



Chapter IV

CHAPTER-4
SEMI STAR GENERALIZED W - CLOSED SETS
IN BITOPOLOGICAL SPACES

The concepts of semi generalized open sets and semi generalized closed sets in a topological space was introduced by Chandrasekhara Rao and Joseph [8]. In this chapter we discuss the generalizations of these concepts due to Chandrasekhara Rao and Narasimhan[13] in Bitopological spaces.

SECTION: 4.1
PRELIMINARIES

Notation: 4.1.1

Let (X, τ_1, τ_2) or simply X denote a bitopological space. For any subset $A \subseteq X$, $\tau_i - \text{int}(A)$ and $\tau_i - \text{cl}(A)$ denote the interior and closure of a set A with respect to a topology τ_i . The closure and interior with respect to the topology τ_i of B relative to A is written as $\tau_i - \text{cl}_A(B)$ and $\tau_i - \text{int}_A(B)$ respectively.

Definition: 4.1.2

A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ - **Generalized semi open** ($\tau_1\tau_2$ - **gs open**) if $F \subseteq \tau_2 - \text{sint}(A)$ whenever $F \subseteq A$ and F is τ_1 - closed in X .

Definition: 4.1.3

A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ - **Generalized semi closed** ($\tau_1\tau_2$ - **gs closed**) if $X - A$ is $\tau_1\tau_2$ - **gs open**.

Definition: 4.1.4

A subset A of a bitopological space (X, τ_1, τ_2) is called **$\tau_1\tau_2$ - generalized w - closed ($\tau_1\tau_2$ - gw closed)** if $\tau_2 - cl_w(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 - open in X .

Definition: 4.1.5

A subset A of a bitopological space (X, τ_1, τ_2) is called **$\tau_1\tau_2$ - generalized w - open ($\tau_1\tau_2$ -gw open)** if $X-A$ is $\tau_1\tau_2$ -gw closed.

Definition: 4.1.6

A subset A of a bitopological space (X, τ_1, τ_2) is called **$\tau_1\tau_2$ - regular generalized w-closed ($\tau_1\tau_2$ -rgw closed)** if $\tau_2-cl_w(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$ - regular open in X

Definition: 4.1.7

A subset A of a bitopological space (X, τ_1, τ_2) is called **$\tau_1\tau_2$ - regular generalized w - open ($\tau_1\tau_2$ -rgw open)** if $X-A$ is $\tau_1\tau_2$ -rgw closed.

SECTION: 4.2**SEMI STAR GENERALIZED W - CLOSED SETS****Definition: 4.2.1**

A set A of a topological space (X, τ) is called **semi star generalized w-closed (s^* gw closed)** if $cl_w(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X .

Definition: 4.2.2

A set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ - **semi star generalized w- closed** ($\tau_1\tau_2$ - **s*gw closed**) if $\tau_2\text{-cl}_w(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -semi open in X .

Example: 4.2.3

Let X be the set of all real numbers \mathbb{R} , $\tau_1 = \{\phi, \mathbb{R}, \mathbb{R} - Q\}$, $\tau_2 = \{\phi, \mathbb{R}, Q\}$, where Q is the set of all rational numbers. Then $\mathbb{R} - Q$ is $\tau_1\tau_2$ - s*gw closed.

Theorem: 4.2.4

Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. Then the following are true.

- (a) If A is τ_2 - w closed, then A is $\tau_1\tau_2$ - s*gw closed.
- (b) If A is τ_1 - semi open and τ_2 - s*gw closed, then A is τ_2 w - closed .
- (c) If A is τ_2 - s*gw - closed, then A is $\tau_1\tau_2$ gw - closed.

Proof:

(a) Suppose that A is τ_2 - w closed. Let $A \subseteq U$ and U is τ_1 - semi open in X . Then $\tau_2\text{-cl}_w(A) = A \subseteq U$. Therefore A is $\tau_1\tau_2$ - s*gw closed.

(b) Suppose that A is τ_1 - semi open and $\tau_1\tau_2$ - s*gw closed. Let $A \subseteq A$ and A is τ_1 - semi open. Then $\tau_2\text{-cl}_w(A) \subseteq A$. Therefore, $\tau_2\text{-cl}_w(A) = A$. Therefore A is τ_2 - w closed.

