α* Closed Sets in Intuitionistic Fuzzy Topological Spaces

By

Gajalaxmi P

(20PMA005)

Supervisor

Dr. S. M. Sudha

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore-641 043

In Partial Fulfilment of the Requirement for the Degree of

Master of Science in Mathematics

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N. Balamani
18.05.2022
Signature of the Head of the Department

18/5/22
Signature of the Supervisor

DECLARATION

I do hereby declare that the thesis entitled " α^* **Closed Sets in Intuitionistic Fuzzy Topological Spaces"** submitted to the Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore for the award of the degree of **MASTER OF SCIENCE** in Mathematics is a record of original work done by me during the period from December 2021 to May 2022 under the guidance and supervision of **Dr. S. M. SUDHA**, Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and it has not previously formed the basis for the award of any degree, diploma, fellowship or associateship or any other similar title of any candidate of any university.

P. Gajalaxmi

Signature of the Candidate

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CONTENT

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INTRODUCTION

Topology:

Topology is one of the most active areas in all of mathematics. Traditionally, it is considered one of the three main areas of pure mathematics (together with algebra and analysis). Recently, topology has also become an important component of applied mathematics, with many mathematicians and scientists employing concepts of topology to model and understand real-world structures and phenomena.

Topology means (Topo-place, logy-study) study of a place. It grew out of geometry, expanding on some of the ideas and loosening some of the structures appearing therein. The word topology, literally, means the study of position or location. It is often described as rubber-sheet geometry.

In traditional geometry, objects such as circles, triangles, planes and polyhedra are considered rigid with well-defined distances between points and well-defined angles between edges or faces. But in topology, distances and angles are irrelevant.

Topology is popularly considered to have begun with Leonhard Eulers's solution to the famous Konigsberg bridges problem in the eighteenth century. Following the initial work of Euler, a number of prominent mathematicians made valuable contributions to the geometry of position over the next century and a half. These included Carl Friedrich Gauss, August Ferdinand Mobius, Johann Listing, Riemann, Felix Klein and Henri Poincare.

Topology as a science, generally believed, formed through the works of the great French mathematician Henri Poincare at the end of the 19^{th} century. The beginnings of topology research may be dated to the work of Riemann in the middle of the 19^{th} century.

General Topology also finds its origin in functional analysis, via calculus of variations and Fourier analysis where precise notions of compactness and convergence are required. Since the early part of the 20^{th} centuries, both Topology and Analysis have fed off each other and this has resulted in some interesting propositions. Historically speaking, topology has followed two principal lines of development.

In homology theory, dimension theory and the study of manifolds, the basic motivation appears to have come from geometry. In these fields, topological spaces are looked upon as generalized geometric configuration, and the emphasis is placed on the structure of the spaces themselves. In the other direction, the main stimulus has been analysis. Continuous functions are the chief objects of interest here and topological spaces are regarded primarily as carriers of such functions and as domains over which they can be integrated.

The objects that we study in topology are called topological spaces. These are sets of point on which a notion of proximity between points is established by specifying a collection of subsets called open sets. The line, the circle, the plane, the sphere, the torus and the Mobius band are all examples of topological spaces.

Topological spaces were first defined by Hausdorff F in 1914 in his seminal "Principles of Set Theory". A topological space is a set K on which we have a topology - a collection of subsets of K which we call the open subsets of K. The only requirements are that both K itself and the empty subset must be among the open sets, that all union of open sets is open and that the intersection of two open sets is open. And also various sets namely β-open sets, b-open sets, semi pre open sets, semi open sets and $\alpha*$ open sets were introduced in topological spaces respectively by Abd El-Monsef et.al. [1], Andrijevic D [2, 3], Levine N [21] and Hatir E et.al [17].

Fuzzy topology:

As an extension of classical set theory, fuzzy set were introduced by Prof. Zadeh L A [35] of University of California at Berkeley in the year 1965. The development of Fuzzy Set Theory, since its introduction, has been dramatic and breathtaking. Thousands of research papers have appeared in various journals such as Fuzzy Sets and Systems and IEEE Transactions on Fuzzy Systems devoted entirely to theoretical and application aspects of fuzzy sets.

Fuzzy Set Theory has numerous applications in various fields like artificial intelligence, automata theory, computer science, control theory, decision making, expert systems, medical diagnosis, and robotics. Fuzzy Set Theory has pervaded almost all the fields of study and its applications have percolated sown to consumer goods level. Apart from this, it is being applied on a major scale in industries through intelligent robots for machine-building and of course for military purposes.

Fuzzy Set Theory is at once a generalization as well as extension of Crisp Set Theory. Thus, the basic theme and ideas of crisp set theory will be reflected in fuzzy set theory also, though in an extended way. The idea of fuzzy set is welcomed because of handful uncertainities and vagueness which cantorian set could not address. The concept of fuzzy set is an extension of the concept of crisp set. Just as a crisp set on a universal set U is defined by its characteristic function from U to $\{0,1\}$, a fuzzy set on a domain U is defined by its membership function from U to [0,1].

Fuzzy theory holds that many things in life are matters of degree. A black and white photo is not just black and white; there are many levels of gray shades which can be observed in a typical picture. As an example, a pixel can have a brightness value between 0 and 255. The value 0 stands for Black, 255 stands for White and every number between 0 and 255 stands for a certain gray level.

Fuzzy topology is constructed based on fuzzy sets. It is an extension of general topology. In 1968, Chang C L [10] gave the topological structure to these fuzzy sets called fuzzy topological spaces and studied the topological properties. Most basic concepts like open set, closed set, neighborhood, interior of a set, continuity and compactness etc. After the introduction of fuzzy topological spaces many researchers like Ganguly S and Saha S [14], Hanafy I M [15], Singal M K and Niti Prakash [28], Thakur S S and Singh S [33] introduced various sets in fuzzy topological spaces.

Intuitionistic fuzzy topology:

Intuitionistic fuzzy set, an extension of fuzzy set, has been introduced by Atanassov K [5] in the year 1986 and this theory has been developed in many papers [6, 7, 8]. Intuitionistic fuzzy set has been found to be more efficient in dealing with vagueness and ambiguity. It is characterized by a membership function (μ_{L}(x)) and a non membership function (ν_{L}(x)) with their sum being less than or equal to one

($0 \leq \mu_L(x) + \nu_L(x) \leq 1$). This relaxes the enforced duality $\nu_L(x) = 1 - \mu_L(x)$ from fuzzy set theory.

Intuitionistic fuzzy set allows one to address the positive and negative side of an imprecise concept separately. Intuitionistic fuzzy sets can be more precisely expressed. For example, the fact that the temperature of a patient changes and other symptoms are not quite clear. There is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object.

Intuitionistic fuzzy set is useful in decision making problems, particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services, etc. Recently various applications of intuitionistic fuzzy set like artificial intelligence, intuitionistic fuzzy expert systems, intuitionistic fuzzy decision making, intuitionistic fuzzy machine learning, intuitionistic fuzzy semantic representations etc., have appeared.

Using the notion of intuitionistic fuzzy sets, in 1997 Coker D [13] has constructed the basic concepts of intuitionistic fuzzy topological spaces. After giving the fundamental definitions and the necessary examples, Coker D [11, 12, 13] introduced the definitions of intuitionistic fuzzy points, intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, intuitionistic fuzzy connectedness and obtained several preservation properties and some characterizations concerning intuitionistic fuzzy connectedness.

Pre semi closed sets and pre semi separation axioms were introduced by Bhattacharjee P and Bhaumil R N [9] in intuitionistic fuzzy topological spaces. The category of Intuitionistic fuzzy topological spaces introduced by Lee S J and Lee E P[20]. Lupianez F G [22, 23] introduced Intuitionistic fuzzy topological operators and Quasi coincidence for intuitionistic fuzzy points. The concept of γ open sets were introduced by Hanafy I M [16] in intuitionistic fuzzy topological spaces. $\gamma*$ generalized closed sets in intuitionistic fuzzy topological spaces was introduced by Riya V M and Jayanthi D [25].

The content of this thesis is split into two chapters. The concept of $\alpha*$ closed sets and $\alpha*$ open sets in intuitionistic fuzzy topological spaces are introduced. Some interesting results and properties are proved and analyzed.

This thesis begins with the introduction and the review of literature. Chapter 1 includes some background materials which are useful for the present study. It throws the insight to the introduction of fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy open sets and intuitionistic fuzzy closed sets.

In Chapter 2, the section 2.1 deals with the introduction of $\alpha*$ closed set in intuitionistic fuzzy topological spaces and suitable examples were given. The newly introduced intuitionistic fuzzy set is compared with the existing intuitionistic fuzzy sets such as intuitionistic fuzzy closed set, intuitionistic fuzzy regular closed set, intuitionistic fuzzy semi closed set, intuitionistic fuzzy pre closed set, intuitionistic fuzzy α closed set, intuitionistic fuzzy β closed set, intuitionistic fuzzy γ closed set and intuitionistic fuzzy semi pre closed set and their inter relation is given by a neat diagram. Also, the union and intersection of this newly introduced intuitionistic fuzzy sets are examined. The section 2.2 deals with the introduction of intuitionistic fuzzy $\alpha*$ open sets in intuitionistic fuzzy topological spaces. This set is compared with other existing open sets and their inter relation is given by a neat diagram. The union and intersection of this intuitionistic fuzzy $\alpha*$ open sets are also verified.

REVIEW OF LITERATURE

A review of literature of recent developments on generalized notions of closed and open sets in topological spaces, fuzzy topological spaces and intuitionistic fuzzy topological spaces are discussed here.

In my thesis work, a new type of closed set called $\alpha*$ closed sets in intuitionistic fuzzy topological spaces is introduced. Their basic properties, preservation propositions, interrelations are established with necessary counter example.

Some of the research articles which I refer for the thesis are given below

1. FUZZY SETS

[Zadeh, L. A., 1965]

In this paper, the author has introduced new set namely fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero to one. Further the author has provided the notions of inclusion, union, intersection, complement etc., with respect to the fuzzy sets.

2. FUZZY TOPOLOGICAL SPACES

[Chang, C. L., 1968]

In this article, the author introduced fuzzy topological space. This concept is considered to be the generalization of general topological spaces. Some basic concepts such as fuzzy open set, fuzzy closed set, interior set, fuzzy neighborhood, fuzzy continuity are discussed.

