

INTRODUCTION

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Decision Making is one of the most complex issues that need scientific analysis of various factors, both tangibles and intangibles like attitude, belief, taste and preferences of people involved. In most of real world situations, usually decision makers are confronted with multiple criteria to be considered before any decision can be made. This is the case of Multi Criteria Decision Making (MCDM); a case with the aim to find the overall preferences among the available alternatives. One of the most popular methods in MCDM is the Technique for Order Preference by Similarity or TOPSIS. TOPSIS was proposed by Hwang and Yoon (1981). In this method, the main concept is that the most preferred alternative should have the shortest distance from the Positive Ideal Solution (PIS) and the longest distance from the Negative Ideal Solution (NIS).

In classical TOPSIS method, the ratings and the weights of the criteria are known precisely. However, under many conditions, crisp data are inadequate to model real-life situations since human judgments including preferences are often vague and cannot estimate his preference with an exact numerical value. A more realistic approach may be to use linguistic assessments instead of numerical values, that is, to suppose that the ratings and weights of the criteria in the problem are assessed by means of linguistic variables. Lingual expressions, for example, low, medium, high, etc. are regarded as the natural representation of the judgment. These characteristics indicate the applicability of fuzzy set theory in capturing the decision makers' preference structure. Fuzzy set theory aids in measuring the ambiguity of concepts that are associated with human being's subjective judgment. Moreover, since in the group decision making, evaluation is resulted from different evaluator's view of linguistic variables, its evaluation must be conducted in an uncertain, fuzzy environment. This is when Fuzzy TOPSIS comes into play where the criteria weights and alternative ratings are given by linguistic variables, expressed by fuzzy numbers.

There are different Fuzzy TOPSIS methods available in the literature. The main aim of this thesis is to study different Fuzzy TOPSIS methods with applications to real life problems

The study is so designed to present the following:

- i. Fuzzy TOPSIS Method
- ii. Modified Fuzzy TOPSIS Method
- iii. Integrated Fuzzy TOPSIS Method
- iv. Intuitionistic Fuzzy TOPSIS Method
- v. Interval-Valued Fuzzy TOPSIS Method
- vi. Fuzzy Flexible TOPSIS Method

The first chapter deals with preliminary definitions and notations.

The second chapter is devoted to the study of the following Fuzzy TOPSIS Methods

- i. Fuzzy TOPSIS Method using Triangular Fuzzy Numbers
- ii. Fuzzy TOPSIS Method using Triangular Fuzzy Numbers with Graded Mean Integration Representation
- iii. Fuzzy TOPSIS Method using Trapezoidal Fuzzy Numbers
- iv. Fuzzy TOPSIS Method based on alpha-level sets.

Section 2.1 deals with the Fuzzy TOPSIS Method using Triangular Fuzzy Numbers. In this method the criteria weights and alternative ratings are given by linguistic variables, expressed by triangular fuzzy numbers. The steps outlining the algorithm of a group multi-criteria decision making using Triangular Fuzzy Numbers are given in detail.

Section 2.2 deals with the Fuzzy TOPSIS Method using Triangular Fuzzy Numbers with Graded Mean Integration Representation

“The graded mean integration representation of a triangular fuzzy number $\tilde{a} = (a, b, c)$ is defined as $P(\tilde{a}) = \frac{1}{6}(a + 4b + c)$ ”

Based on the graded mean integration representation of triangular fuzzy numbers and the canonical representation of addition and multiplication on triangular fuzzy numbers, the procedure of fuzzy TOPSIS method can be performed in an efficient way. This property can reduce the computational complexity in the decision making process. This method is illustrated in detail in section 2.2.

In section 2.3, a Fuzzy TOPSIS method using trapezoidal fuzzy numbers is presented.

Section 2.4 deals with Fuzzy TOPSIS method based on alpha-level sets.

The most commonly used fuzzy numbers in Fuzzy TOPSIS methods are triangular and trapezoidal fuzzy numbers. According to Zadeh’s extension principle(1991), a fuzzy number \tilde{m} can also be expressed by its intervals, namely

$$\tilde{m} = \bigcup_{\alpha} \alpha m_{\alpha}, 0 < \alpha \leq 1$$

where

$$\begin{aligned} m_{\alpha} &= \{x \in X | \mu_{\tilde{m}}(x) \geq \alpha\} \\ &= [\min\{x \in X | \mu_{\tilde{m}}(x) \geq \alpha\}, \max\{x \in X | \mu_{\tilde{m}}(x) \geq \alpha\}] \end{aligned}$$

m_{α} is referred to as α -cuts of the fuzzy number \tilde{m} .

In 2006, Ying-Ming Wang and Taha Elhag combined the TOPSIS method and the extension principle to solve Fuzzy Multi Criteria Decision Making problems and presented a fuzzy TOPSIS method based on alpha level sets. This method is illustrated in detail in this section.