(c) Suppose that A is $\tau_1\tau_2$ - s*gw closed. Let $A \subseteq U$ and U is τ_1 - open in X . Then U is τ_1 - semi open in X and $\tau_2\text{-cl}_w(A) \subseteq U$. Therefore A is $\tau_1\tau_2$ - gw closed.

Theorem: 4.2.5

Every $\tau_1\tau_2 - s^*g$ closed set is $\tau_1\tau_2 - s^*gw$ closed and every τ_2 - closed set is $\tau_1\tau_2 - s^*gw$ closed.

Remark: 4.2.6

From the Theorem 4.2.4, Theorem 4.2.5 and above definitions, the following relations.

$$\begin{array}{ccc}
 \tau_1\tau_2 - s^*gw \text{ closed} \Rightarrow \tau_1\tau_2 - gw \text{ closed} \Rightarrow \tau_1\tau_2 - rgw \text{ closed} & & \\
 \uparrow & \uparrow & \uparrow \\
 \tau_2 - w \text{ closed} & \tau_1\tau_2 - g \text{ closed} \Rightarrow \tau_1\tau_2 - rg \text{ closed} & \\
 \uparrow & \uparrow & \\
 \tau_2 - \text{closed} \Rightarrow \tau_1\tau_2 - s^*g \text{ closed} \Rightarrow \tau_1\tau_2 - gs \text{ closed} & & \\
 \downarrow & \downarrow & \\
 \tau_2 - \text{semi closed} \Rightarrow \tau_1\tau_2 - sg \text{ closed} & &
 \end{array}$$

Theorem: 4.2.7

If A is $\tau_1\tau_2 - s^*gw$ closed in X and $A \subseteq B \subseteq \tau_2 - cl_w(A)$, then B is $\tau_1\tau_2 - s^*gw$ closed.

Proof:

Suppose that A is $\tau_1\tau_2 - s^*gw$ closed in X and $A \subseteq B \subseteq \tau_2 - cl_w(A)$. Let $B \subseteq U$ and U is τ_1 - semi open in X. Then $A \subseteq U$. Since A is $\tau_1\tau_2 - s^*gw$ closed, $\tau_2 - cl_w(A) \subseteq U$. Since $B \subseteq \tau_2 - cl_w(A)$, $\tau_2 - cl_w(B) \subseteq \tau_2 - cl_w(A) \subseteq U$. Hence B is $\tau_1\tau_2 - s^*gw$ closed.

Theorem: 4.2.8

If A and B are $\tau_1\tau_2 - s^*gw$ closed sets then so is $A \cup B$.

Proof:

Suppose that A and B are $\tau_1\tau_2 - s^*gw$ closed sets. Let U be $\tau_1 -$ semi open in X and $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $\tau_1\tau_2 - s^*gw$ closed sets, $\tau_2 - cl_w(A) \subseteq U$ and $\tau_2 - cl_w(B) \subseteq U$. Consequently $\tau_2 - cl_w(A \cup B) \subseteq U$. Therefore, $A \cup B$ is $\tau_1\tau_2 - s^*gw$ closed.

Theorem: 4.2.9

Let $B \subseteq A \subseteq X$ where A is $\tau_1 -$ open and $\tau_1\tau_2 - s^*gw$ closed in X . Then B is $\tau_1\tau_2 - s^*gw$ closed relative to A if and only if B is $\tau_1\tau_2 - s^*gw$ closed relative to X .

Proof:

Suppose that $B \subseteq A \subseteq X$ where A is $\tau_1 -$ open and $\tau_1\tau_2 - s^*gw$ closed. Suppose that B is $\tau_1\tau_2 - s^*gw$ closed relative to A . Let $B \subseteq U$ and U is $\tau_1 -$ semi open in X . Since $A \subseteq X$, A is $\tau_1 -$ open, $A \cap U$ is $\tau_1 -$ semi open in X . Consequently, $A \cap U$ is $\tau_1 -$ semi open in A . Since $B \subseteq A$, $B \subseteq U$, $B \subseteq A \cap U$. Since B is $\tau_1\tau_2 - s^*gw$ closed relative to A , $\tau_2 - cl_w(B_A) \subseteq A \cap U$. Therefore, $\tau_2 - cl_w(B_A) \subseteq U$. Since A is $\tau_1 -$ open, $\tau_2 - cl_w(B_A) = \tau_2 - cl_w(B) \cap A = \tau_2 - cl_w(B) \subseteq U$. Hence B is $\tau_1\tau_2 - s^*gw$ closed relative to X .