3. **A NOTE ON FUZZY SEMI OPEN SETS IN FUZZY TOPOLOGICAL SPACES**

[Ganguly, S. and Saha, S., 1986]

In this article, the authors introduced the concepts of semi-T_i (i = 0, 1, 2) spaces and semi-R_i (i = 0, 1) spaces. They investigated some fuzzy topological properties under the newly introduced concepts. They also introduced and investigated some mappings involving semi-open sets on fuzzy topological spaces.

4. **A DECOMPOSITION OF CONTINUITY**

[Hatir, E., Noiri, T. and Yuksel, S., 1996]

In this article, the authors introduced the notions of $\alpha*$ sets and C-continuity and obtained another decomposition of continuity.

5. **INTERIOR AND CLOSURE OF FUZZY OPEN SETS IN FUZZY TOPOLOGICAL TM- SYSTEM**

[Annalakshmi, M. and Chandramouleeswaran, M., 2015]

In this article, the authors discussed the notion of interior and closure of fuzzy open sets in a fuzzy topological TM-system. They have analyzed their properties and obtained some interesting propositions.

6. **FUZZY α SETS AND α CONTINUOUS MAPS**

[Singal, M. K. and Niti Rajvanshi., 1992]

In this article, the authors generalized the notion of α sets to fuzzy spaces. They also introduced the concepts of fuzzy α continuous and fuzzy α open mappings and discussed their relations with fuzzy continuous and other weaker forms of fuzzy continuous mappings.

7. ON FUZZY γ-OPEN SETS AND FUZZY γ-CONTINUITY

[Hanafy, I. M., 2002]

In this paper, the author introduced fuzzy γ open sets and fuzzy γ continuity in intuitionistic fuzzy topological spaces and studied their basic properties.

8. INTUITIONISTIC FUZZY SETS

[Atanassov, K., 1986]

In this article, the author provided the notion of intuitionistic fuzzy sets. This is considered to be the generalization of fuzzy sets. The highlight of this particular article is that some relations and operations concerning classical sets are extended to intuitionistic fuzzy sets.

9. AN INTRODUCTION TO INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Coker, D., 1997]

In this article, the author has introduced intuitionistic fuzzy topological spaces. The notions of intuitionistic fuzzy interior and intuitionistic fuzzy closure are being provided and this is followed by the discussion of some important properties concerning them. Furthermore, the notion of intuitionistic fuzzy continuity is provided.

10. INTUITIONISTIC FUZZY SEMI PRE OPEN SETS AND INTUITIONISTIC FUZZY SEMI-PRE CONTINUOUS MAPPINGS

[Young Bae Jun and Seok-zun song., 2005]

In this paper, the authors have introduced the notions of intuitionistic fuzzy semi pre open sets and its continuous mapping. Also they investigated the connections of these notions with other existing intuitionistic fuzzy topological concepts.

11. GENERALIZED PRE CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Rajarajeswari, P. and Senthil Kumar, L., 2011]

In this article, the author introduced intuitionistic fuzzy generalized pre-closed sets and intuitionistic fuzzy generalized pre-open sets. They also studied its properties. They also provided some applications of intuitionistic fuzzy generalized pre-closed sets.

12. RELATION BETWEEN SEMI PRE CLOSED SETS AND BETA CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Jayanthi, D., 2013]

This article is considered to be a significant one as it clearly distinguishes intuitionistic fuzzy semi pre closed sets and beta closed sets. The author has additionally provided the relationship between these two sets.

13. NOWHERE DENSE SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Thakur, S. S. and Dhavaseelan, R., 2015]

In this article, the authors introduced the concepts of intuitionistic fuzzy nowhere dense sets and their characterizations are studied.

14. ON INTUITIONISTIC FUZZY β GENERALIZED CLOSED SETS

[Saranya, M. and Jayanthi, D., 2016]

This article consists of the notion of intuitionistic fuzzy β generalized closed sets. The authors have analyzed some of their properties and obtained some interesting propositions. Also, the relationship between this new class of sets and some of the previously existing sets are discussed.

15. β** GENERALIZED CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Sudha, S. M. and Jayanthi, D., 2020]

In this article, the authors introduced the notion of intuitionistic fuzzy β** generalized closed sets and investigated some of their properties and produced some characterization propositions.

16. INTUITIONISTIC FUZZY ALPHA-CONTINUITY AND INTUITIONISTIC FUZZY PRECONTINUITY

[Joung Kon Jeon, Young Bae Jun and Jin Han Park., 2005]

In this article, a characterization of intuitionistic fuzzy α open set was introduced and the conditions for an intuitionistic fuzzy set to be an intuitionistic fuzzy α open set are provided. They also gave the characterizations of intuitionistic fuzzy pre continuous mappings.

17. INTUITIONISTIC FUZZY γ-CONTINUITY

[Hanafy, I. M., 2009]

In this article, the author introduced the concepts of γ- open sets and γ- continuity in intuitionistic fuzzy topological spaces. The author obtained some other results related to topological concepts also.

CHAPTER 1

CHAPTER 1

PRELIMINARIES

Definition 1.1: [5] An ***intuitionistic fuzzy set*** (IFS) L is of the form

$$L = \{ \langle k, \mu_L(k), \nu_L(k) \rangle : k \in K\},$$

where the functions μ_L: K → [0,1] and ν_L: K → [0,1] denotes the degree of membership ($\mu_L(k)$) and the degree of non-membership ($\nu_L(k)$) of each element k ∈ K to the set L, respectively, and $0 \leq \mu_L(k) + \nu_L(k) \leq 1$ for each k ∈ K. An intuitionistic fuzzy set L in K is simply denoted by L = ⟨k, μ_L, ν_L⟩ instead of denoting L = {⟨k, $\mu_L(k), \nu_L(k)$⟩: k ∈ K}.

Definition 1.2: [5] Let L and M be two IFSs of the form

$$L = \{ \langle k, \mu_L(k), \nu_L(k) \rangle : k \in K\}$$

and

$$M = \{ \langle k, \mu_M(k), \nu_M(k) \rangle : k \in K\}$$

Then, (i) L ⊆ M if and only if $\mu_L(k) \leq \mu_M(k)$ and $\nu_L(k) \geq \nu_M(k)$ for all k ∈ K,

(ii) L = M if and only if L ⊆ M and L ⊇ M,

(iii) $L^c = \{ \langle k, \nu_L(k), \mu_L(k) \rangle : k \in K \}$

(iv) $L \cup M = \{ \langle k, \mu_L(k) \cup \mu_M(k), \nu_L(k) \cap \nu_M(k) \rangle : k \in K\}$,

(v) $L \cap M = \{ \langle k, \mu_L(k) \cap \mu_M(k), \nu_L(k) \cup \nu_M(k) \rangle : k \in K\}$

The intuitionistic fuzzy sets $0_\sim = \langle k, 0, 1 \rangle$ and $1_\sim = \langle k, 1, 0 \rangle$ are the empty set and the whole set of K respectively.

Definition 1.3: [13] An ***intuitionistic fuzzy topology*** (IFT) on K is a family τ of IFSs in K satisfying the below conditions:

(a) $0_\sim, 1_\sim \in \tau$,

(b) $H_1 \cap H_2 \in \tau$ for any $H_1, H_2 \in \tau$,

(c) $\cup H_i \in \tau$ for any family $\{ H_i : i \in J \} \in \tau$.

The pair (K,τ) is called an ***intuitionistic fuzzy topological space*** (IFTS) and any IFS in τ is known as an ***intuitionistic fuzzy open set*** (IFOS) in K. The complement L^c of an IFOS L in an IFTS (K,τ) is called an ***intuitionistic fuzzy closed set*** (IFCS) in K.

Definition 1.4: [32] Two IFSs L and M are said to be ***q-coincident*** (L_qM) if and only if there exists an element k ∈ K such that $\mu_L(k) > \nu_M(k)$ or $\nu_L(k) < \mu_M(k)$.

Definition 1.5: [32] Two IFSs L and M are said to be ***not q- coincident*** ($L_{\bar{q}}M$) if and only if $L \subseteq M^c$.

Definition 1.6: [11] An ***intuitionistic fuzzy point*** (IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of K given by

$$p_{(\alpha,\beta)}(k) = \begin{cases} (\alpha,\beta) & if\ k = p, \\ (0,1) & otherwise. \end{cases}$$

An IFP $p_{(\alpha,\beta)}$ is said to belong to a set L if $\alpha \leq \mu_L$ and $\beta \geq \nu_L$.

Definition 1.7: [26] An IFS L in (K,τ) is an ***intuitionistic fuzzy Q-set*** if int(cl(L)) = cl(int(L)).

Definition 1.8: [13] Let (K,τ) be an IFTS and L = $\langle$ k, μ_L, ν_L $\rangle$ be an IFS in K. Then the ***intuitionistic fuzzy interior and intuitionistic fuzzy closure*** are defined by

$$\text{int}(L) = \cup\{ Q \,/\, Q \text{ is an IFOS in K and } Q \subseteq L\},$$

$$\text{cl}(L) = \cap\{ R \,/\, R \text{ is an IFCS in K and } L \subseteq R\}.$$

For any IFS L in (K,τ), we have

(i) $\text{cl}(L^c) = (\text{int}(L))^c$ or $\text{cl}\overline{(L)} = \overline{(int(L))}$

(ii) $\text{int}(L^c) = (\text{cl}(L))^c$ or $\text{int}\overline{(L)} = \overline{(cl(L))}$

Definition 1.9: [11] Let (K,τ) be an IFTS and L = ⟨ k, μ_L, ν_L ⟩ be an IFS in K. Then ***intuitionistic fuzzy kernel*** of L is the intersection of all IFOSs containing L.

Definition 1.10: [31] An IFS L in (K,τ) is called an ***intuitionistic fuzzy nowhere dense set*** if there exists no IFOS U such that U ⊆ cl(L). That is int(cl(L)) = $0_{\sim}$.

Result 1.11: [31] Let L be an IFS in K. If L is an ***intuitionistic fuzzy nowhere dense set*** in K, then int(L) = $0_{\sim}$.

