Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – August 2020
VI Semester

Class : III UG **Time : 2 Hours**
Major : Mathematics **Max. Marks : 50**

15BMAC22 REAL ANALYSIS II

Part A **10 x 1 = 10**
Choose the Correct Answer

1. A real-valued function f which is continuous on a metric space S is said to be two-valued on S if
   a. $f(S) \subseteq \{0,1\}$ b. $f(S) \supseteq \{0,1\}$
   c. $f(S) \subset \{0,1\}$ d. $f(S) \supset \{0,1\}$

2. If f be a function from a metric space S to Euclidean space R^k and is continuous on a compact subset X of S then
   a. f is connected on X b. f is closed on X
   c. f is bounded on X d. none of the above

3. The function f defined by $f(x) = 1/x$ if $x \neq 0, f(0) = A$ (neither $f(0+)\ nor\ f(0-)$ exists), has an
   a. removable jump discontinuity at 0 b. irremovable discontinuity at 0
   c. jump discontinuity at 0 d. none of the above

4. Let f be strictly increasing and continuous on a compact interval $[a, b]$ then f^{-1} on the interval $[f(a), f(b)]$ is
   a. continuous and decreasing b. continuous and increasing
   c. continuous and strictly decreasing d. continuous and strictly increasing

5. If f is constant on (a, b), then
   a. $\acute{f}$=0 on (a, b) b. $f' \neq 0$ on (a, b)
   c. f' is constant on (a, b) d. none of the above

6. If f and g are continuous on $[a, b]$ and have equal finite derivatives in (a, b), then $f - g$ is
   a. increasing on $[a, b]$ b. decreasing on $[a, b]$
   c. constant on $[a, b]$ d. none of the above

7. Assume f' exists and is continuous at end points a and b, if $f'(x) \neq 0$ for all x in (a, b) then f is
   a. monotonic on $[a, b]$ b. strictly monotonic on $[a, b]$
   c. discontinuous on $[a, b]$ d. none of the above

8. If f' takes only negative values (finite or infinite) in (a, b), then f is
   a. constant on $[a, b]$ b. strictly increasing on $[a, b]$
   c. strictly decreasing on $[a, b]$ d. none of the above

9. If f is monotonic on $[a, b]$, then
   a. f in of bounded variation on $[a, b]$
   b. the set of discontinuities of f is uncountable
   c. f in unbounded in $[a, b]$
   d. none of the above

10. f is constant on $[a, b]$ if and only if
    a. $V_f(a, b) < 0$ b. $V_f(a, b) > 0$
    c. $V_f(a, b) \neq 0$ d. $V_f(a, b) = 0$

Part B 3 x 6 = 18

Answer any **Three** questions

Each answer should not exceed 400 words or two pages

11. Let $f: S \to M$ be a function from a metric space S to another metric space M. Show that continuous image of a connected set is connected under f.

12. State and prove Bolzano's theorem.

13. If f is increasing function on $[a,b]$ then show that $f(c+)$ and $f(c-)$ both exist for each c in (a,b) and $f(c-) \leq f(c) \leq f(c+)$. show also that , at the endpoints $f(a) \leq f(a+)$and $f(b-) \leq f(b)$.

14. Define Strictly increasing function and prove that if f is strictly increasing on a set S in R, then show that s f^{-1} exists and is strictly increasing on $f(S)$.

15. If f is defined on (a,b) and differentiable at a point c in (a,b), then, prove that there is a function f^* (depending on f and on c) which is continuous at c and which satisfies the equation $f(x) - f(c) = (x-c)f^*(x)$, for all x in (a,b), with $f^*(c) = f'(c)$. Show conversely, that if there is a function f^*, continuous at c, which satisfies the equation then f is differentiable at c and $f'(c) = f^*(c)$.

16. Define derivatives, one-sided derivatives and infinite derivatives.

17. Let f be defined on an open interval (a,b) and assume that for some c in (a,b) $f'(c) > 0$or $f'(c) = +\infty$. Then show that there is a $1-ball\ B(c) \subseteq (a,b)$ in which $f(x) > f(c) if\ x > c$,and $f(x) < f(c) if\ x < c$.

18. State and prove Generalized Mean-Value theorem.

19. If f is monotonic on $[a,b]$, then show that the set of discontinuities of f is countable.

20. Let f be of bounded variation on $[a,b]$. Let V be defined on $[a,b]$ by $V(x) = V_f(a,x)$ if $a < x \leq b, V(a) = 0$. Then, show that
    i) $V$is an increasing function on $[a,b]$.
    ii) $V - f$is an increasing function on $[a,b]$.

Part C 2 x 11 = 22

Answer any **Two** questions

Each answer should not exceed 800 words or four pages

21. Show that, a metric space S is connected if and only if every two-valued function on S is constant.

22. Let $f: S \to T$ be a function from one metric space (S, d_S) to another (T, d_T). Then show that f is continuous on $S if\ and\ only\ if$ for every open set Yin T, the inverse image $f^{-1}(Y)$ is open in S.

23. State and prove Heine theorem.

24. State and prove Fixed-point theorem.

25. Assume f and g are defined on (a,b) and differentiable at c. Then show that $f+g, f-g$ and $f.g$ are also differentiable at c . This is also true of $f|g$ if $g(c) \neq 0$. The derivatives at c are given by the following formulas:
    i) $(f \pm g)'(c) = f'(c) \pm g'(c)$,
    ii) $(f.g)'(c) = f(c)g'(c) + f'(c)g(c)$,
    iii) $(f|g)'(c) = \frac{g(c)f'(c) - f(c)g'(c)}{g(c)^2}$, provided $g(c) \neq 0$.

26. State and prove Chain rule.

27. State and prove Intermediate-value theorem for derivatives.

28. i) Assume f' exists and is monotonic on an open interval (a, b).
Then show that f' is continuous on (a, b).
ii) State and prove Rolle's theorem.

29. State and prove additive property of total variation.

30. Let f be of bounded variation on $[a, b]$. If $x \in (a, b]$, let $V(x) = V_f(a, x)$ and put $V(a) = 0$. Then show that every point of continuity of f is also a point of continuity of V. Prove the converse also .

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