Conversely, suppose that B is $\tau_1\tau_2 - s^*gw$ closed relative to X . Let $B \subseteq U$ and U is $\tau_1 -$ semi open in A . Since $A \subseteq X$, U is $\tau_1 -$ semi open in X . Since B is $\tau_1\tau_2 - s^*gw$ closed relative to X , $\tau_2 - cl_w(B) \subseteq U$.

Now, $\tau_2 - cl_w(B_A) = \tau_2 - cl_w(B) \cap A = \tau_2 - cl_w(B) \subseteq U$. Therefore, B is $\tau_1\tau_2 - s^*gw$ closed relative to A .

Corollary: 4.2.10

If A is $\tau_1\tau_2 - s^*gw$ closed, $\tau_1 -$ semi open in X and F is $\tau_2 - w$ closed in X , then $A \cap F$ is $\tau_2 - w$ closed in X .

Proof:

Since A is $\tau_1\tau_2$ - s^* gw closed, τ_1 - semi open in X , A is τ_2 - w closed in X . {By Theorem 4.2.4 (b)}. Since F is τ_2 - w - closed in X , $A \cap F$ is τ_2 - w - closed in X .

Theorem: 4.2.11

If a set A is $\tau_1\tau_2$ - s^* gw closed in X then τ_2 - $cl_w(A) - A$ contains no nonempty τ_1 - semi closed set.

Proof:

Suppose that A is $\tau_1\tau_2$ - s^* gw closed in X . Let F be τ_1 - semi closed and $F \subseteq \tau_2 - cl_w(A) - A$. Since F be τ_1 - semi closed, F^C is τ_1 - semi open. Since $F \subseteq \tau_2 - cl_w(A) - A$, $F \subseteq \tau_2 - cl_w(A)$ and $A \subseteq F^C$. Since A is $\tau_1\tau_2$ - s^* gw closed in X , $\tau_2 - cl_w(A) \subseteq F^C$. Consequently, $F = \phi$. Hence $\tau_2 - cl_w(A) - A$ contains no nonempty τ_1 - semi closed set.

Corollary: 4.2.12

Let A be $\tau_1\tau_2$ - s^* gw closed. Then A is τ_2 - w - closed if and only if $\tau_2 - cl_w(A) - A$ is τ_1 - semi closed.

Proof:

Suppose that A is $\tau_1\tau_2$ - s^* gw closed. Since A is τ_2 - w - closed, we have $\tau_2 - cl_w(A) = A$. Then $\tau_2 - cl_w(A) - A = \phi$ is τ_1 - semi closed. Conversely, Suppose that A is $\tau_1\tau_2$ - s^* gw closed and $\tau_2 - cl_w(A) - A$ is τ_1 - semi closed. Since A is $\tau_1\tau_2$ - s^* gw closed, $\tau_2 - cl_w(A) - A$ contains no nonempty τ_1 - semi closed set {by Theorem 2.10}. Since $\tau_2 - cl_w(A) - A$ is itself τ_1 - semi closed, $\tau_2 - cl_w(A) - A = \phi$. Then $\tau_2 - cl_w(A) = A$. Hence A is τ_2 - w - closed.

Theorem: 4.2.13

If A is $\tau_1\tau_2 - s^*gw$ closed and $A \subseteq B \subseteq \tau_2 - cl_w(A)$ then $\tau_2 - cl_w(B) - B$ contains no nonempty τ_1 - semi closed set.

Proof:

Let A be $\tau_1\tau_2 - s^*gw$ closed and $A \subseteq B \subseteq \tau_2 - cl_w(A)$. Then B is $\tau_1\tau_2 - s^*gw$ closed. {By Theorem 4.2.7}. Hence $\tau_2 - cl_w(B) - B$ contains no nonempty τ_1 - semi closed set. { By Theorem 4.2.11 }

SECTION: 4.3**SEMI STAR GENERALIZED W - OPEN SETS****Definition: 4.3.1**

A set A of a topological space (X, τ_1, τ_2) is called **$\tau_1\tau_2 -$ semi star generalized w open ($\tau_1\tau_2 - s^*gw$ open)** if and only if A^C is $\tau_1\tau_2 - s^*gw$ Closed .