Definition 1.12: [19] An IFS L = ⟨ k, μ_L, ν_L ⟩ in an IFTS (K,τ) is said to be an

(i) ***intuitionistic fuzzy regular open set*** (IFROS) if int(cl(L)) = L
(ii) ***intuitionistic fuzzy semi open set*** (IFSOS) if L ⊆ cl(int(L))
(iii) ***intuitionistic fuzzy pre open set*** (IFPOS) if L ⊆ int(cl(L))
(iv) ***intuitionistic fuzzy α open set*** (IFαOS) if L ⊆ int(cl(int(L)))
(v) ***intuitionistic fuzzy β open set*** (IFβOS) if L ⊆ cl(int(cl(L)))

Definition 1.13: [19] An IFS L = ⟨ k, μ_L, ν_L ⟩ in an IFTS (K,τ) is said to be an

(i) ***intuitionistic fuzzy regular closed set*** (IFRCS) if cl(int(L)) = L
(ii) ***intuitionistic fuzzy semi closed set*** (IFSCS) if int(cl(L)) ⊆ L
(iii) ***intuitionistic fuzzy pre closed set*** (IFPCS) if cl(int(L)) ⊆ L
(iv) ***intuitionistic fuzzy α closed set*** (IFαCS) if cl(int(cl(L))) ⊆ L
(v) ***intuitionistic fuzzy β closed set*** (IFβCS) if int(cl(int(L))) ⊆ L

Definition 1.14: [16] An IFS L = ⟨ k, μ_L, ν_L ⟩ in an IFTS (K,τ) is said to be an

(i) ***intuitionistic fuzzy γ open set*** (IFγOS) if L ⊆ cl(int(L)) ∪ int(cl(L))
(ii) ***intuitionistic fuzzy γ closed set*** (IFγCS) if cl(int(L)) ∩ int(cl(L)) ⊆ L

Definition 1.15: [34] An IFS L = ⟨ k, μ_L, ν_L ⟩ in an IFTS (K,τ) is said to be an

(i) ***intuitionistic fuzzy semi-preopen set*** (IFSPOS) if there exists M ∈ IFPOS(O) such that M ⊆ L ⊆ cl(M)
(ii) ***intuitionistic fuzzy semi-preclosed set*** (IFSPCS) if there exists an IFPCS M such that int(M) ⊆ L ⊆ M

Definition 1.16: [13] Let (K,τ) be an IFTS and L, M be IFSs in K. Then the following properties hold:

(i) int(L) ⊆ L

(ii) L ⊆ cl(L)

(iii) L ⊆ M ⇒ int(L) ⊆ int(M)

(iv) L ⊆ M ⇒ cl(L) ⊆ cl(M)

(v) int(int(L)) = int(L)

(vi) cl(cl(L)) = cl(L)

(vii) int(L ∩ M) = int(L) ∩ int(M)

(viii) cl(L ∪ M) = cl(L) ∪ cl(M)

(ix) int($1_\sim$) = $1_\sim$

(x) cl($0_\sim$) = $0_\sim$

Result 1.17: [4] Let (K,τ) be an intuitionistic fuzzy topological space and L, M be intuitionistic fuzzy sets in K. Then the following properties hold:

(i) int(L ∪ M) ⊇ int(L) ∪ int(M)

(ii) cl(L ∩ M) ⊆ cl(L) ∩ cl(M)

CHAPTER 2

CHAPTER 2

2.1 α^* Closed Sets in Intuitionistic Fuzzy Topological Spaces

Here we have introduced intuitionistic fuzzy α^* closed sets and established the inter-relation between intuitionistic fuzzy α^* closed sets and already existing intuitionistic fuzzy closed sets in intuitionistic fuzzy topological spaces.

Definition 2.1.1: An IFS L of an IFTS (K,τ) is said to be an ***intuitionistic fuzzy α^* closed set*** (IFα^*CS) if cl(int(cl(L))) = cl(L) whenever L ⊆ U and U is an IFOS in (K,τ).

Example 2.1.2: Let K = {a,b}, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K, where $H_1 = \langle$ k, $(0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$, $H_2 = \langle$ k, $(0.8_a, 0.6_b), (0.2_a, 0.4_b) \rangle$ then (K,τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set L = $\langle$ k, $(0.4_a, 0.3_b), (0.6_a, 0.7_b) \rangle$ in K, then L is an intuitionistic fuzzy α^* closed set as,

$$\begin{aligned} cl(int(cl(L))) &= cl(int(H_1^c)) \\ &= cl(H_1) \\ &= H_1^c \\ &= cl(L) \end{aligned}$$

Hence, cl(L) = cl(int(cl(L))) and L is an IFα^*CS in K.

Proposition 2.1.3: Every IFRCS is an IFα^*CS in (K,τ) but the reverse is not true in general.

Proof: Let L be an IFRCS in (K,τ) then

$$cl(int(L)) = L \qquad (1)$$

Since every IFRCS is an IFCS, we have

$$cl(L) = L \qquad (2)$$

Now, $cl(int(cl(L))) = cl(int(L))$ [From (2)]

$= L$ [From (1)]

$= cl(L)$ [From (2)]

Hence, $cl(int(c(L))) = cl(L)$.

Therefore, L is an IFα*CS in (K,τ).

Example 2.1.4: Let $K = \{a,b\}$, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle k, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$, $H_2 = \langle k, (0.8_a, 0.5_b), (0.2_a, 0.5_b) \rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set $L = \langle k, (0.5_a, 0.3_b), (0.5_a, 0.5_b) \rangle$ in K, then L is an intuitionistic fuzzy α* closed set as,

$$cl(int(cl(L))) = cl(int(H_1^c))$$

$$= cl(H_1)$$

$$= H_1^c$$

$$= cl(L)$$

But, L is not an IFRCS in K as

$$cl(int(L)) = cl(0_\sim)$$

$$= 0_\sim$$

$$cl(int(L)) \neq L$$

Remark 2.1.5: Every IFCS in (K,τ) is independent of every IFα*CS in (K,τ).

Example 2.1.6: Let $K = \{a,b\}$, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle k, (0.2_a, 0.3_b), (0.8_a, 0.7_b) \rangle$, $H_2 = \langle k, (0.3_a, 0.4_b), (0.7_a, 0.6_b) \rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = ⟨ k, (0.3_a, 0.2_b), (0.7_a, 0.7_b) ⟩ in K, then L is an intuitionistic fuzzy α* closed set as,

$$cl(int(cl(L))) = cl(int(H_2^c))$$

$$= cl(H_2)$$

$$= H_2^c$$

$$= cl(L)$$

But L is not an IFCS in K as,

$$cl(L) = H_2^c$$

$$\neq L$$

Example 2.1.7: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.4_a, 0.5_b), (0.6_a, 0.5_b) ⟩, H_2 = ⟨ k, (0.5_a, 0.6_b), (0.5_a, 0.4_b) ⟩, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = ⟨ k, (0.5_a, 0.4_b), (0.5_a, 0.6_b) ⟩ in K, then L is an IFCS as,

$$cl(L) = H_2^c$$

$$= L$$

But L is not an IFα*CS in K as,

$$cl(int(cl(L))) = cl(int(H_2^c))$$

$$= cl(0_\sim)$$

$$= 0_\sim$$

$$\neq cl(L)$$

Remark 2.1.8: Every IFSCS in an IFTS (K,τ) is independent of every IFα*CS in (K,τ).

Example 2.1.9: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, (0.3_a, 0.6_b), (0.7_a, 0.4_b) $\rangle$, $H_2 = \langle$ k, (0.5_a, 0.9_b), (0.5_a, 0.1_b) $\rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = $\langle$ k, (0.2_a, 0.5_b), (0.8_a, 0.4_b) $\rangle$ in K, then L is an intuitionistic fuzzy α* closed set as,

$$\begin{aligned} cl(int(cl(L))) &= cl(int(1_\sim)) \\ &= cl(1_\sim) \\ &= 1_\sim \\ &= cl(L) \end{aligned}$$

But L is not an IFSCS in K as,

$$\begin{aligned} int(cl(L)) &= int(1_\sim) \\ &= 1_\sim \\ &\nsubseteq L \end{aligned}$$

Therefore, int(cl(L)) $\nsubseteq$ L.

Example 2.1.10: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, (0.8_a, 0.4_b), (0.2_a, 0.6_b) $\rangle$, $H_2 = \langle$ k, (0.6_a, 0.3_b), (0.4_a, 0.7_b) $\rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = $\langle$ k, (0.2_a, 0.4_b), (0.8_a, 0.6_b) $\rangle$ in K, then L is an IFSCS as,

$$\begin{aligned} int(cl(L)) &= int(H_1^c) \\ &= 0_\sim \\ &\subseteq L \end{aligned}$$

But L is not an IFα*CS in K as,

$$cl(int(cl(L))) = cl(int(H_1^c))$$

$$= cl(0_\sim)$$

$$= 0_\sim \neq cl(L)$$

Remark 2.1.11: Every IFγCS in (K,τ) is independent of every IFα*CS in (K,τ).

Example 2.1.12: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, $(0.8_a, 0.4_b)$, $(0.2_a, 0.6_b)$ $\rangle$, $H_2 = \langle$ k, $(0.6_a, 0.3_b)$, $(0.4_a, 0.7_b)$ $\rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = $\langle$ k, $(0.7_a, 0.3_b)$, $(0.3_a, 0.7_b)$ $\rangle$ in K, then L is an intuitionistic fuzzy α* closed set as,

$$cl(int(cl(L))) = cl(int(1_\sim))$$

$$= cl(1_\sim)$$

$$= 1_\sim$$

$$= cl(L)$$

But L is not an IFγCS in K as,

$$cl(int(L)) \cap int(cl(L)) = cl(H_2) \cap int(1_\sim)$$

$$= 1_\sim \cap 1_\sim$$

$$= 1_\sim$$

$$\nsubseteq L$$

Example 2.1.13: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.3_a, 0.6_b), (0.7_a, 0.4_b) ⟩, H_2 = ⟨ k, (0.5_a, 0.9_b), (0.5_a, 0.1_b) ⟩, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = ⟨ k, (0.5_a, 0.4_b), (0.5_a, 0.6_b) ⟩ in K, then L is an IFγCS as,

$$\begin{aligned} cl(int(L)) \cap int(cl(L)) &= cl(0_\sim) \cap int(H_1^c) \\ &= 0_\sim \cap 0_\sim \\ &= 0_\sim \\ &\subseteq L \end{aligned}$$

But L is not an IFα*CS in K as,

$$\begin{aligned} cl(int(cl(L))) &= cl(int(H_1^c)) \\ &= cl(0_\sim) \\ &= 0_\sim \\ &\neq cl(L) \end{aligned}$$

Remark 2.1.14: Every IFPCS in (K,τ) is independent of every IFα*CS in (K,τ).

