

Avinashilingam Institute for Home Science And Higher Education for Women
(Deemed to be University) Coimbatore - 641043

Master's Degree Examination - November 2018
Semester - I

Class : I PG
Major: Mathematics

Time : 3 Hours
Max Marks :60

17MMAC01- ADVANCED ALGEBRA

Part A

10x½ = 5

Choose the correct answer

1. A mapping ϕ from a group G into a \bar{G} is said to be homomorphism if for all $a, b \in G$, $\phi(ab) = \dots$
 - a. $\phi(a)\phi(b)$
 - b. $\phi(a) + \phi(b)$
 - c. $\phi(a) - \phi(b)$
 - d. $\frac{\phi(a)}{\phi(b)}$
2. If $a, b \in G$ then b is said to be a conjugate of a in G if there exists an element $c \in G$, such that $b = \dots$
 - a. $c^{-1}ac$
 - b. $b^{-1}ab$
 - c. a
 - d. ϕ
3. The number of partitions of 4 is -----
 - a. 2
 - b. 3
 - c. 5
 - d. 7
4. Number of 11 Sylow subgroups of a group of order $11^2 13^2$ is -----
 - a. 1
 - b. 2
 - c. 3
 - d. 4
5. Consider a polynomial $(x^3 - 2)$ in $F[x]$ then $F[x] / (x^3 - 2)$ is a-----
 - a. group
 - b. sub group
 - c. Ring
 - d. Field
6. A polynomial is said to be integer monic if all its co-efficient are integers and its highest co-efficient is -----
 - a. 1
 - b. 0
 - c. ∞
 - d. n
7. If $a \in K$ is algebraic of degree n over F , then $[F(a): F] = \dots$
 - a. 1
 - b. 0
 - c. ∞
 - d. n
8. The extension K of F is called ----- of F if every element in K is algebraic over F
 - a. an algebraic extension
 - b. a complex extension
 - c. an algebraic number
 - d. an algebraic integer
9. If $p(x) \in F[x]$, then an element a lying in some extension field of F is called a root of $p(x)$ if $p(a) = \dots$
 - a. 1
 - b. 0
 - c. ∞
 - d. n
10. The degree of the splitting field of polynomial $x^5 - 1$ is ----- over the field of rational number F .
 - a. 2
 - b. 3
 - c. 4
 - d. 5

Part B

5X 4 =20

Answer the following

- 11.a. Show that $N(a)$ is a subgroup of G
(Or)
b. If $O(G) = p^2$ where p is prime number, then G is abelian
- 12 .a. Suppose G is the direct product of N_1, N_2, \dots, N_n . Then evaluate $N_i \cap N_j = (e)$ for $i \neq j$
and $ab = ba$ if $a \in N_i, b \in N_j$
(Or)
b. If G is a finite group, p is a prime and $p^m / O(G), p^{m+1} / O(G)$, then prove that any two subgroups of G of order p^m are conjugate.
13. a. If $f(x), g(x)$ are two non-zero elements of $F[x]$, then show that
 $\deg(f(x)g(x)) = \deg f(x) + \deg g(x)$
(Or)
b. If $f(x), g(x)$ are primitive polynomials, then prove that $f(x)g(x)$ is a primitive polynomial
- 14.a. If a is any algebraic number, prove that there is positive integer n such that,
 na is an algebraic integer
(Or)
b. If a, b in K are algebraic over F then show that $a \pm b, ab$ and a/b (if $b \neq 0$) are all algebraic over F of degree at most mn
- 15.a. If $f(x) \in F[x]$, then there is a finite extension E of F in which $f(x)$ has a root.
Moreover $[E:F] \leq \deg f(x)$
(Or)
b. Let F be the field of rational numbers. Determine the degree of splitting field of polynomials $x^4 + 1$

Part C

5X 7 =35

Answer all questions

16. a. Discuss Sylow's theorem for abelian groups
(Or)
b. If G is a finite group, then $c_a = O(G) / O(N(a))$
- 17.a. Explain that the number of p - Sylow subgroups in G for a given prime is of the form $1 + kp$
(Or)
b. Let G be a group and suppose that G is the internal direct product of N_1, N_2, \dots, N_n . Let $T = N_1 \times N_2 \times \dots \times N_n$. Then prove that G and T are isomorphic

18. a. Describe the Division algorithm

(Or)

b. Analyze the Eisenstein criteria

19. a. Interpret, if L is an algebraic extension of K and if K is an algebraic extension of F then L is an algebraic extension of F

(Or)

b. Analyze that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F .

20.a. Operate that a polynomial of degree n over a field can have at most n roots in any extension field

(Or)

b. If $p(x)$ is a polynomial in $F[x]$ of degree $n \geq 1$ and is irreducible over F , then there is an extension E of F , such that $[E: F] = n$, in which $p(x)$ has a root.
