Specimen Format for Thesis of the Month

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Department:	Mathematics
Branch /Area:	Topology
Sub Subject Heading:	-----
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Abstract (within 300 words):

The scope of this thesis is to explore some of the existing neutrosophic variants and introduce some new types of neutrosophic variants. The notion of extended Pythagorean neutrosophic set, neutrosophic spherical set, and Fermatean neutrosophic set has been examined and the concepts of topology, rough set, operators, and measures have been developed and analysed. The idea of the proposed logic is extended to define Fermatean neutrosophic cubic set to manage high levels of uncertainty and vagueness and also to introduce Fermatean temporal neutrosophic set to deal with time moments. Further, the thesis combines rough set concept with Fermatean temporal neutrosophic set to construct a new class of rough set called Fermatean temporal neutrosophic rough set. A new class of aggregation operators for neutrosophic variant has been developed and used in a **CO**mbinative **D**istance-based **AS**sessment (**CODAS**) evaluation method. Furthermore, tangent metric neutrosophic spherical distance measure and tangent metric Fermatean neutrosophic distance measure are formulated and applied to the **T**echnique for **O**rder **P**reference by **S**imilarity to **I**deal **S**olution (**TOPSIS**) method. Illustrative examples have been provided to validate and compare the defined aggregation operators and distance measures.

INTRODUCTION

The necessity to handle uncertainty in real-time problems has been an unending research challenge that has created different methodologies and theories. Fuzzy Set (FS) was introduced by Zadeh L. A. (1965). Fuzzy set and its extensions such as Intuitionistic Fuzzy Set (IFS), introduced by Atanassov K. T. (1986), type-2 fuzzy set initiated by Karnik N. N. (1999), and interval-valued fuzzy set introduced by Liang Q. et al. (2000), etc., enables one to work in uncertain and ambiguous situation as well as to solve unwell-defined problem and problem with incomplete information. Fuzzy logic extends classical logic by assigning a membership function ranging between 0 and 1 to the variables. IFS allow both membership and non-membership to define uncertainties where FS allows membership function alone.

The extensions of the intuitionistic fuzzy set include, Pythagorean fuzzy set (Yager R. R, 2013), which expands membership and non-membership degrees by raising their powers to 2, to handle uncertainties, and spherical fuzzy set (Kutlu Gundogdu, 2019), which further extends these degrees by employing hesitant value to manage uncertainties more effectively than Pythagorean fuzzy set.

The cubic set, introduced by Jun Y.B. et al. (2012), generalises fuzzy and interval-valued fuzzy sets to better model the real-life scenarios where data is non-exact. Cubic set is crucial for handling such data, as it combines both exact and non-exact values, which is represented through intervals.

The neutrosophic set (Smarandache F., 1998) incorporates an additional component, 'indeterminacy,' allowing a more comprehensive representation of uncertainty. In recent days, several variants of neutrosophic set have been introduced. Pythagorean neutrosophic set, introduced by Jansi R. et al, (2019), extends Pythagorean fuzzy set by defining $0 \leq T^2 + I^2 + F^2 \leq 2$. Fermatean neutrosophic set has been introduced by Antony Crispin Sweety C. and Jansi R. (2021) to model high uncertainty and incomplete information by defining $0 \leq T^3 + I^3 + F^3 \leq 2$.

The concept of rough set, introduced by Pawlak Z. (1982), approximates a crisp set by defining lower and upper approximations. Next to fuzzy set, it is highly effective with diverse applications, handling uncertainty for attribute reduction and pattern recognition without prior probability. Antony Crispin Sweety C. and Arockiarani I. (2017) combined rough and neutrosophic sets to develop the notion of neutrosophic rough set and explored their properties.

Classical topology relying on open sets to define a topological space, studies the properties of a space that are preserved under continuous deformations which includes continuity, compactness, and connectedness. Fuzzy topology modifies classical topology by introducing uncertainty, allowing elements to have varying degrees of membership.

Decision-making is the process of selecting the best option among several alternatives to achieve a specific goal or to derive solution for a problem. Fuzzy and neutrosophic methods are essential as they effectively handle uncertainty, vagueness, and incomplete information in decision making approaches.

Building on the above concepts, this research aims to explore some of the existing neutrosophic variants and introduce some new types of neutrosophic variants.

BRIEF DESCRIPTION ON THE STATE-OF-ART OF THE RESEARCH TOPIC

The notion of extended Pythagorean neutrosophic set, neutrosophic spherical set, and Fermatean neutrosophic set has been explored and the concepts of topology, rough set, operators, and measures has been analysed. The idea of the proposed logic is extended to define Fermatean neutrosophic cubic set to manage high levels of uncertainty and vagueness and also to introduce Fermatean temporal neutrosophic set to deal with time moments. Further, the research combines rough set concept with Fermatean temporal neutrosophic set to construct a new class of rough set called Fermatean temporal neutrosophic rough set. The decision-making methodologies that address high uncertainty, has been developed for the defined neutrosophic variants. A new class of neutrosophic variant aggregation operators has been developed and used in a **C**ombinative **D**istance-based **AS**sessment (**CODAS**) evaluation method. Furthermore, tangent metric neutrosophic spherical distance measure and tangent metric Fermatean neutrosophic distance measure are formulated and applied to the **T**echnique for **O**rder **P**reference by **S**imilarity to **I**deal **S**olution (**TOPSIS**). Illustrative examples have been provided to validate and compare the defined aggregation operators and distance measures.