Example 2.1.15: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.8_a, 0.4_b), (0.2_a, 0.6_b) ⟩, H_2 = ⟨ k, (0.6_a, 0.3_b), (0.4_a, 0.7_b) ⟩, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = ⟨ k, (0.8_a, 0.3_b), (0.2_a, 0.7_b) ⟩ in K, then L is an intuitionistic fuzzy α* closed set as,

$$\begin{aligned} cl(int(cl(L))) &= cl(int(1_\sim)) \\ &= cl(1_\sim) \\ &= 1_\sim \end{aligned}$$

$= cl(L)$

But L is not an IFPCS in K as, $cl(int(L)) = cl(H_2) = 1_\sim \nsubseteq L$.

Example 2.1.16: Let $K = \{a,b\}$, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle k, (0.8_a, 0.4_b), (0.2_a, 0.6_b) \rangle$, $H_2 = \langle k, (0.6_a, 0.3_b), (0.4_a, 0.7_b) \rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set $L = \langle k, (0.1_a, 0.3_b), (0.6_a, 0.7_b) \rangle$ in K, then L is an IFPCS as,

$$cl(int(L)) = cl(0_\sim)$$

$$= 0_\sim$$

$$\subseteq L$$

But L is not an IFα*CS in K as,

$$cl(int(cl(L))) = cl(int(H_2^c))$$

$$= cl(0_\sim)$$

$$= 0_\sim$$

$$\neq cl(L)$$

Remark 2.1.17: Every IFαCS in (K,τ) is independent of every IFα*CS in (K,τ).

Example 2.1.18: Let $K = \{a,b\}$, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle k, (0.5_a, 0.7_b), (0.5_a, 0.3_b) \rangle$, $H_2 = \langle k, (0.3_a, 0.6_b), (0.7_a, 0.4_b) \rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set $L = \langle k, (0.4_a, 0.5_b), (0.5_a, 0.4_b) \rangle$ in K, then L is an intuitionistic fuzzy α* closed set as,

$$cl(int(cl(L))) = cl(int(1_\sim))$$

$$= cl(1_\sim)$$

$= 1_{\sim}$

$= cl(L)$

But L is not an IFαCS in H as,

$$cl(int(cl(L))) = cl(int(1_{\sim}))$$

$$= cl(1_{\sim})$$

$$= 1_{\sim} \nsubseteq L$$

Example 2.1.19: Let K = {a,b}, τ = {$0_{\sim}$, H_1, H_2, $1_{\sim}$} be an IFT on K where $H_1 = \langle$ k, (0.8_a, 0.4_b), (0.2_a, 0.6_b) $\rangle$, $H_2 = \langle$ k, (0.6_a, 0.3_b), (0.4_a, 0.7_b) $\rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = $\langle$ k, (0.1_a, 0.3_b), (0.6_a, 0.7_b) $\rangle$ in K, then L is an IFαCS as,

$$cl(int(cl(L))) = cl(int(H_2^c))$$

$$= cl(0_{\sim})$$

$$= 0_{\sim}$$

$$\subseteq L$$

But L is not an IFα*CS in K as,

$$cl(int(cl(L))) = cl(int(H_2^c))$$

$$= cl(0_{\sim})$$

$$= 0_{\sim}$$

$$\neq cl(L)$$

Remark 2.1.20: Every IFβCS in (K,τ) is independent of every IFα*CS in (K,τ).

Example 2.1.21: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, (0.8_a, 0.4_b), (0.2_a, 0.6_b) $\rangle$, $H_2 = \langle$ k, (0.6_a, 0.3_b), (0.4_a, 0.7_b) $\rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = $\langle$ k, (0.8_a, 0.3_b), (0.2_a, 0.7_b) $\rangle$ in K, then L is an intuitionistic fuzzy α* closed set as,

$$\begin{aligned} cl(int(cl(L))) &= cl(int(1_\sim)) \\ &= cl(1_\sim) \\ &= 1_\sim \\ &= cl(L) \end{aligned}$$

But L is not an IFβCS in K as,

$$\begin{aligned} int(cl(int(L))) &= int(cl(H_2)) \\ &= cl(1_\sim) \\ &= 1_\sim \\ &\nsubseteq L \end{aligned}$$

Example 2.1.22: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, (0.8_a, 0.4_b), (0.2_a, 0.6_b) $\rangle$, $H_2 = \langle$ k, (0.6_a, 0.3_b), (0.4_a, 0.7_b) $\rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = $\langle$ k, (0.2_a, 0.4_b), (0.8_a, 0.6_b) $\rangle$ in K, then L is an IFβCS as,

$$\begin{aligned} int(cl(int(L))) &= int(cl(0_\sim)) \\ &= int(0_\sim) \\ &= 0_\sim \subseteq L \end{aligned}$$

But L is not an IFα*CS in K as,

$$\begin{aligned} cl(int(cl(L))) &= cl(int(H_1^c)) \\ &= cl(0_\sim) \\ &= 0_\sim \\ &\neq cl(L) \end{aligned}$$

Remark 2.1.23: Every IFSPCS in (K,τ) is independent of every IFα*CS in (K,τ).

Example 2.1.24: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.8_a, 0.4_b), (0.2_a, 0.6_b) ⟩, H_2 = ⟨ k, (0.6_a, 0.3_b), (0.4_a, 0.7_b) ⟩, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy sets L = ⟨ k, (0.7_a, 0.3_b), (0.3_a, 0.7_b) ⟩ and M = ⟨ k, (0.8_a, 0.3_b), (0.2_a, 0.7_b) ⟩ in K, then L and M are intuitionistic fuzzy α* closed sets as,

$$\begin{aligned} cl(int(cl(L))) &= cl(int(1_\sim)) \\ &= cl(1_\sim) \\ &= 1_\sim \\ &= cl(L) \end{aligned}$$

$$\begin{aligned} cl(int(cl(M))) &= cl(int(1_\sim) \\ &= cl(1_\sim) \\ &= 1_\sim \\ &= cl(M) \end{aligned}$$

But L is not an IFSPCS in K as, M is not an IFPCS as

$$cl(int(M)) = cl(H_2) = 1_\sim \nsubseteq M.$$

Example 2.1.25: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.3_a, 0.6_b), (0.7_a, 0.4_b) ⟩, H_2 = ⟨ k, (0.5_a, 0.9_b), (0.5_a, 0.1_b) ⟩, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy sets L = ⟨ k, (0.4_a, 0.4_b), (0.6_a, 0.6_b) ⟩ and M = ⟨ k, (0.5a, 0.4_b), (0.5_a, 0.6_b) ⟩ in K, then L is an IFSPCS as, M is an IFPCS as,

$$\begin{aligned} cl(int(M)) &= cl(0_\sim) \\ &= 0_\sim \\ &\subseteq M \end{aligned}$$

And,

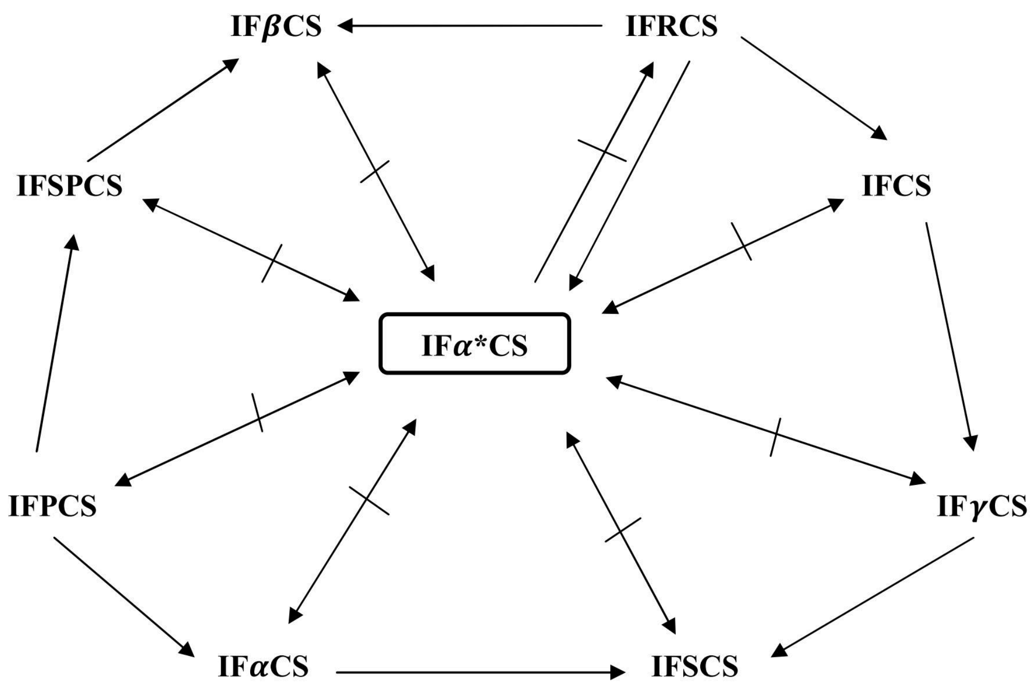
$$\begin{aligned} int(M) &= 0_\sim \\ &\subseteq L \subseteq M \end{aligned}$$

But L and M are not IFα*CS in K as,

$$\begin{aligned} cl(int(cl(L))) &= cl(int(H_1^c)) \\ &= cl(0_\sim) \\ &= 0_\sim \\ &\neq cl(L) \end{aligned}$$

$$\begin{aligned} cl(int(cl(M))) &= cl(int(H_1^c)) \\ &= cl(0_\sim) \\ &= 0_\sim \\ &\neq cl(M) \end{aligned}$$

In the following diagram, we have provided the inter-relation between various types of intuitionistic fuzzy closedness.



Proposition 2.1.26: Let L and M be any two IFα*CS in an IFTS (K,τ), then L ∪ M is also an IFα*CS in (K,τ).

Proof:

Given: L and M are IFα*CSs in (K,τ)

(i.e), cl(int(cl(L))) = cl(L) (1)

and

cl(int(cl(M))) = cl(M) (2)

Now, consider

cl(int(cl(L ∪ M))) ⊆ cl(cl(L ∪ M))

= cl(L ∪ M)

Therefore, cl(int(cl(L ∪ M))) ⊆ cl(L ∪ M) (3)

Now,

$$cl(int(cl(L \cup M))) = cl(int(cl(L) \cup cl(M)))$$

$$\supseteq cl(int(cl(L)) \cup int(cl(M)))$$

$$= cl(int(cl(L))) \cup cl(int(cl(M)))$$

$$= cl(L) \cup cl(M) \qquad \text{[From (1) and (2)]}$$

$$= cl(L \cup M)$$

Therefore, $cl(int(cl(L \cup M))) \supseteq cl(L \cup M)$ (4)

Now, from (3) and (4)

$$cl(int(cl(L \cup M))) = cl(L \cup M)$$

which implies $L \cup M$ is an IFα*CS in (K,τ).

