra α generalized continuous mapping.

Section 4.3

Neutrosophic completely α generalized continuous mappings

In this section we have introduced neutrosophic completely α generalized continuous mappings and studied some of their properties.

Definition 4.3.1:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic completely α generalized continuous mapping if $f^{-1}(A)$ is a neutrosophic regular closed set in (X, τ) for every neutrosophic α generalized closed set A of (Y, σ) .

Proposition 4.3.2:

Every neutrosophic completely α generalized continuous mapping is a neutrosophic α generalized continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic completely α generalized continuous mapping. Let A be a neutrosophic closed set in Y . Since every neutrosophic closed set is a neutrosophic α generalized closed set, A is a neutrosophic α generalized closed set in Y . Then $f^{-1}(A)$ is a neutrosophic regular closed set in X . Since every neutrosophic regular closed set is a neutrosophic α generalized closed set in X , $f^{-1}(A)$ is a neutrosophic α generalized closed set in X . Hence f is a neutrosophic α generalized continuous mapping.

Example 4.3.3:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.2, 0.3), (0.2, 0.2), (0.7, 0.6) \rangle$ and $G_2 = \langle y, (0.4, 0.4), (0.1, 0.1), (0.5, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic α generalized continuous mapping but not a neutrosophic completely α generalized continuous mapping, since G_2^c is a neutrosophic α generalized closed set in Y , but

$f^{-1}(G_2^c) = \langle x, (0.5, 0.6), (0.1, 0.1), (0.4, 0.4) \rangle$ is not a neutrosophic regular closed set in X as $NCl(NInt(f^{-1}(G_2^c))) = NCl(G_1) = G_1^c \neq f^{-1}(G_2^c)$.

Proposition 4.3.4:

Every neutrosophic completely α generalized continuous mapping is a neutrosophic continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic completely α generalized continuous mapping. Let A be a neutrosophic closed set in Y . Since every neutrosophic closed set is a neutrosophic α generalized closed set, A is a neutrosophic α generalized closed set in Y . Then $f^{-1}(A)$ is a neutrosophic regular closed set in X . Since every neutrosophic regular closed set is a neutrosophic closed set, $f^{-1}(A)$ is a neutrosophic closed set in X . Hence f is a neutrosophic continuous mapping.

Example 4.3.5:

Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, G_1, G_2, 1_N\}$ and $\sigma = \{0_N, G_3, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.5, 0.4), (0.2, 0.2), (0.5, 0.6) \rangle$, $G_2 = \langle x, (0.2, 0.3), (0.2, 0.2), (0.8, 0.7) \rangle$ and $G_3 = \langle y, (0.2, 0.3), (0.2, 0.2), (0.8, 0.7) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic continuous mapping but not a neutrosophic completely α generalized continuous mapping, since G_3^c is a neutrosophic α generalized closed set in Y , but $f^{-1}(G_3^c) = \langle x, (0.8, 0.7), (0.2, 0.2), (0.2, 0.3) \rangle$ is not a neutrosophic regular closed set in X as $NCl(NInt(f^{-1}(G_3^c))) = NCl(G_1) = G_1^c \neq f^{-1}(G_3^c)$.

Proposition 4.3.6:

Every neutrosophic completely α generalized continuous mapping is a neutrosophic α continuous mapping but not conversely in general.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic completely α generalized continuous mapping. Let A be a neutrosophic closed set in Y . Since neutrosophic closed set is a neutrosophic α generalized closed set, A is a neutrosophic α generalized closed set in Y . Then $f^{-1}(A)$ is a neutrosophic regular closed set in X . Since every neutrosophic regular closed set is a neutrosophic α closed set, $f^{-1}(A)$ is a neutrosophic α closed set in X . Hence f is a neutrosophic α continuous mapping.