DEFINITION OF THE PROBLEM

Mitigation the informational limitations in neutrosophic set variants through the development of new variants and a holistic, integrated framework encompassing topological properties, approximation spaces, and decision-making approaches for addressing high uncertainty.

OBJECTIVES AND SCOPE

- To introduce the concept of neutrosophic variants and explore their mathematical properties and axioms to establish a strong theoretical foundation for handling high uncertainty.
- To investigate the incorporation of temporal components with topological structures and also to define cubic sets in the Fermatean neutrosophic environment.
- To construct the concept of approximation spaces within Fermatean temporal neutrosophic sets, develop their rough topology, and introduce an attribute reduction technique for efficient processing of uncertain and complex datasets.
- To develop comprehensive distance measures, such as the Tangent Metric Neutrosophic Spherical Distance Measure and the Tangent Metric Fermatean Neutrosophic Distance Measure, to improve uncertainty quantification and similarity analysis.
- To formulate aggregation operators, including arithmetic mean and geometric mean, to enhance the applicability of neutrosophic variants in multi-criteria decision-making methodologies under high uncertainty.

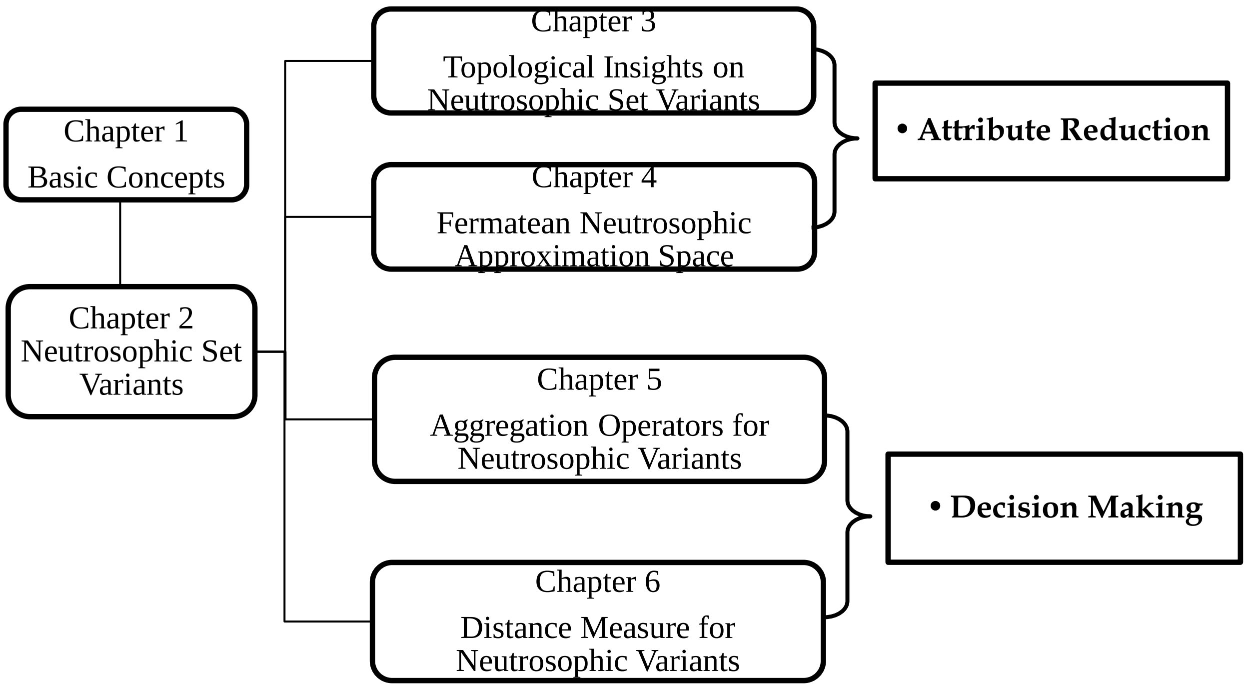
METHODOLOGY

- **Set-Theoretic Approach** is used to define new neutrosophic set variants and examine their relationships.
- **Axiomatic Approach** is applied to develop frameworks based on fundamental rules, ensuring logical consistency.
- **Algebraic Approach** involves applying algebraic operations and structures for analysis, problem-solving, and modelling.
- **Comparison Approach** is used to examine proposed systems and identify patterns.
- **Operator Formulation Technique** is employed to define new operators and verify their fundamental properties to ensure their effective analysis and decision-making.

ORIGINAL CONTRIBUTION

FRAMEWORK OF THE STUDY

The following diagram illustrates the key concepts discussed in this thesis.