Remark 2.1.27: The intersection of any two IFα*CSs need not be an IFα*CS in IFTS (K,τ).

Example 2.1.28: Let $K = \{a,b\}$, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle k, (0.6_a, 0.8_b), (0.4_a, 0.2_b) \rangle$, $H_2 = \langle k, (0.5_a, 0.5_b), (0.4_a, 0.4_b) \rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy sets $L = \langle k, (0.5_a, 0.4_b), (0.4_a, 0.5_b) \rangle$ and $M = \langle k, (0.4_a, 0.6_b), (0.5_a, 0.2_b) \rangle$ in K, then L and M are IFα*CSs in (K,τ).

Consider,

$$cl(int(cl(L))) = cl(int(1_\sim))$$

$$= cl(1_\sim)$$

$$= 1_\sim$$

$$= cl(L)$$

Therefore, L is an IFα*CS in (K,τ).

And consider,

$$cl(int(cl(M))) = cl(int(1_{\sim}))$$
$$= cl(1_{\sim})$$
$$= 1_{\sim}$$
$$= cl(M)$$

Therefore, M is an IFα*CS in (K,τ).

Now, consider the intersection of L and M as,

$$L \cap M = \langle\, k, (0.4_a, 0.4_b), (0.5_a, 0.5_b) \,\rangle \text{ in } K$$

Then,

$$cl(int(cl(L \cap M))) = cl(int(H_2^c))$$
$$= cl(0_{\sim})$$
$$= 0_{\sim}$$
$$\neq cl(L \cap M)$$

Therefore,

L ∩ M is not an IFα*CS in (K,τ).

Proposition 2.1.29: If L is both an IFCS and an IFα*CS in an IFTS (K,τ), then L is an IFRCS in (K,τ).

Proof: Given: L is an IFCS in (K,τ)

i.e., $cl(L) = L$ (1)

Given: L is an IFα*CS in (K,τ)

i.e., $cl(int(cl(L))) = cl(L)$ (2)

Now, consider,

$$cl(int(L)) = cl(int(cl(L))) \quad \text{[from (1)]}$$

$$= cl(L) \quad \text{[from (2)]}$$

$$= L \quad \text{[from (1)]}$$

Therefore, $cl(int(L)) = L$

Hence, L is an IFRCS in (K,τ).

Proposition 2.1.30: For an IFS L in an IFTS (K,τ), the following are equivalent:

(i) L is an IFα*CS

(ii) $L \subseteq cl(int(cl(L)))$

Proof:

(i) ⇒ (ii) Let L be an IFα*CS in (K,τ).

i.e., $cl(int(cl(L))) = cl(L)$

$\supseteq L$

Therefore, $L \subseteq cl(int(cl(L)))$

(ii) ⇒ (i)

Let $L \subseteq cl(int(cl(L)))$

Then,

$$cl(L) \subseteq cl(cl(int(cl(L))))$$

$$= cl(int(cl(L)))$$

Therefore,

$$cl(L) \subseteq cl(int(cl(L))) \quad (1)$$

Now, consider,

$$cl(int(cl(L))) \subseteq cl(cl(L))$$

$$= cl(L)$$

Therefore, $cl(int(cl(L))) \subseteq cl(L) \quad (2)$

From (1) and (2),

$$cl(int(cl(L))) = cl(L)$$

Hence, L is an IFα*CS in (K,τ).

Proposition 2.1.31: Every intuitionistic fuzzy clopen set in an IFTS (K,τ) is an IFα*CS in (K,τ) but not conversely in general.

Proof: Let L be an intuitionistic fuzzy clopen set in (K,τ), then

$$cl(L) = L \quad (1)$$

and $int(L) = L \quad (2)$

Now, consider

$$cl(int(cl(L))) = cl(int(L)) \quad \text{[from (1)]}$$

$$= cl(L) \quad \text{[from (2)]}$$

Hence,

$$cl(int(cl(L))) = cl(L)$$

Therefore, L is an IFα*CS in (K,τ).

Example 2.1.32: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, (0.5_a, 0.7_b), (0.5_a, 0.3_b) $\rangle$, $H_2 = \langle$ k, (0.3_a, 0.6_b), (0.7_a, 0.4_b) $\rangle$, then (K,τ) is an IFTS.

Now, consider the intuitionistic fuzzy set L = ⟨ k, $(0.2_a, 0.6_b)$, $(0.8_a, 0.4_b)$ ⟩ in K, then L is an intuitionistic fuzzy α* closed set as,

$$\begin{aligned} cl(int(cl(L))) &= cl(int(1_\sim)) \\ &= cl(1_\sim) \\ &= 1_\sim \\ &= cl(L) \end{aligned}$$

But L is not an intuitionistic fuzzy clopen set in K as,

$$\begin{aligned} int(L) &= 0_\sim \\ &\neq L \end{aligned}$$

and

$$\begin{aligned} cl(L) &= 1_\sim \\ &\neq L \end{aligned}$$

Proposition 2.1.33: An IFS L is an IFαCS in an IFTS (K,τ) if L is an IFCS and IFα*CS in (K,τ).

Proof: Let L be an IFCS in (K,τ), then

$$cl(L) = L \qquad (1)$$

Let L be an IFα*CS in (K,τ), then

$$cl(int(cl(L))) = cl(L) \qquad (2)$$

From (2),

$$\begin{aligned} cl(int(cl(L))) &\subseteq cl(L) \\ &= L \qquad \text{[from (1)]} \end{aligned}$$

Therefore, cl(int(cl(L))) ⊆ L and L is an IFαCS in (K,τ).

Proposition 2.1.34: For an intuitionistic fuzzy clopen set L in (K,τ), the following conditions are equivalent:

(i) L is an IFRCS,

(ii) L is an IFα*CS and an IF Q-set.

Proof: Let L be an intuitionistic fuzzy clopen set,

i.e., $cl(L) = L$ (1)

and $int(L) = L$ (2)

(i) ⇒ (ii) Let L be an IFRCS, then

$$cl(int(L)) = L \quad (3)$$

Since, L is an intuitionistic fuzzy clopen set, by Proposition 2.1.31,

L is an IFα*CS in (K,τ)

Now, consider,

$$int(cl(L)) = int(L) \quad \text{[from (1)]}$$

$$= L \quad \text{[from (2)]}$$

$$= cl(int(L)) \quad \text{[from (3)]}$$

Therefore,

$$cl(int(L)) = int(cl(L))$$

L is an IF Q-set in (K,τ).

(ii) ⇒ (i) Let L be an IFα*CS and IF Q-set in (K,τ)

$$cl(int(cl(L))) = cl(L) \quad (4)$$

and

$$cl(int(L)) = int(cl(L)) \quad (5)$$

Consider (4),

$$cl(int(cl(L))) = cl(L)$$

$$cl(cl(int(L))) = L \quad \text{[from (1) \& (5)]}$$

$$cl(int(L)) = L$$

Hence, L is an IFRCS in (K,τ).

Proposition 2.1.35: An IFCS L of an IFTS (K,τ) is an IFα*CS if $L_{\bar{q}}M \Rightarrow [cl(int(cl(L))) = cl(L)]_{\bar{q}}M$ for every IFCS M of K.

Proof: Let M be an IFCS

Let $L_{\bar{q}}M$, then $L \subseteq M^c$

Then, $cl(int(cl(L))) = cl(L)$

$cl(int(cl(L))) = L$

$\subseteq M^c$

Therefore, $[cl(int(cl(L))) = cl(L)]_{\bar{q}}M$.

Proposition 2.1.36: An IFS L of an IFTS (K,τ) is an IFγCS if L is both IFCS and IFα*CS in (K,τ).

Proof: Let L be an IFCS, then

$$cl(L) = L \quad (1)$$

Let L be an IFα*CS, then

$$cl(int(cl(L))) = cl(L) \quad (2)$$

Consider,

$$cl(int(L)) \cap int(cl(L)) = cl(int(cl(L))) \cap int(L) \quad \text{[from (1)]}$$

$= cl(L) \cap int(L)$ [from (2)]

$= int(L) \subseteq L$

Therefore, $cl(int(L)) \cap int(cl(L)) \subseteq L$

Hence, L is an IFγCS in (K,τ).

Proposition 2.1.37: If an IFCS L is both nowhere dense and IFα*CS in (K,τ), then L is IFγCS in K.

Proof: Let L be an IFCS, then

$$cl(L) = L \quad (1)$$

Let L be nowhere dense, then

$$int(cl(L)) = 0_\sim \quad (2)$$

Let L be an IFα*CS, then

$$cl(int(cl(L))) = cl(L) \quad (3)$$

Consider,

$$cl(int(L)) \cap int(cl(L)) = cl(int(cl(L))) \cap int(cl(L)) \quad \text{[from (1)]}$$

$= cl(L) \cap 0_\sim$ [from (2) & (3)]

$= 0_\sim \subseteq L$

Hence, L is an IFγCS in (K,τ).

Proposition 2.1.38: If an IFCS L is both IFβCS and IFα*CS in (K,τ), then L is an IFSCS in K.

Proof: Let L be an IFCS, then

$$cl(L) = L \quad (1)$$

Let L be an IFβCS, then

$$\text{int}(\text{cl}(\text{int}(L))) \subseteq L \qquad (2)$$

Let L be an IFα*CS, then

$$\text{cl}(\text{int}(\text{cl}(L))) = \text{cl}(L) \qquad (3)$$

Consider (2),

$$\text{int}(\text{cl}(\text{int}(L))) \subseteq L$$

$$\text{int}(\text{cl}(\text{int}(\text{cl}(L)))) \subseteq L \qquad \text{[from (1)]}$$

$$\text{int}(\text{cl}(L)) \subseteq L \qquad \text{[from (3)]}$$

Hence, L is an IFSCS in (K,τ).

Proposition 2.1.39: If an IFS L is both IF Q-set and IFα*CS in (K,τ), then L is an IFSOS in K.

