Review of Literature

The concept of open sets is a powerful tool in defining topological spaces. The idea of developing various forms of open sets originated from the notion of regular open sets by Stone (1937) in his classical paper on Boolean rings. This notion was stronger than that of open sets. Having topological structures as the suitable mathematical models for the formulation of both quantitative and qualitative data, several mathematicians were interested in developing concepts through the topological frame.

Levine (1963) developed and studied the concept of semi-open sets and semi-closed sets. The notion of semi-open sets were weaker than that of the open sets. Njastad (1965) established α-closed sets which was properly placed between closed and semi-closed sets. Velicko (1968) introduced δ-open sets, which are stronger than open sets, in order to investigate the characterization of H-closed spaces in terms of arbitrary filterbases and showed that the collection of all δ-open sets is a topology on X such that $\tau_\delta \subseteq \tau$. The family τ_s is the so called semi-regularization of τ. In a semi-regular space, $\tau = \tau_s$. Since then δ-open sets have been widely used in order to introduce new spaces and functions. Jankovic (1985), Noiri (1980) and Caldas (1999) continued the work of Velicko and obtained interesting results.

Levine (1970) introduced the idea of g-closed sets, which was another weaker form of closed sets. He proved that the notions of compactness, countably compactness, para compactness and normality were all g-closed hereditary. g-closed sets were of highly efficient in the characterization of topological spaces satisfying lower separation axioms. Since then intensive research on the field of generalized closed sets was undertaken. The

following sets were developed out of this fashion.

Crossley S. G. et al. analyzed the notion of semi-closure using semi-closed sets in 1971. Mashhour et al (1982) defined the notion of pre-open sets in topological spaces and obtained various properties. Using pre-open sets, they introduced and investigated modified continuous functions called pre-continuous functions and weak pre-continuous functions. Bhattacharya et al. (1987) investigated semi-generalized closed (briefly sg-closed) sets. Sundaram et al. (1991) studied the notion of sg-continuous functions and characterized semi-$T_{1/2}$ spaces. Regular generalized closed sets were investigated by Palaniappan et al. (1993).

Title : On δ-Generalized Closed Sets and $T_{3/4}$ -Spaces.

Authors : Julian Dontchev and Maximilian Ganster (1996)

Inference observed : Through the semi regularization of a given topology and the associated δ-closure operator, a stronger form of g-closedness, properly placed between δ-closedness and g-closedness called δ-generalised closed sets, briefly denoted by δg-closed sets was introduced in this article. Also a new separation axiom, namely, $T_{3/4}$, which is properly placed between $T_{1/2}$ and T_1 -spaces was defined and proved that every δg-closed set coincides with δ-closed set in this space. Further, the concept of δg-continuous and δg-irresolute functions were introduced and investigated in this paper.

Title : On Generalized δ-Closed Sets and Almost Weakly Hausdorff Spaces.

Authors : Julian Dontchev, Arokiarani, I. and Balachandran, K. (2000)

Inference observed : In this article, two new classes of generalized closed sets namely, $g\delta$-closed sets and δg^*-closed sets were introduced and new characterizations of almost weakly Hausdorff spaces and thus of the digital line were explored. Both concepts were based on the δ-closure operator which was initiated by Veliko (1968). Further, a stronger form of semi-regularity, called T_δ-spaces was introduced and it was proved that it is equal to semi-regularity plus almost weakly Hausdorffness. Also $g\delta$-continuous and $g\delta$-irresolute functions were introduced and studied in this paper.

Title : On δgs-closed sets and almost weakly Hausdorff Spaces

Authors : Jin Han Park, Dae Seob and Bu Young Lee (2007)

Inference observed : In this paper, a new class of sets namely, δgs-closed sets are introduced and their characterizations in almost weakly Hausdorff Space are obtained. δgs-closed sets were developed using the concepts of δ-semi closure and δ-open sets. Further, δgs-continuity is established and its relationship with various other forms of continuity is discussed.

Title : On generalized δ-semiclosed sets in topological spaces

Authors : Jin Han Park, Dae Seob and Saadati, R. (2007)

Inference observed : The aim of this paper was to introduce the class of $g\delta s$-closed sets and obtain the characterizations of $T_{3/4}$-space due to Dontchev and Ganster. They also introduced the notion of $g\delta s$-continuity and investigated the relationships between it and other types of continuity.