Example: 4.3.2

In Example 4.2.3, Q is $\tau_1\tau_2 - s^*gw$ open

Theorem: 4.3.3

A set A is $\tau_1\tau_2 - s^*gw$ open if and only if $F \subseteq \tau_2 - int_w(A)$ whenever F is τ_1 - semi closed and $F \subseteq A$.

Proof:

Suppose that A is $\tau_1\tau_2 - s^*gw$ open. Suppose that F is τ_1 - semi closed and $F \subseteq A$. Then F^C is τ_1 - semi open and $A^C \subseteq F^C$. Since A^C is $\tau_1\tau_2 - s^*gw$ closed, $\tau_2 - cl_w(A^C) \subseteq F^C$. Since $\tau_2 - cl_w(A^C) = [\tau_2 - int_w(A)]^C$, we have $F \subseteq \tau_2 - int_w(A)$.

Conversely, suppose that $F \subseteq \tau_2 - \text{int}_w(A)$ whenever F is τ_1 - semi closed and $F \subseteq A$. Then $A^C \subseteq F^C$ and F^C is τ_1 - semi open. Since $F \subseteq \tau_2 - \text{int}_w(A)$, and $\tau_2 - \text{cl}_w(A^C) = [\tau_2 - \text{int}_w(A)]^C$, $\tau_2 - \text{cl}_w(A^C) \subseteq U$. Then A^C is $\tau_1\tau_2 - s^*gw$ closed. Consequently A is $\tau_1\tau_2 - s^*gw$ open.

Theorem: 4.3.4

If A and B are separated $\tau_1\tau_2 - s^*gw$ open sets, then $A \cup B$ is $\tau_1\tau_2 - s^*gw$ open set.

Proof:

Suppose A and B are separated $\tau_1\tau_2 - s^*gw$ open sets. Let F be τ_1 semi closed and $F \subseteq A \cup B$. Since A and B are separated, $\tau_1 - \text{cl}(A) \cap B = A \cap \tau_1 - \text{cl}(B) = \phi$ and $\tau_2 - \text{cl}(A) \cap B = A \cap \tau_2 - \text{cl}(B) = \phi$. Then, $F \cap \tau_2 - \text{cl}(A) \subseteq (A \cup B) \cap \tau_2 - \text{cl}(A) = A$. Similarly, we can prove $F \cap \tau_2 - \text{cl}(B) \subseteq B$. Since F is τ_1 - semi closed, $F \cap \tau_1 - \text{cl}(A)$ and $F \cap \tau_1 - \text{cl}(B)$ are τ_1 - semi closed. Since A and B are $\tau_1\tau_2 - s^*gw$ open, $F \cap \tau_2 - \text{cl}(A) \subseteq \tau_2 - \text{int}_w(A)$ and $F \cap \tau_2 - \text{cl}(B) \subseteq \tau_2 - \text{int}_w(B)$. Now, $F = F \cap (A \cup B) \subseteq \{F \cap \tau_2 - \text{cl}(A)\} \cup \{F \cap \tau_2 - \text{cl}(B)\} \subseteq \tau_2 - \text{int}_w(A \cup B)$. Therefore, $A \cup B$ is $\tau_1\tau_2 - s^*gw$ open.

Theorem: 4.3.5

If A and B are $\tau_1\tau_2 - s^*gw$ open sets, then so is $A \cap B$.

Proof.

Suppose that A and B are $\tau_1\tau_2 - s^*gw$ open sets. Let F be τ_1 - semi closed and $F \subseteq A \cap B$. Then, $F \subseteq A$ and $F \subseteq B$. Since A and B are $\tau_1\tau_2 - s^*gw$ open, $F \subseteq \tau_2 - \text{int}_w(A)$ and $F \subseteq \tau_2 - \text{int}_w(B)$. Hence $F \subseteq \tau_2 - \text{int}_w(A \cap B)$. Consequently $A \cap B$ is $\tau_1\tau_2 - s^*gw$ open set.

Theorem: 4.3.6

If A is $\tau_1\tau_2$ - s^* gw open in X and $\tau_2\text{-int}_w(A) \subseteq B \subseteq A$, then B is $\tau_1\tau_2$ - s^* gw open.