Proof: Let L be an IF Q-set, then

$$\text{cl}(\text{int}(L)) = \text{int}(\text{cl}(L)) \qquad (1)$$

Let L be an IFα*CS, then

$$\text{cl}(\text{int}(\text{cl}(L))) = \text{cl}(L) \qquad (2)$$

Consider (2),

$$\text{cl}(\text{int}(\text{cl}(L))) = \text{cl}(L)$$

$$\text{cl}(\text{cl}(\text{int}(L))) = \text{cl}(L) \qquad \text{[from (1)]}$$

$$\text{cl}(\text{int}(L)) = \text{cl}(L)$$

$$\supseteq L$$

$$\text{cl}(\text{int}(L)) \supseteq L$$

Hence, L is an IFSOS in (K,τ).

Proposition 2.1.40: The following are equivalent for an IFS L of an IFTS (K,τ).

(i) L is both IFαCS and IF α*CS

(ii) L is an IFRCS in (K,τ).

Proof:

(i) ⇒ (ii) Let L be an IFαCS,

$$cl(int(cl(L))) \subseteq L \qquad (1)$$

Let L be an IFα*CS,

$$cl(int(cl(L))) = cl(L) \qquad (2)$$

From (1) & (2),

$$cl(L) \subseteq L \qquad (3)$$

$$L \subseteq cl(L) \qquad (4)$$

From (3) & (4),

$$L = cl(L) \qquad (5)$$

Consider,

$$cl(int(L)) = cl(int(cl(L))) \qquad \text{[from (5)]}$$

$$= cl(L) \qquad \text{[from (2)]}$$

$$= L \qquad \text{[from (5)]}$$

Hence, cl(int(L)) = L

Therefore, L is an IFRCS in (K,τ).

(ii) ⇒ (i)Let L be an IFRCS,

$$cl(int(L)) = L \quad (6)$$

By Proposition 2.1.3,

L is an IFα*CS in (K,τ).

$$\text{i.e.,} \quad cl(int(cl(L))) = cl(L) \quad (7)$$

WKT, Every IFRCS is an IFCS,

$$\text{i.e.,} \quad cl(L) = L \quad (8)$$

From (7) & (8),

$$cl(int(cl(L))) = L$$

$$\subseteq L$$

Hence, L is an IFαCS in (K,τ).

2.2 α^* Open Sets in Intuitionistic Fuzzy Topological Spaces

Here we have introduced intuitionistic fuzzy α^* open sets in intuitionistic fuzzy topological spaces and established the inter-relation between intuitionistic fuzzy α^* open sets and already existing intuitionistic fuzzy sets in intuitionistic fuzzy topological spaces.

Definition 2.2.1: An IFS L of an IFTS (K,τ) is said to be an intuitionistic fuzzy α^* open set (IFα^*OS) if int(cl(int(L))) = int(L) whenever L $\subseteq$ U and U is an IFOS in (K,τ).

Example 2.2.2: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K, where $H_1 = \langle$ k, (0.4_a, 0.5_b), (0.6_a, 0.5_b) $\rangle$, $H_2 = \langle$ k, (0.5_a, 0.6_b), (0.5_a, 0.4_b) $\rangle$, then (K,τ) is an IFTS. The IFS L = $\langle$k, (0.4_a, 0.6_b), (0.6_a, 0.4_b)$\rangle$ in K is an IFα^*OS.

Remark 2.2.3: Every IFROS is an IFα^*OS in (K,τ) but the reverse is not true in general.

Proof: Let L be an IFROS in (K,τ) then int(cl(L)) = L. Since, every IFROS is IFOS, we have int(L) = L. Consider, int(cl(int(L))) = int(cl(L)) = L = int(L). Therefore, L is an IFα^*OS in (K,τ).

Example 2.2.4: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, (0.5_a, 0.4_b), (0.5_a, 0.6_b) $\rangle$, $H_2 = \langle$ k, (0.8_a, 0.6_b), (0.2_a, 0.4_b) $\rangle$, then (K,τ) is an IFTS. The IFS L = $\langle$ k, (0.4_a, 0.3_b), (0.6_a, 0.7_b) $\rangle$ in K is an IFα^*OS but L is not an IFROS in K.

Remark 2.2.5: Every IFOS is independent of every IFα^*OS in (K,τ).

Example 2.2.6: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, (0.5_a, 0.4_b), (0.5_a, 0.6_b) $\rangle$, $H_2 = \langle$ k, (0.8_a, 0.6_b), (0.2_a, 0.4_b) $\rangle$, then (K,τ) is an IFTS. The IFS L = $\langle$ k, (0.5_a, 0.3_b), (0.5_a, 0.5_b) $\rangle$ in K is an IFα^*OS but L is not an IFOS in K.

Example 2.2.7: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, (0.4_a, 0.5_b), (0.6_a, 0.5_b) $\rangle$, $H_2 = \langle$ k, (0.5_a, 0.6_b), (0.5_a, 0.4_b) $\rangle$, then (K,τ) is an IFTS. The IFS L = $\langle$ k, (0.5_a, 0.6_b), (0.5_a, 0.4_b) $\rangle$ in K is an IFOS but L is not an IFα^*OS in K.

Remark 2.2.8: Every IFSOS in (K,τ) is independent of every IFα*OS in (K,τ).

Example 2.2.9: Let K = {a,b}, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle$ k, $(0.4_a, 0.5_b), (0.6_a, 0.5_b) \rangle$, $H_2 = \langle$ k, $(0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, then (K,τ) is an IFTS. The IFS L = $\langle$ k, $(0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$ in K is an IFα*OS but L is not an IFSOS in K.

Example 2.2.10: Let K = {a,b}, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle$ k, $(0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$, $H_2 = \langle$ k, $(0.8_a, 0.6_b), (0.2_a, 0.4_b) \rangle$, then (K,τ) is an IFTS. The IFS L = $\langle$ k, $(0.8_a, 0.6_b), (0.2_a, 0.4_b) \rangle$ in K is an IFSOS but L is not an IFα*OS in K.

Remark 2.2.11: Every IFγOS in (K,τ) is independent of every IFα*OS in (K,τ).

Example 2.2.12: Let K = {a,b}, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle$ k, $(0.4_a, 0.5_b), (0.6_a, 0.5_b) \rangle$, $H_2 = \langle$ k, $(0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, then (K,τ) is an IFTS. The IFS L = $\langle$ k, $(0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$ in K is an IFα*OS but L is not an IFγOS in K.

Example 2.2.13: Let K = {a,b}, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle$ k, $(0.2_a, 0.3_b), (0.8_a, 0.7_b) \rangle$, $H_2 = \langle$ k, $(0.3_a, 0.4_b), (0.7_a, 0.6_b) \rangle$, then (K,τ) is an IFTS. The IFS L = $\langle$ k, $(0.3_a, 0.3_b), (0.7_a, 0.7_b) \rangle$ in K is an IFγOS but L is not an IFα*OS in K.

Remark 2.2.14: Every IFPOS in (K,τ) is independent of every IFα*OS in (K,τ).

Example 2.2.15: Let K = {a,b}, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle$ k, $(0.3_a, 0.6_b), (0.7_a, 0.4_b) \rangle$, $H_2 = \langle$ k, $(0.5_a, 0.9_b), (0.5_a, 0.1_b) \rangle$, then (K,τ) is an IFTS. The IFS L = $\langle$ k, $(0.3_a, 0.4_b), (0.7_a, 0.6_b) \rangle$ in K is an intuitionistic fuzzy $\alpha*$ open set but L is not an IFPOS in K.

Example 2.2.16: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.2_a, 0.3_b), (0.8_a, 0.7_b) ⟩, H_2 = ⟨ k, (0.3_a, 0.4_b), (0.7_a, 0.6_b) ⟩, then (K,τ) is an IFTS. The IFS L = ⟨ k, (0.3_a, 0.3_b), (0.7_a, 0.7_b) ⟩ in K is an IFPOS but L is not an IFα*OS in K.

Remark 2.2.17: Every IFαOS in (K,τ) is independent of every IFα*OS in (K,τ).

Example 2.2.18: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.3_a, 0.6_b), (0.7_a, 0.4_b) ⟩, H_2 = ⟨ k, (0.5_a, 0.9_b), (0.5_a, 0.1_b) ⟩, then (K,τ) is an IFTS. The intuitionistic fuzzy set L = ⟨ k, (0.3_a, 0.4_b), (0.7_a, 0.6_b) ⟩ in K is an intuitionistic fuzzy α* open set but L is not an IFαOS in K.

Example 2.2.19: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.5_a, 0.4_b), (0.5_a, 0.6_b) ⟩, H_2 = ⟨ k, (0.8_a, 0.6_b), (0.2_a, 0.4_b) ⟩, then (K,τ) is an IFTS. The intuitionistic fuzzy set L = ⟨ k, (0.8_a, 0.6_b), (0.2_a, 0.4_b) ⟩ in K is an IFαOS but L is not an IFα*OS in K.

Remark 2.2.20: Every IFβOS in (K,τ) is independent of every IFα*OS in (K,τ).

Example 2.2.21: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.3_a, 0.6_b), (0.7_a, 0.4_b) ⟩, H_2 = ⟨ k, (0.5_a, 0.9_b), (0.5_a, 0.1_b) ⟩, then (K,τ) is an IFTS. Now, the intuitionistic fuzzy set L = ⟨ k, (0.3_a, 0.4_b), (0.7_a, 0.6_b) ⟩ in K is an intuitionistic fuzzy α* open set but L is not an IFβOS in K.

Example 2.2.22: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.5_a, 0.4_b), (0.5_a, 0.6_b) ⟩, H_2 = ⟨ k, (0.8_a, 0.6_b), (0.2_a, 0.4_b) ⟩, then (K,τ) is an IFTS. Now, the intuitionistic fuzzy set L = ⟨ k, (0.8_a, 0.6_b), (0.2_a, 0.4_b) ⟩ in K is an IFβOS but L is not an IFα*OS in K.