In Chapter III, a modified fuzzy TOPSIS method proposed by Soroush Saghafian and Reza Hejazi (2005) is studied. This method includes modifications in Fuzzy Multiple Criteria Decision-Making theory to strengthen the

comprehensiveness and reasonableness of the decision making process using Fuzzy TOPSIS.

Chapter IV deals with an integrated Fuzzy TOPSIS method to improve the quality of decision making for ranking alternatives. The proposed fuzzy TOPSIS method mainly accounts for the classification of criteria, the integrated weights of criteria and sub-criteria, and the performance values of decision matrix. In this model, the criteria are classified into subjective criteria and objective ones. The fuzzy analytic hierarchy process approach and the entropy weighting method are used to solve the subjective weights and objective ones. In addition, the adjusted integration weights are measured by combining these two methods. The performance values of subjective criteria and of objective ones will be obtained by linguistic expressions and objective evaluation values, respectively. Furthermore, the graded mean integration representation method and the modified distance method proposed by Heieh and Chen (1999) are employed to the integrated fuzzy TOPSIS method. The details of this method are given in this chapter.

Chapter V deals with the concept of TOPSIS for solving multiple criteria decision making problems which is extended with Intuitionistic Fuzzy data that is expressed in terms of Intuitionistic Fuzzy Sets. Intuitionistic Fuzzy Set, introduced by Atanassov (1986) is characterised by three functions expressing the degree of belongingness, the degree of non-belongingness and the degree of hesitation. The concept of Intuitionistic Fuzzy Sets is a generalization of fuzzy sets. In this method the intuitionistic separation between alternatives can be measured by distance measures and similarity measures, which are dual concepts. Sixteen different intuitionistic separation equations are given to measure the distance between Intuitionistic Fuzzy Sets. All intuitionistic separation measures are divided into four categories depending on their characteristics such as the square root, maximum and parameters. Using one of these separation measures, the relative closeness to the intuitionistic ideal solution is calculated and the preference order of alternatives is ranked.

In Chapter VI, the concept of TOPSIS method has been extended for interval valued data. Interval arithmetic was first suggested by Dwyer(1951). Development of interval arithmetic as a formal system and evidence of its value as a computational device was provided by Moore (1979, 1996). In 2008, Ting-Yu Chen and Cheuh-Yung Tsao used interval valued fuzzy numbers to develop a method for solving multi criteria decision making problems. This method is illustrated in this chapter.

Chapter VII is devoted to Fuzzy Flexible TOPSIS which is proposed with the objective of improving the Fuzzy TOPSIS ability to deal with uncertainty through the combination of the mathematical process involved in the original Fuzzy TOPSIS with the expert empirical knowledge. Originally, Fuzzy TOPSIS is an optimization technique that uses linguistic terms to evaluate the importance of attributes and their values. In order to improve their ability to deal with vagueness, an attribute that represents the evaluation of a Fuzzy Rules Based System is added to the Fuzzy TOPSIS. The main purpose of this modification is to add the capabilities of fuzzy rules based systems to Fuzzy TOPSIS. Thus, the analysis by Fuzzy Flexible TOPSIS will allow the empirical knowledge of the expert, represented by fuzzy rules, also be considered in the decision making and optimization process, in addition to definitions of the membership functions of the fuzzy sets. The use of optimization techniques associated with the empirical knowledge of experts, allows a hybrid analysis of the multi-criteria problems where the process of decision making requires the use of human sensitivity, which often can be expressed by a fuzzy rules base. Thus, the behaviour of the system may have greater influence, or not, than the rules defined by the expert. This fuzzy flexible method is explained in this chapter.

Application of all these Fuzzy TOPSIS methods in real-life problems facilitates in deriving scientific and logical solutions.

Chapter VIII deals with an application of intuitionistic fuzzy TOPSIS method. In this chapter the author of this thesis made an attempt to develop a decision making model for choosing a suitable mode of transport by using Intuitionistic Fuzzy TOPSIS method.

In real-life situations, the decision makers are confronted with unquantifiable, incomplete and non-obtainable information that make precise judgment impossible. The ever increasing alternatives and the expectations of the beneficiaries make the decision making more complicated. What is good for one is not the same for others. When a decision goes wrong, one finds it very difficult to compromise for the outcome. On contrary, the same goes right he feels comfortable. Nowadays, decision making is not the problem of administrators alone, every common man is bound to take right decision on many routine affairs like food, shelter, education for children, purchase of durables, healthcare, conveyance and so on. So the author of this thesis intended to develop a mathematical model for decision making regarding the selection of right mode of public transport by women commuters using Intuitionistic Fuzzy TOPSIS method.

It is worth mentioning that the author has sent an article entitled “Preference of Mode of Public Transport – A Decision making Model Using Intuitionistic Fuzzy TOPSIS Method” for publication in *International Journal of Mathematical Archive* and the same is likely to be accepted for publication.