The content of the thesis is presented in six chapters.

CHAPTER I:

Chapter I has got an overture to our study which includes the requirements of versatile sets which are of great use in the sub sequel of the thesis.

CHAPTER II:

Exploring neutrosophic variants is essential for handling more complex and high uncertainties what traditional neutrosophic sets cannot offer. To address these uncertainties, Chapter II focus developing new types neutrosophic variants.

The first section defines neutrosophic variants in three ways by redefining the structure of the neutrosophic set within a dependency-based frame work. It introduces an extended form of the Pythagorean neutrosophic set and neutrosophic spherical set, examines their fundamental properties, and compare them with Fermatean neutrosophic set.

The second section defines the concept of Fermatean neutrosophic cubic set and its properties. The next section combines the notion of a temporal component (i.e., time component) with Fermatean neutrosophic set, and introduces Fermatean temporal neutrosophic set, and defines some proposition.

CHAPTER III:

Chapter III defines and analyses the topological structure of the established neutrosophic set variants. It develops the concept of topology for neutrosophic variants in the framework of the experts, Chang, Lowen, and Sostak. The chapter is divided into three sections. The first section defines the topological structures of Pythagorean, spherical, and Fermatean neutrosophic topology in the sense of Chang and Lowen, along with an exploration of their properties.

Furthermore, the second section **introduces** the concept of **Fermatean neutrosophic gradation of openness**. Within this framework, gradation assigns a degree of openness to each set, effectively capturing its openness. Additionally, it **defines Fermatean neutrosophic gradation of closeness,** subspaces, gradation-preserving maps, bases and subbases, product neutrosophic topological spaces, neutrosophic compactness, and **explores** Tychonoff's theorem.

The third section introduces a new class of topology called Fermatean neutrosophic temporal topology. This topology is defined within the frameworks established by Chang, Lowen, and Sostak, accompanied by an exploration of its properties.

CHAPTER IV:

Chapter IV constructs the approximation space for the defined neutrosophic variants and developed the concept of rough topology. This chapter is structured into three sections. The first section introduces Fermatean neutrosophic rough set, defines level cuts, and presents relevant propositions and theorems. The second section defines Fermatean temporal neutrosophic rough set and develops Fermatean temporal neutrosophic rough topology, exploring their topological properties.

The third section establishes a working rule for identifying core attributes by integrating the neutrosophic variants from Chapter II, the topology from Chapter III, and the rough topology introduced in this chapter. Additionally, an attribute reduction technique using a Fermatean temporal neutrosophic set is developed and provided with an example.

CHAPTER V:

Chapter V develops aggregation operators to enhance decision-making using defined neutrosophic variants from Chapter II. Aggregation operators integrate multiple decision-maker's inputs into a single outcome. This chapter applies the CODAS method in conjunction with aggregation operators. CODAS, a multi-criteria decision-making approach, evaluate the alternatives based on their relative distances.

The first section formulates aggregation operators, including arithmetic and geometric operators, for the neutrosophic spherical set and Fermatean neutrosophic set. These operators are classified based on the interconnections among criteria, and their properties are analysed.

The second section applies aggregation operators to the CODAS technique, introducing the Fermatean/Spherical neutrosophic CODAS method for scenarios characterized by high uncertainty, vagueness, or conflicting opinions. A working model is also provided to depict the proposed technique.

CHAPTER VI:

Chapter VI introduces a distance measure for neutrosophic variants. A distance measure is a mathematical function that quantifies the closeness or separation between two objects in a given space. This chapter is structured into three sections. The first section formulates distance measures for neutrosophic variants, such as the Tangent Metric Neutrosophic Spherical Distance Measure (TMNSDM) and the Tangent Metric Fermatean

Neutrosophic Distance Measure (TMFNDM). Additionally, relevant propositions and theorems are discussed.

The second section compares the proposed distance measures with existing distance measures such as Euclidean Distance, Normalized Euclidean Distance, Hamming Distance, Normalized Hamming Distance Sine Metric Single-Valued Neutrosophic Distance Measure, supported by an illustrative example.

The third section applies the defined distance measures and aggregation operators to the TOPSIS method, enhancing decision-making in high-uncertain and vague scenarios. TOPSIS, a widely used MCDM technique, ranks alternatives based on their relative distances. A working model of the proposed TOPSIS method is also provided.

CONCLUSION

This research has made significant contributions to neutrosophic set theory by introducing new variants and their corresponding mathematical frameworks. By integrating topology, rough set theory, and decision-making methodologies, it provides a robust approach to handling high uncertainty in various applications. The proposed distance measures enhance analytical precision, improving the effectiveness of decision-making frameworks. Furthermore, the study opens new directions for future research, particularly in extending neutrosophic models to real time problems.

The future research directions based on this study are outlined as follows:

- Exploring lower and higher separation axioms, convergence properties, and compactness to enhance research in functional analysis and topology-driven solutions.
- Implementing the validated theoretical concepts, initially demonstrated through assumed numerical examples, in a practical, real-time application in pattern recognition and machine learning.

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