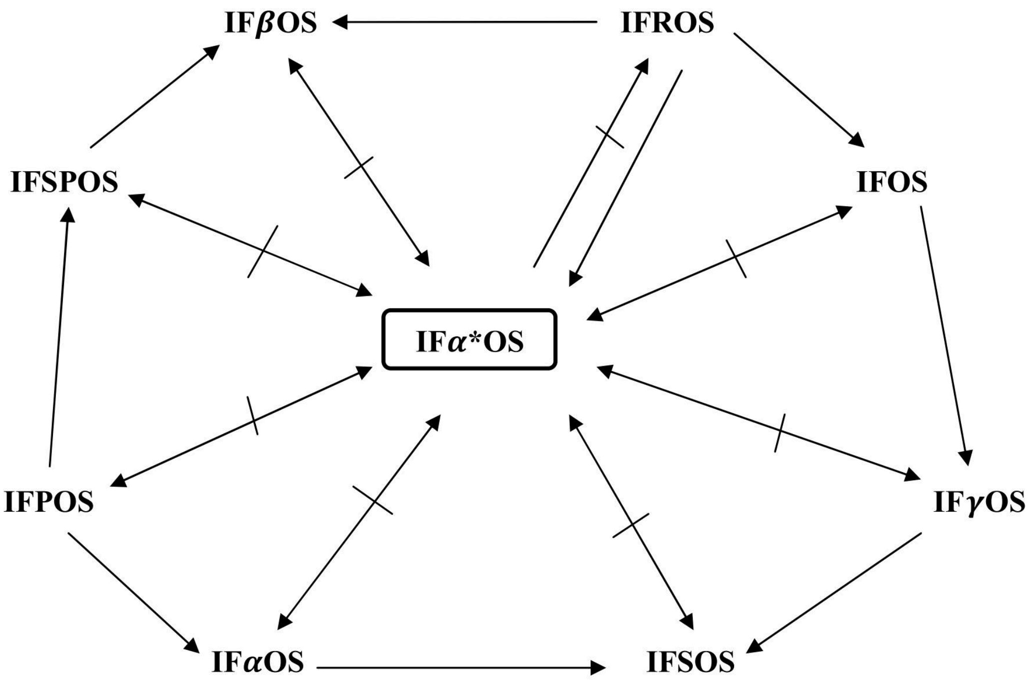
Remark 2.2.23: Every IFSPOS in (K,τ) is independent of every IFα*OS in (K,τ).

Example 2.2.24: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨ k, (0.3_a, 0.6_b), (0.7_a, 0.4_b) ⟩, H_2 = ⟨ k, (0.5_a, 0.9_b), (0.5_a, 0.1_b) ⟩, then (K,τ) is an IFTS. The intuitionistic fuzzy sets L = ⟨ k, (0.5_a, 0.4_b), (0.5_a, 0.6_b) ⟩ and

$M = \langle$ k, $(0.3_a, 0.4_b), (0.7_a, 0.6_b) \rangle$ in K are intuitionistic fuzzy α^* open sets but L is not an IFSPOS in K as M is not an IFPOS.

Example 2.2.25: Let $K = \{a,b\}$, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K where $H_1 = \langle$ k, $(0.2_a, 0.3_b), (0.8_a, 0.7_b) \rangle$, $H_2 = \langle$ k, $(0.3_a, 0.4_b), (0.7_a, 0.6_b) \rangle$, then (K,τ) is an IFTS. Consider, the intuitionistic fuzzy sets $L = \langle$ k, $(0.3_a, 0.3_b), (0.7_a, 0.6_b) \rangle$ and $M = \langle$ k, $(0.3_a, 0.3_b), (0.7_a, 0.7_b) \rangle$ in K, then L is an intuitionistic fuzzy semi pre open set as, M is an IFPOS and $M \subseteq L \subseteq cl(M)$. But, L and M are not IFα^*OSs in K.

In the following diagram, we have provided the inter-relation between various types of intuitionistic fuzzy open sets with IFα^*OS.



Remark 2.2.26: The union of any two IFα^*OS in (K,τ) need not be an IFα^*OS in (K,τ).

Example 2.2.27: Let $K = \{a,b\}$, $\tau = \{0_\sim, H_1, H_2, 1_\sim\}$ be an IFT on K, where $H_1 = \langle$ k, $(0.4_a, 0.5_b), (0.6_a, 0.5_b) \rangle$, $H_2 = \langle$ k, $(0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, then (K,τ) is an IFTS. The IFSs $L = \langle$ k, $(0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$ and $M = \langle$ k, $(0.4_a, 0.6_b), (0.6_a, 0.4_b) \rangle$ in K are intuitionistic fuzzy α^* open sets but the union of L and M, $L \cup M = \langle$ k, $(0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$ is not an IFα^*OS in (K,τ).

Proposition 2.2.28: The intersection of any two IFα*OS is an IFα*OS in an IFTS (K,τ).

Proof: Let L and M be any two IFα*OS in (K,τ), then int(cl(int(L))) = int(L) and int(cl(int(M))) = int(M). Consider, int(cl(int(L ∩ M))) = int(cl[int(L) ∩ int(M)]) ⊆ int[cl(int(L)) ∩ cl(int(M))] = int(cl(int(L))) ∩ int(cl(int(M))) = int(L) ∩ int(M). Therefore, int(cl(int(L ∩ M))) ⊆ int(L) ∩ int(M). Now, int(L ∩ M) = int(int(L ∩ M)) ⊆ int(cl(int(L ∩ M))). Therefore, int(L ∩ M) ⊆ int(cl(int(L ∩ M))). Hence, int(L ∩ M) = int(cl(int(L ∩ M))) which implies L ∩ M is an IFα*OS in (K,τ).

Proposition 2.2.29: If L is both an IFOS and IFα*OS in (K,τ), then L is an IFROS in (K,τ).

Proof: Let L be an IFOS in (K,τ), then int(L) = L. Let L be an IFα*OS in (K,τ), then int(cl(int(L))) = int(L). Consider, int(cl(L)) = int(cl(int(L))) = int(L) = L. Hence, L is an IFROS in (K,τ).

Proposition 2.2.30: For an IFS L in (K,τ), then the following are equivalent:

(i) L is an IFα*OS

(ii) int(cl(int(L))) ⊆ L

Proof: (i) ⇒ (ii) Let L be an IFα*OS in (K,τ), then int(cl(int(L))) = int(L) ⊆ L. Hence, int(cl(int(L))) ⊆ L.

(ii) ⇒ (i) Let int(cl(int(L))) ⊆ L, then int(int(cl(int(L)))) ⊆ int(L) which implies int(cl(int(L))) ⊆ int(L). Now, int(cl(int(L))) ⊇ int(int(L)) = int(L). Therefore, int(cl(int(L))) ⊇ int(L). Hence, int(cl(int(L))) = int(L) that is L is an IFα*OS in (K,τ).

Proposition 2.2.31: If L is an intuitionistic fuzzy clopen set in (K,τ), then L is an IFα*OS in (K,τ) but the converse is not true in general.

Proof: Let L be an intuitionistic fuzzy clopen set in (K,τ), then int(L) = L and cl(L) = L. Consider, int(cl(int(L))) = int(cl(L)) = int(L). Hence, L is an IFα*OS in (K,τ).

Example 2.2.32: Let K = {a,b}, τ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle$ k, (0.4_a, 0.5_b), (0.6_a, 0.5_b) $\rangle$, $H_2 = \langle$ k, (0.5_a, 0.6_b), (0.5_a, 0.4_b) $\rangle$, then (K,τ) is an IFTS. The intuitionistic fuzzy set L = $\langle$ k, (0.4_a, 0.5_b), (0.5_a, 0.4_b) $\rangle$ in K is an IFα*OS. but L is not an intuitionistic fuzzy clopen set.

Proposition 2.2.33: An IFS L of an IFTS (K,τ) is an IFROS if and only if L is an IFOS and IFα*OS in (K,τ).

Proof: Necessity: Suppose L is an IFROS in (K,τ). WKT, Every IFROS is IFOS. Therefore, L is an IFOS in (K,τ). By Remark 2.2.3, L is an IFα*OS in (K,τ).

Sufficiency: Let L be an IFOS, then int(L) = L. Let L be an IFα*OS, then int(cl(int(L))) = int(L). Consider, int(cl(L)) = int(cl(int(L))) = int(L) = L. Hence, L is an IFROS in (K,τ).

Proposition 2.2.34: If an IFS L is an intuitionistic fuzzy nowhere dense set in K, then L is an IFα*OS in (K,τ).

Proof: Let L be an nowhere dense set in K, then int(L) = $0_\sim$. Consider int(cl(int(L))) = int(cl($0_\sim$)) = int($0_\sim$) = $0_\sim$ = int(L). Hence, L is an IFα*OS in (K,τ).

Proposition 2.2.35: For an intuitionistic fuzzy clopen set L in (K,τ), the following conditions are equivalent:

(i) L is an IFROS,

(ii) L is an IFα*OS and an IF Q-set.

Proof: Let L be an intuitionistic fuzzy clopen set, i.e., cl(L) = L and int(L) = L

(i) ⇒ (ii) Let L be an IFROS, then int(cl(L)) = L. Since, L is an intuitionistic fuzzy clopen set, by Proposition 2.2.31, L is an IFα*OS in (K,τ). Now, cl(int(L)) = cl(L) = L = int(cl(L)). Therefore, cl(int(L)) = int(cl(L)) which implies L is an IF Q-set in (K,τ).

(ii) ⇒ (i) Let L be an IFα*OS and IF Q-set in (K,τ), then int(cl(int(L))) = int(L) and cl(int(L)) = int(cl(L)). Consider, int(cl(int(L))) = int(L) ⇒ int(int(cl(L))) = L ⇒ int(cl(L)) = L. Hence, L is an IFROS in (K,τ).

Proposition 2.2.36: An IFS L is both IFαOS and IFα*OS in an IFTS (K,τ) if and only if L is an IFROS in (K,τ).

Proof: Necessity: Suppose L is both IFαOS and IFα*OS, L $\subseteq$ int(cl(int(L))) and int(cl(int(L))) = int(L). Therefore, L $\subseteq$ int(L). W.K.T., int(L) $\subseteq$ L. Therefore, L = int(L). Consider, int(cl(L)) = int(cl(int(L))) = int(L) = L. Hence, L is an IFROS in (K,τ).

Sufficiency: Let L be an IFROS, int(cl(L)) = L. By Remark 2.2.3, L is an IFα*OS in (K,τ). (i.e.,) int(cl(int(L))) = int(L). W.K.T., Every IFROS in (K,τ) is an IFOS in (K,τ) which is int(L) = L. Therefore, int(cl(int(L))) = L which implies L $\subseteq$ int(cl(int(L))). Hence, L is an IFαOS in (K,τ).

Proposition 2.2.37: If L is an IFOS in (K,τ), then L is an IFα*CS in an IFTS (K,τ).

Proof: Let L be an IFOS in (K,τ), then, int(L) = L. Consider, cl(int(cl(L))) = cl(cl(L)) = cl(L). Hence, L is an IFα*CS in (K,τ).

Proposition 2.2.38: If L is an IFROS in (K,τ), then L is an IFα*CS in an IFTS (K,τ).