Proof:

Suppose that A is $\tau_1\tau_2$ - s^* gw open in X and $\tau_2\text{-int}_w(A) \subseteq B \subseteq A$. Let F be τ_1 -semi closed and $F \subseteq B$. Since $F \subseteq B$, $B \subseteq A$, $F \subseteq A$. Since A is $\tau_1\tau_2$ - s^* gw open, $F \subseteq \tau_2\text{-int}_w(A)$. Since $\tau_2\text{-int}_w(A) \subseteq B$, $\tau_2\text{-int}_w(A) \subseteq \tau_2\text{-int}_w(B)$. Then $F \subseteq \tau_2\text{-int}_w(B)$. Therefore, B is $\tau_1\tau_2$ - s^* gw open set.

Theorem: 4.3.7

If a set A is $\tau_1\tau_2$ - s^* gw closed in X then $\tau_2\text{-cl}_w(A) - A$ is $\tau_1\tau_2$ - s^* gw open.

Proof:

Suppose that A is $\tau_1\tau_2$ - s^* gw closed in X . Let F be τ_1 -semi closed and $F \subseteq \tau_2\text{-cl}_w(A) - A$. Since A is $\tau_1\tau_2$ - s^* gw closed in X , $\tau_2\text{-cl}_w(A) - A$ contains no nonempty τ_1 -semi closed set. Since $F \subseteq \tau_2\text{-cl}_w(A) - A$, $F = \emptyset \subseteq \tau_2\text{-int}_w[\tau_2\text{-cl}_w(A) - A]$. Therefore, $\tau_2\text{-cl}_w(A) - A$ is $\tau_1\tau_2$ - s^* gw open.

Theorem: 4.3.8

If a set A is $\tau_1\tau_2$ - s^* gw open in a bitopological space (X, τ_1, τ_2) , then $G = X$ whenever G is τ_1 -semi open and $\tau_2\text{-int}_w(A) \cup A^C \subseteq G$.

Proof:

Suppose that A is $\tau_1\tau_2$ - s^* gw open in a bitopological space (X, τ_1, τ_2) and G is τ_1 -semi open and $\tau_2\text{-int}_w(A) \cup A^C \subseteq G$. Then $G^C \subseteq \{\tau_2\text{-int}_w(A)\} \cup A^C \}^C = \tau_2\text{-cl}_w(A^C) - A^C$. Since G^C is τ_1 -semi closed and A^C

is $\tau_1\tau_2 - s^*gw$ closed , $\tau_2 - clw(A^C) - A^C$ contains no nonempty τ_1 -semi closed set in X { By Theorem 4.2.11 }. Therefore, $G^C = \phi$. Hence $G = X$.

Theorem: 4.3.9

The intersection of a $\tau_1\tau_2 - s^*gw$ open set and a $\tau_1 - w$ open set is always $\tau_1\tau_2 - s^*gw$ open.

Proof:

Suppose that A is $\tau_1\tau_2 - s^*gw$ open and B is $\tau_1 - w$ open. Then B^C is $\tau_2 - w$ closed. Therefore, B^C is $\tau_1\tau_2 - s^*gw$ closed. {By Theorem 4.2.4 (a)}. Hence B is $\tau_1\tau_2 - s^*gw$ open. Consequently, $A \cap B$ is $\tau_1\tau_2 - s^*gw$ open. { By Theorem 4.3.5 }

Theorem: 4.3.10

If $A \times B$ is $\tau_1\tau_2 - s^*gw$ open subset of $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$ then A is $\tau_1\tau_2 - s^*gw$ open subset in (X, τ_1, τ_2) and B is $\sigma_1\sigma_2 - s^*gw$ open subset in (Y, σ_1, σ_2) .

Proof:

Let F be a τ_1 - semi closed subset of (X, τ_1, τ_2) and let G be a σ_1 -semi closed subset of (Y, σ_1, σ_2) such that $F \subseteq A$ and $G \subseteq B$. Then $F \times G$ is

$\tau_1 \times \sigma_1$ - semi closed in $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$ such that $F \times G \subseteq A \times B$.

