Chapter V

CHAPTER – V

CIRCULANT TRAPEZOIDAL FUZZY NUMBER MATRICES

Definition : 5.1

A **circulant trapezoidal fuzzy number matrix** is defined as follows

$$\begin{bmatrix} <a_1^{I}, a_1^{II}, a_1^{III}, a_1^{iv}> & <a_2^{I}, a_2^{II}, a_2^{III}, a_2^{iv}> & \ldots\ldots\ldots\ldots & <a_n^{I}, a_n^{II}, a_n^{III}, a_n^{iv}> \\ <a_n^{I}, a_n^{II}, a_n^{III}, a_n^{iv}> & <a_1^{I}, a_1^{II}, a_1^{III}, a_1^{iv}> & \ldots\ldots\ldots\ldots & <a_{n-1}^{I}, a_{n-1}^{II}, a_{n-1}^{III}, a_{n-1}^{iv}> \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots \\ <a_2^{I}, a_2^{II}, a_2^{III}, a_2^{iv}> & <a_3^{I}, a_3^{II}, a_3^{III}, a_3^{iv}> & \ldots\ldots\ldots\ldots & <a_1^{I}, a_1^{II}, a_1^{III}, a_1^{iv}> \end{bmatrix}$$

Theorem : 5.2

Adj A is also circulant,for a circulant trapezoidal fuzzy number matrix A.

Proof

Let adj A = B

If $\ell \in n_j$ iff $k \oplus \ell \in n_{k \oplus j}$ $\forall$ i, j, k $\in$ {1, 2, ..., n}

$n_{k \oplus j} = k \oplus n_j$ where $k \oplus n_j = \{ k \oplus 1, k \oplus 2, \ldots, k \oplus n / k \oplus j\}$

Define $\phi : S_{njni} \to S_{n(k \oplus j)\, n(k \oplus i)}$ defined by $\phi(\pi) = k \oplus \pi = \sigma$ is an isomorphism.

$$b_{(k \oplus i)(k \oplus j)} = \bigvee_{\sigma \in S_{n(k \oplus j) n(k \oplus i)}} \left[\bigwedge_{k \oplus \ell \in n_{k \oplus j}} a_{(k \oplus \ell)\, \sigma(k \oplus \ell)} \right]$$

$$= \bigvee_{\pi \in S_{njni}} \left[\bigwedge_{\ell \in n_j} a_1\, \pi(\ell) \right]$$

$$= b_{ij}$$

Thus B = adj A is circulant.

Theorem : 5.3

A circulant trapezoidal fuzzy number matrix A is symmetric iff $\tilde{A}_{1j} = \tilde{A}_{1(n-i+2)}$ for every i $\in$ {1, 2, ..., n}.

Proof

Let the circulant trapezoidal fuzzy matrix A be symmetric then

$$\tilde{A}_{1i} = \tilde{A}_{(1\oplus k)(i\oplus k)}$$

$$= \tilde{A}_{i1}$$

$$= \tilde{A}_{(i\oplus k)(1\oplus k)} \text{ for every } i, k \in \{1, 2, \ldots, n\}$$

take $k = n - i$

$$\tilde{A}_{1i} = \tilde{A}_{(i\oplus n-i)(1\oplus n-i)}$$

$$= \tilde{A}_{n(1+n-i)}$$

$$= \tilde{A}_{1(n+2-i)}$$

Conversely let $\tilde{A}_{1i} = \tilde{A}_{1(n-i+2)}$ for every $i \in \{1, 2, \ldots, n\}$.

$$\tilde{A}_{i1} = \tilde{A}_{(i\oplus k)(1\oplus k)} \text{ for every } i, k \in \{1, 2, \ldots, n\}.$$

take $k = n - i$

$$\tilde{A}_{i1} = \tilde{A}_{(i\oplus n-i)(1\oplus n-i)}$$

$$= \tilde{A}_{n(n-i+1)}$$

$$= \tilde{A}_{1(n-i+2)}$$

$$= \tilde{A}_{1i}$$

$\therefore$ A is symmetric.

Theorem : 5.4

A adj A is idempotent for a circulant trapezoidal fuzzy number matrix A.

Proof

Let C = A adj A, since C is transitive then $c_{ij}^{(2)} \leq c_{ij}$ for every $i, j \in \{1, 2, \ldots, n\}$.

$$c_{ij}^{(2)} = \bigvee_{k=1}^{n} (c_{ik} \wedge c_{kj}) \geq c_{ii} \wedge c_{ij} = c_{ij} \ \forall\, i, j \in \{1, 2, \ldots, n\}$$

where $\vee$, $\wedge$ are fuzzy union and intersection respectively.

Hence $c_{ij}^{(2)} = c_{ij}$ and C = A adj A is idempotent [C is weakly reflexive].

Theorem : 5.5

The determinant | A | of a circulant fuzzy matrix A is the largest element in A.

Proof

Let $a_{1\ell} \geq a_{1i}$ for every $i \in \{1, 2, \ldots, n\}$, $a_{1\ell}$ is the largest element in A.

$$| A | = \vee_{\sigma \in S_n} \left[\wedge_{k=1}^{n} a_{k\,\sigma(k)} \right]$$

$$= \wedge_{k=1}^{n} a_{k\,\pi(k)}$$

for some $\pi \in S_n = a_{1\pi(1)} \wedge a_{2\pi(2)} \wedge \ldots\ldots \wedge a_{n\pi(n)}$, $\pi(1) = 1$, A is circulant, we get

$$A_{1\ell} = a_{2(\ell \oplus 1)} = a_{3(\ell \oplus 2)} = \ldots\ldots = a_{n(\ell \oplus n-1)}$$

$$= a_1$$

Hence proved.

Theorem : 5.6

For a circulant trapezoidal fuzzy number matrix A(adj A) = (adj A) A.

Proof

Let C = A(adj A), D = (adj A) A.

$$c_{ij} = \sum_{k=1}^{n} a_{ik} \, | A_{jk} |$$

$$\geq a_{ii} \, | A_{ji} |$$

$$= | A_{ji} |$$

$$d_{ij} = \sum_{k=1}^{n} | A_{ki} | \, a_{kj}$$

$$\geq | A_{ji} | \, a_{jj}$$

$$= | A_{ji} |$$

Thus $c_{ij} = d_{ij}$

$\therefore$ A(adj A) = (adj A) A.

Remark : 5.7

For a square fuzzy matrix adj A = A(adj A) = (adj A) A but for a circulant trapezoidal fuzzy number matrices A(adj A) = (adj A) A.