Example 4.3.7:

Let $X=\{a,b\}$ and $Y=\{u,v\}$. Then $\tau = \{0_N, G_1, G_2, 1_N\}$ and $\sigma = \{0_N, G_3, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x, (0.5, 0.4), (0.2, 0.2), (0.5, 0.6) \rangle$, $G_2 = \langle x, (0.2, 0.3), (0.2, 0.2), (0.8, 0.7) \rangle$ and $G_3 = \langle y, (0.2, 0.3), (0.2, 0.2), (0.8, 0.7) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a neutrosophic α continuous mapping but not a neutrosophic completely α generalized continuous mapping, since G_3^c is a neutrosophic α generalized closed set in Y , but $f^{-1}(G_3^c) = \langle x, (0.8, 0.7), (0.2, 0.2), (0.2, 0.3) \rangle$ is not a neutrosophic regular closed set in X as $NCl(NInt(f^{-1}(G_3^c))) = NCl(G_1) = G_1^c \neq f^{-1}(G_3^c)$.

Proposition 4.3.8:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic completely α generalized continuous mapping then f is a neutrosophic α continuous mapping.

Proof:

Let A be a neutrosophic closed set in Y . Since every closed set is a neutrosophic α generalized closed set, A is a neutrosophic α generalized closed set in Y . By hypothesis, $f^{-1}(A)$ is a neutrosophic regular closed set and hence $f^{-1}(A)$ is a neutrosophic α closed set in X . Then f is a neutrosophic α continuous mapping.

Proposition 4.3.9:

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic completely α generalized continuous mapping if and only if the inverse image of each neutrosophic α generalized open set in Y is a neutrosophic regular open set in X .

Proof:**Necessity:**

Let A be a neutrosophic α generalized open set in Y . Then A^c is a neutrosophic α generalized closed set in Y . Since f is neutrosophic completely α generalized continuous mapping, $f^{-1}(A^c)$ is neutrosophic regular closed set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic regular open set in X .

Sufficiency:

Let A be a neutrosophic α generalized closed set in Y . This implies A^c is neutrosophic α generalized open set in Y . By hypothesis, $f^{-1}(A^c)$ is a neutrosophic regular open set in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic regular closed set in X . Hence f is a neutrosophic completely α generalized continuous mapping.

Proposition 4.3.10:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic completely α generalized continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is a neutrosophic α generalized irresolute mapping, $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic completely α generalized continuous mapping.

Proof:

Let A be a neutrosophic α generalized open set in Z . Since g is a neutrosophic α generalized irresolute mapping, $g^{-1}(A)$ is a neutrosophic α generalized open set in Y . Since f is a neutrosophic completely α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic regular open set in X . Since

$(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$, $g \circ f$ is a neutrosophic completely α generalized continuous mapping.

Proposition 4.3.11:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic completely α generalized continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is a neutrosophic α generalized continuous mapping, $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic α generalized continuous mapping.

Proof:

Let A be a neutrosophic open set in Z . Since g is a neutrosophic α generalized continuous mapping, $g^{-1}(A)$ is a neutrosophic α generalized open set in Y . Since f is a neutrosophic completely α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic regular open set in X . Hence $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized open set in X as $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$, $g \circ f$ is a neutrosophic α generalized continuous mapping.

Proposition 4.3.12:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is a neutrosophic completely α generalized continuous mapping, $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is a neutrosophic α generalized irresolute mapping.

Proof:

Let A be a neutrosophic α generalized open set in Z . Since g is a neutrosophic completely α generalized continuous mapping, $g^{-1}(A)$ is a neutrosophic regular open set in Y . Since neutrosophic regular open set is a neutrosophic open set, $g^{-1}(A)$ is a neutrosophic open set in Y . Since f is a neutrosophic continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic open set in X and hence $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized open set in X as $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$. Then $g \circ f$ is a neutrosophic α generalized irresolute mapping.

Summary and conclusion

In **Chapter 1**, the preliminary definitions are discussed.

In **Chapter 2**, neutrosophic α generalized continuous mappings and neutrosophic α generalized irresolute mappings are introduced and their basic properties are investigated.

In **Chapter 3**, neutrosophic contra α continuous mappings, neutrosophic contra α generalized continuous mappings and neutrosophic contra α generalized irresolute mappings are introduced and their characterizations are obtained.

In **Chapter 4**, neutrosophic almost α generalized continuous mappings, neutrosophic almost contra α generalized continuous mappings and neutrosophic completely α generalized continuous mappings are introduced and their basic properties are obtained.

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