

**Avinashilingam Institute for Home Science and Higher Education for
Women, Coimbatore-641043
Bachelor's Degree Examination - November 2017**

I Semester

**Class : I UG
Major : Mathematics**

**Time : 3 Hrs
Max.Marks : 100**

15BMAC02 Classical Algebra and Theory of Numbers

Part – A

10x1=10

Choose the correct answer

1. If an equation is unaltered when 'x' is changed into its reciprocal then it is called ----- equation.
a. Quadratic
b. Cubic
c. Reciprocal
d. Linear
2. Descartes' rule of negative roots gives whether an equation $f(x)=0$ has ----- roots or not.
a. Real
b. Imaginary
c. District
d. Equal
3. Two similar matrices have ----- eigen values.
a. Same
b. Different
c. Reciprocal
d. None of the above
4. Is $A A^T = A^T A = I$, then A is called ----- matrix.
a. Unit
b. Orthogonal
c. Symmetric
d. Square
5. $\phi(720) =$ -----
a. 191
b. 192
c. 190
d. 189
6. $\phi(191) =$ -----
a. 190
b. 191
c. 192
d. 193
7. Is p is a prime greater than 3, then A^{p-2} is a multiple of -----
a. p
b. p^2
c. $p-2$
d. $p+1$
8. The number of prime numbers are -----
a. odd
b. even
c. finite
d. infinite
9. If p is prime than $\phi(p^r) =$ -----
a. $p^r (1 - \frac{1}{p})$
b. $p^r (1 - p)$
c. $p (1 - \frac{1}{p^r})$
d. $p^r (1 - \frac{1}{p^r})$
10. If p is a prime, then $(p-r)! + 1 \equiv$ -----
a. $\text{mod}(p)$
b. $0(\text{mod } p)$
c. $-1(\text{mod } p)$
d. $1(\text{mod } p)$

Answer the following

Answer should not exceed 400 words or two pages

11. a. Find the quotient and remainder when $3x^3 + 8x^2 + 8x + 12$ is divided by $x - 4$.
(or)

11. b. Diminish by 2 the roots of the equation $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$.

12. a. Show that $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$ is orthogonal.

(or)

12. b. Find A^{-1} for the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ using Cayley – Hamilton theorem.

13. a. Find the smallest number with 18 divisions.

(or)

13. b. Show that $n(n+1)(2n+1)$ is divisible by 6.

14. a. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

(or)

14. b. Show that every integer which is a perfect cube is of the form $7p$ or $7p \pm 1$.

15. a. State and prove Fermat's theorem.

(or)

15. b. Show that $(18)! + 1$ is divisible by 437.

Part – C

Answer the following

Answer should not exceed 800 words or 4 pages

16. a. Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.
(or)

16. b. Determine completely the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$ by Descartes's Rule of sign.

17. a. Determine the eigen values and eigen vectors of the matrix $\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$
(or)

17. b. Find the characteristic equation of the matrix $\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$, and determine A^n in terms of A .

18. a. Find the value of $\phi(N)$ and show that $\phi(ab) = \phi(a)\phi(b)$ where a and b are prime to one-another.

(or)

18. b. Prove that the product of ' r ' consecutive integers is divisible by $r!$.

19. a. i) find the remainder obtained in dividing 2^{46} by 47.

ii) If x, y, z be three consecutive integers, show that $(\Sigma x)^3 - 3 \Sigma x^3$ is divisible by 108.

(or)

19. b. Derive the criteria of divisibility of a number by 3, 9, 11 from the properties of congruences.

20. a. State and prove Wilson's Theorem.

(or)

20. b. If $M = 1.3.5 \dots (p-2)$ where p is odd prime, then show that $M^2 \equiv (-1)^{\frac{p+1}{2}} \pmod{p}$.