Title : On δ-Semigeneralized Closed Sets in Topology

Authors : Rajamani, M. and Padmanaban, S. (2011)

Inference observed : In this paper, a new class of sets called δ-Semigeneralized Closed Sets, briefly δ-sg-closed is developed. Some of its basic properties are studied and its relation with sg-closed sets and δg-closed sets are analyzed. Additionally, δ-sg-continuity, δ-sg-irresoluteness and study some of their fundamental properties are obtained.

According to Bourbaki (1966) a subset of a topological space is called a locally closed if it is the intersection of an open set and a closed set. Stone (1980) has used the term FG for a locally closed subset. Using the concept of locally closedness, Ganster and Reilly (1989) obtained different notions of generalized continuity. Balachandran et. al., (1996) introduced the concept of generalized locally closed sets and obtained different notions of generalized continuity.

Maki (1986) initiated the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel(= saturated set) which means, the intersection of all open supersets of A. Arenas et al. (1997) introduced the notion of λ-closed sets and λ-open sets by involving Λ-sets and closed sets, which enabled them to investigate some fine results. Caldas et al. (2007) introduced the notions of λ-derived, λ-border, λ-frontier and λ-exterior of a set and established that their properties were analogous to those of open sets. Gilbert Rani (2011) studied the notion of Λ^{λ}-closed sets using λ-open and λ-open sets.

Caldas and Dontchev (2000) developed Maki's work by introducing Λ_s-sets and V_s-sets. They developed new forms namely $g\Lambda_s$-sets and gV_s-sets. These notions were helpful in obtaining characterizations of *semi*-T_1 spaces, *semi*-R_0 spaces and *semi*-$T_{1/2}$ spaces. Ganster et al. (2002) introduced pre-Λ-set and pre-V-set and obtained some fundamental properties related to these notions. They also investigated the topologies defined by these families of sets.

Georgiou et al. (2004) presented the notions of Λ_δ-sets, (Λ, δ)-sets and (Λ, δ)-continuity in topological spaces. They introduced and investigated the concepts of (Λ, δ)-compactness and (Λ, δ)-connectedness. It was shown that these two concepts were preserved by δ-continuous functions. Characterizations of δ-T_i axioms were also presented in their work.

Caldas et al. (2007) defined the notions of Λ_α-sets, (Λ, α)-sets and (Λ, α)-continuity using the notions of α-open and α-closed sets. Some new lower separation axioms were obtained along with the study of continuity, compactness, connectedness concepts with regard to the new notions. Noiri et al. (2002) discussed the properties of (θ, s)-continuous functions.

Caldas et al. (2008) investigated two more classes of sets namely Λ_g-closed sets and Λ_g-open sets and inspected their properties. They portrayed that Λ_g-closed sets are prop-

erly placed between closed and g-closed sets, meaning that they are weaker than closed sets but stronger than g-closed sets. He also investigated the notion of Λ-g-closed sets and studied the relationship between Λ_g-closed sets and Λ-g-closed sets.

Separation Axioms

Separation axioms is one of the most important and interesting concept in topological spaces. One of the most well known low separation axioms is the T_1 separation axiom in which singleton sets are closed. Several topological spaces that fail tobe T_1 are very often of significant importance in the study of the geometric and topological properties of digital images. Several new separation axioms were defined in the course of the investigation of generalized closed sets.

Maheswari et al. discussed some separation axioms in 1975 and investigated Feebly T_1-spaces in 1978. Dunhan (1977) showed that $T_{1/2}$ -spaces are precisely the spaces in which singletons are open or closed. Sivakamasundari investigated gG-axioms in topological spaces in 2011. Submaximal spaces spaces and door spaces were analyzed by Dontchev (1995). Jain (1980) discussed the properties of δT_i-spaces.

Grill

Grill on topological spaces was introduced by Chouquet in 1947. Roy et al. studied the ideas of Principal Grill and Grill topological spaces in 2009 and 2007 respectively. Thron (1973) discussed the concept of proximity structure and grill. Recently, Rodyna (2012) developed the theory of Grill using δ-sets in topological spaces.