By assumption $A \times B$ is $\tau_1\tau_2 - s^*gw$ open in $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$ and

so $F \times G \subseteq \tau_2 - int_w(A \times B) \subseteq \tau_2 - int_w(A) \times \tau_2 - int_w(B)$. Therefore

$F \subseteq \tau_2 - int_w(A)$ and $G \subseteq \tau_2 - int_w(B)$. Hence A and B are $\tau_1\tau_2 - s^*gw$ open.

SECTION: 4.4

PAIRWISE SEMI STAR GENERALIZED $w-T_{1/2}$ SPACES

Definition: 4.4.1

A space (X, τ) is called $T_{1/2}$ if every g - closed set is closed.

Definition: 4.4.2

A space (X, τ) is a **regular generalized $w - T_{1/2}$** (simply, $rgw - T_{1/2}$) if every rgw - closed set is w - closed.

Definition: 4.4.3

A space (X, τ_1, τ_2) is called **pairwise $T_{1/2}$** if every $\tau_1 - g$ closed set is τ_2 - closed and or every $\tau_2 - g$ closed set is τ_1 - closed.

Definition: 4.4.4

A space (X, τ_1, τ_2) is a **pairwise regular generalized $w - T_{1/2}$** (simply, pairwise $rgw - T_{1/2}$) if every $\tau_1\tau_2 - rgw$ closed set in (X, τ_1, τ_2) is $\tau_2 - w$ closed and $\tau_2\tau_1 - rgw$ closed set in (X, τ_1, τ_2) is $\tau_1 - w$ closed.

Definition: 4.4.5

A space (X, τ) is a **semi star generalized $w - T_{1/2}$** (simply, $s^*gw - T_{1/2}$) if every s^*gw - closed set is w - closed.

Definition: 4.4.6

A space (X, τ_1, τ_2) is a **pairwise semi star generalized $w - T_{1/2}$** (Simply, pairwise $s^*gw - T_{1/2}$) if every $\tau_1\tau_2 - s^*gw$ closed set in (X, τ_1, τ_2) is $\tau_2 - w$ closed and $\tau_2\tau_1 - s^*gw$ closed set in (X, τ_1, τ_2) is $\tau_1 - w$ closed.

Theorem: 4.4.7

For a space (X, τ_1, τ_2) , the following are equivalent.

- (a) X is pairwise $s^*gw - T_{1/2}$,
- (b) Every singleton is either τ_i - semi closed or $\tau_j - w$ open, $i \neq j$.

Proof:

(a) \Rightarrow (b)

Suppose $\{x\}$ is not a τ_1 - semi closed subset for some $x \in X$. Then $X - \{x\}$ is not τ_1 - semi open and hence X is the only τ_1 - semi open set containing $X - \{x\}$. Therefore $X - \{x\}$ is $\tau_1\tau_2 - s^*gw$ closed. Since (X, τ_1, τ_2) is $\tau_1\tau_2 - s^*gw - T_{1/2}$ space, $X - \{x\}$ is $\tau_2 - w$ closed and thus $\{x\}$ is $\tau_2 - w$ open.

(b) \Rightarrow (a)

Let A be an $\tau_1\tau_2 - s^*gw$ closed subset of (X, τ_1, τ_2) and $x \in \tau_2 - cl_w(A)$. We show that $x \in A$.

Case: 1 If $\{x\}$ is τ_1 - semi closed and x does not belong to A , then $x \in \tau_2 - cl_w(A) - A$. Thus $\tau_2 - cl_w(A) - A$ contains a nonempty τ_1 - semi closed set $\{x\}$, which is a contradiction to the fact that A is $\tau_1\tau_2 - s^*gw$ closed. So $x \in A$.

Case: 2 If $\{x\}$ is $\tau_2 - w$ open, since $x \in \tau_2 - cl_w(A)$, then for every $\tau_2 - w$ open set U containing x , $U \cap A \neq \emptyset$. But $\{x\}$ is $\tau_2 - w$ open, then $\{x\} \cap A \neq \emptyset$. Hence, $x \in A$. So in both cases $x \in A$. Therefore A is $\tau_2 - w$ closed.

Similarly we can prove every $\tau_2\tau_1 - s^*gw$ closed set in (X, τ_1, τ_2) is $\tau_1 - w$ closed. Hence X is pairwise $s^*gw - T_{1/2}$.

Theorem: 4.4.8

Every pairwise $rgw - T_{1/2}$ space is pairwise $s^*gw - T_{1/2}$ space