Proof: Let L be an IFROS in (K,τ), then int(cl(L)) = L. Consider, cl(int(cl(L))) = cl(L). Hence, L is an IFα*CS in (K,τ).

Proposition 2.2.39: If an IFS L is both IFCS and IFα*CS in (K,τ), then L is an IFγOS in (K,τ).

Proof: Let L be an IFCS, then, cl(L) = L. Let L be an IFα*CS, then cl(int(cl(L))) = cl(L). Consider, int(cl(L)) $\cup$ cl(int(L)) = int(L) $\cup$ cl(int(cl(L))) = int(L) $\cup$ cl(L) = cl(L) $\supseteq$ L. Therefore, L $\subseteq$ int(cl(L)) $\cup$ cl(int(L)). Hence, L is an IFγOS in (K,τ).

Proposition 2.2.40: If an IFCS L is both nowhere dense and IFα*CS in (K,τ), then L is IFγOS in (K,τ).

Proof: Let L be an IFCS, then cl(L) = L. Let L be nowhere dense, then int(cl(L)) = $0_\sim$. Let L be an IFα*CS, then cl(int(cl(L))) = cl(L). Consider, cl(int(L)) $\cup$ int(cl(L)) = cl(int(cl(L))) $\cup$ $0_\sim$ = cl(L) $\cup$ $0_\sim$ = cl(L) $\supseteq$ L. Therefore, L $\subseteq$ cl(int(L)) $\cup$ int(cl(L)). Hence, L is an IFγOS in (K,τ).

Proposition 2.2.41: If an IFCS L is both IFαOS and IFα*CS in (K,τ), then L is an IFPOS in K.

Proof: Let L be an IFCS, then cl(L) = L. Let L be an IFα*CS, then cl(int(cl(L))) = cl(L). Let L be an IFαOS, then L ⊆ int(cl(int(L))). Consider, L ⊆ int(cl(int(cl(L)))) = int(cl(L)). Therefore, L ⊆ int(cl(L). Hence, L is an IFPOS in K.

SUMMARY AND CONCLUSION

SUMMARY AND CONCLUSION

In the literature it is observed that there are large numbers of ways to generalize the closed sets in intuitionistic fuzzy topological space.

The concept of "Intuitionistic fuzzy sets" was first published by Atanassov K in the year 1986. Using the notion of intuitionistic fuzzy sets, Coker D introduced the idea of intuitionistic fuzzy topological spaces in the year 1997.

In this thesis, we have introduced intuitionistic fuzzy $\alpha*$ closed sets (intuitionistic fuzzy $\alpha*$ open sets) and we made an attempt to compare intuitionistic fuzzy $\alpha*$ closed sets and intuitionistic fuzzy $\alpha*$ open sets in intuitionistic fuzzy topological spaces with some other existing intuitionistic fuzzy closed (open) sets, intuitionistic fuzzy regular closed (open) sets, intuitionistic fuzzy semi closed (open) sets, intuitionistic fuzzy pre closed (open) sets, intuitionistic fuzzy α closed (open) sets, intuitionistic fuzzy β closed (open) sets, intuitionistic fuzzy γ closed (open) sets, intuitionistic fuzzy semi pre closed (open) sets. Also, the union and intersection of intuitionistic fuzzy $\alpha*$ closed(open) sets are examined.

The future research directions based on this research work may be extended as follows:

The notion of intuitionistic fuzzy $\alpha*$ closed sets can be studied for continuous, connectedness, separation axioms, homeomorphisms, compactness in intuitionistic fuzzy topological spaces. It can also be extended to bitopological spaces, supra topological spaces and nano topological spaces.

REFERENCE

REFERENCES

[1] **Abd El-Monsef, M. E.**, **El-Deeb, S. N.** and **Mahmoud, R. A.**, β-open Sets and β-continuous mappings, Bulletin of the Faculty of Science Assiut University, Vol.12, 1983, 77 – 90.

[2] **Andrijevic, D.,** b-open sets, Matematicki Vesnik, Vol. 48, 1996, 59 – 64.

[3] **Andrijevic, D.**, On Semi Pre Open Sets, Matematicki Vesnik, Vol.38, 1986, 24 – 32.

[4] **Annalakshmi, M.** and **Chandramouleeswaran, M**., Interior and Closure of Fuzzy open sets in Fuzzy Topological TM-System, Global Journal of Pure and Applied Mathematics, Vol. 11, No. 5, 2015, 3157 – 3164.

[5] **Atanassov, K.**, Intuitionistic Fuzzy Sets, Fuzzy sets and Systems, 1986, 87 – 96.

[6] **Atanassov, K.**, More on intuitionistic fuzzy sets, Fuzzy sets and Systems, Vol.33, 1989, 37 – 45.

[7] **Atanassov, K.,** New operations of intuitionistic fuzzy sets-III, Fuzzy Sets and Systems, Vol. 61, 1994, 137 – 142.

[8] **Atanassov, K.**, On four intuitionistic fuzzy topological operators, Mathware and soft computing, Vol. 8, 2001, 65 – 70.

[9] **Bhattacharjee, P.** and **Bhaumil, R. N.,** Pre-Semi-closed sets and Pre-Semi Separation axioms in Intuitionistic fuzzy topological spaces, General Mathematics Notes, Vol. 8, 2012, 11 – 17.

[10] **Chang, C. L.**, Fuzzy Topological Spaces, Journal of Mathematical Analysis and Applications, Vol. 24, 1968, 182 – 190.

[11] **Coker, D.** and **Demirci, M.,** On Intuitionistic Fuzzy Points, Notes on intuitionistic fuzzy sets, vol. 1, 1995, 79 – 84.

[12] **Coker, D.**, A Note on Intuitionistic Sets and Intuitionistic Points, Turkish Journal of Mathematics, Vol. 20, 1996, 343 – 351.

[13] **Coker, D.**, An introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy sets and Systems, 1997, 81 – 89.

[14] **Ganguly, S.** and **Saha, S.**, A note on fuzzy semi open sets in fuzzy topological spaces, Fuzzy Sets and Systems, Vol. 18, 1986, 83 – 96.

[15] **Hanafy, I. M.,** On fuzzy γ-open sets and fuzzy γ-continuity, Journal of Fuzzy Mathematics, Vol. 10, 2002, 9-19.

[16] **Hanafy, I. M.**, Intuitionistic Fuzzy γ- Continuity, The Canadian Mathematical Bulletin, Vol. 52, 2009, 544 – 554.

[17] **Hatir, E., Noiri, T.** and **Yuksel, S.**, A decomposition of continuity, Acta Mathematica Hungarica, Vol. 70, 1996, 145 – 150.

[18] **Jayanthi, D.**, Relation between Semi Pre Closed Sets and Beta Closed Sets in Intuitionistic Fuzzy Topological Spaces, 9th International Workshop on IFSs, Bankska Bystrics, Vol. 19, 2013, No.2, 53 – 56.

[19] **Joung Kon Jeon, Young Bae Jun** and **Jin Han Park.,** Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy precontinuity, International Journal of Mathematics and Mathematical Sciences, Vol. 19, 2005, 3091 – 3101.

[20] **Lee, S. J.** and **Lee, E. P.**, The category of Intuitionistic Fuzzy topological spaces, Bulletin of the Korean Mathematical Society, Vol. 37, 2000, 63 – 76.

[21] **Levine, N.**, Semi-open sets and semi-continuity in topological spaces, The American Mathematical Monthly, Vol. 70, 1963, 36 – 41.

[22] **Lupianez, F. G.**, Intuitionistic fuzzy topological operators and topology, International Journal of Pure and Applied Mathematics, Vol. 17, 2004, 35 – 40.

[23] **Lupianez, F. G.**, Quasi – coincidence for intuitionistic fuzzy points, International Journal of Mathematics and Mathematical Sciences, vol. 10, 2005, 1539-1542.

[24] **Rajarajeswari, P.** and **Senthil Kumar, L.**, Generalized pre closed sets in intuitionistic fuzzy topological spaces, International Journal of Fuzzy Mathematics and Systems, Vol.1, 2011, 253 – 262.

[25] **Riya, V. M.** and **Jayanthi, D.**, On Intuitionistic Fuzzy γ^* Generalized Closed Sets, Advances in Fuzzy Mathematics, Vol. 12, 2017, 389 – 410.

[26] **Santhi, R.** and **Jayanthi, D.**, Generalized Semi-Pre Connectedness in Intuitionistic Fuzzy Topological Space, Annals of Fuzzy Mathematics and Informatics, Vol. 3, 2012, 243 – 253.

[27] **Saranya, M.** and **Jayanthi, D.**, On Intuitionistic Fuzzy β Generalized Closed Sets, International Journal of Computational Engineering Research, 2016, Vol. 6, 37 – 42.

[28] **Singal, M. K.** and **Niti Prakash.**, Fuzzy pre open sets and fuzzy pre separation axioms, Fuzzy Sets and Systems, Vol. 44, 1991, 273 – 281.

[29] **Singal, M. K.** and **Niti Rajvanshi.**, Fuzzy α sets and α continuous maps, Fuzzy Sets and Systems, Vol. 48, 1992, 383 – 390.

[30] **Sudha, S. M.** and **Jayanthi, D.**, β^{**} Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces, Advances in Mathematics Scientific Journal, 2020, 667 – 677.

[31] **Thakur, S. S.** and **Dhavaseelan, R.**, Nowhere dense sets in Intuitionistic Fuzzy Topological Spaces, Proceedings of National Seminar on Recent Developments in Topology, 2015, 11 – 12.

[32] **Thakur, S. S.** and **Rekha Chaturvedi.**, Regular Generalized Closed set in Intuitionistic Fuzzy Topological Spaces, Universitatea Din Bacau, Studii Si CercetariStiintifice, Seria, Vol. 16, 2006, 257 – 272.

[33] **Thakur, S. S.** and **Singh, S.**, On fuzzy semi pre open sets and fuzzy semi pre continuity, Fuzzy Sets and Systems, Vol. 67, 1998, 383 – 391.

[34] **Young Bae Jun** and **Seok-zun song.**, Intuitionistic Fuzzy Semi Pre Open Sets and Intuitionistic Fuzzy Semi-Pre Continuous Mappings, Journal of Applied Mathematics and Computing, Vol. 19, 2005, 467-474.

[35] **Zadeh, L. A.**, Fuzzy sets, Information and Control, Vol. 8, 1965, 338 – 353.