Continuity

With continuity being one of the core concepts of topology, various authors have generalized many types of continuity concepts. g-closed and g-continuous functions were studied by Cueva (1993). Raychaudhuri et al. (1993) developed the concepts of δ-almost continuity and δ-preopen sets in topological spaces. Semi-generalized closed maps and generalized semi-closed maps were studied by Devi et al. (1993). Levine established the idea of decomposition of continuity in 1961. Dontchev et al. (1996) portrayed some interesting decomposition of continuity and some of its weaker forms in which various forms of continuities were decomposed and characterizations of extremely disconnected spaces were obtained.

In 1989, Ganster and Reilly introduced the notion of LC-continuous functions via the concept of locally closed sets. Dontchev (1996) introduced a stronger form of LC-continuity called contra-continuity. Caldas and Jafari (2001) analyzed some vital properties of β-continuous functions. The idea of contra precontinuous functions was analyzed by Jafari et al. (2002). Noiri (1989) developed the notion of almost continuous functions. Baker used the notion of β-closed sets to study the so called contra almost β-continuous functions in 2006. The notion of $g\delta s$-open sets in topological space was applied to present a new class of functions called contra $g\delta s$-continuous functions, as a new generalization of contra continuity by Benchalli et al. (2012). He also studied another new class of functions called almost contra $g\delta s$-continuous functions as a generalization of contra continuity. Maio et al. (1988) investigated various weak and strong forms of irresolute functions.

Homeomorphism

A homeomorphism is a map between spaces that preserves all topological properties. Intuitively, given some sort of geometric object, a topological property is a property of the object that remains unchanged after the object has been stretched or deformed in

some way. Formally, a homeomorphism between two topological spaces and is a bijection $f : A \to B$ such that $f$is continuous and $f^{-1} : B \to A$ is also continuous. Then, topological properties are defined to be those properties of topological spaces that are preserved under homeomorphism. Recall that continuous maps are essentially those which send points close to one another in the domain to points close to one another in the codomain. Generalized homeomorphisms in topological spaces was studied by Maki (1991).

Compactness

The notion of weakly-compact spaces was analyzed by Singal (1969), α-compact spaces was investigated by Maheswari et al. (1985) and their properties were discussed by Noiri (1988). The notion of $g\alpha$-compact spaces was accomplished by Saraf (2000).

Image Processing

Khalimsky et al. (1990) related the theory of connected topologies on finite ordered sets to computer graphics. Face recognition is one of the most addressed topics in computer vision and pattern recognition research communities. Closed-set face identification problem, assigning test images to a set of known subjects, and face verification, comparing test images with the ones from claimed identity to check whether the claim is correct or not, have been extensively studied. Hua Gu et al. (2003) worked on facial feature extraction in Image processing. Smith et al. introduced SUSAN approach in low level image processing in 1997. Yaoyu Cheng-Shuxian Zhang et al. studied Edge Detection Algorithm Based on SUSAN Operation on Auto Hub Image. Open set face recognition using transduction was studied by Li et al. (2005).

Fuzzy Sets

Research on the theory of fuzzy sets has been witnessing an exponential growth both within mathematics and in its applications ranging from traditional mathematical subjects like logic, topology, algebra, analysis, pattern recognition, information theory, artificial intelligence, operation research, neural networks, planning and so on. Consequently fuzzy set theory has emerged as a potential area of inter disciplinary research. Many investigations have been carried out, in the general theoretical field and also in different applied areas, based on this concept.

The study of fuzzy sets was initiated by Zadeh (1965). Since the generalization of the usual notion of a set into a fuzzy set by Zadeh in his classic paper of 1965, many abstract structures were generalized using fuzzy sets. Fuzzy topological spaces were introduced by Chang (1968). Azad (1981) introduced the concept of fuzzy regular open sets and fuzzy regular closed sets in fuzzy topological spaces. Petricevic (1991) introduced the concept of fuzzy δ-open sets and fuzzy δ-closed sets in fuzzy topological spaces. Somewhat fuzzy δ-continuous functions were studied by Thangaraj (2015).