CHAPTER - I

CHAPTER I

FUNDAMENTAL DEFINITIONS AND NOTATIONS

Definition: 1.1 [68]

Let U be a non-empty set. A ***fuzzy Set*** in U is a function with domain U and values in the closed unit interval I = [0, 1].

(or)

A ***fuzzy set A*** in a universe of discourse X is characterized by a membership function $\mu_A(x)$ which associates with each element x in X a real number in the interval [0,1].The function value $\mu_A(x)$ is termed the grade of membership of x in A.

Definition: 1.2 [58]

A fuzzy set A of the universe of discourse X is ***convex*** if and only if for all x_1 and x_2 in X we have $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \text{Min}(\mu_A(x_1), \mu_A(x_2))$ where $\lambda \in [0,1]$.

Definition: 1.3 [58]

A fuzzy set A of the universe of discourse X is called a ***normal*** fuzzy set implying that $\exists x \in X : \mu_A(x) = 1$

Definition: 1.4 [58]

A ***fuzzy number*** n is a fuzzy subset in the universe of discourse X that is both convex and normal.

Definition: 1.5 [58]

The ***α-cut of a fuzzy number*** n is defined as $n^\alpha = \{x_i : \mu_n(x_i) \geq \alpha,\ x_i \in X\}$ where $\alpha \in [0,1]$.

n^α is a non-empty bounded closed interval contained in X and it can be denoted by $n^\alpha = [n_l, n_u]$ where n_l and n_u are the lower and upper bounds of the closed interval, respectively.

Definition: 1.6 [10]

D is called a ***fuzzy matrix***, if all its entries are fuzzy numbers.

Definition: 1.7 [61]

An ***Intuitionistic Fuzzy Set (IFS)*** A in a nonempty set U (a universe of discourse) is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in U\}$, where the functions $\mu_A(x) : U \rightarrow [0,1]$, $\nu_A(x) : U \rightarrow [0,1]$, denotes the degree of membership and degree of non-membership of each element $x \in U$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in U$.

Definition: 1.8 [61]

For each element $x \in X$, the ***Intutionistic Index*** of x in A is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ where $\pi_A(x) \in [0,1]$ for every $x \in X$. $\pi_{A_i}(x)$ reflects the fact that decision makers may not always be certain that the non-possession strength is just equal to 1 minus the possession strength. Thus, $\pi_{A_i}(x)$ denotes the degree of hesitation.

Definition: 1.9 [6]

Let Int([0, 1]) stands for the set of all closed subintervals of [0, 1]. An ***Interval-valued Fuzzy Set (IVFS)*** A in a nonempty and a finite set X (a universe of discourse) is an object having the form $A = \{(x, M_A(x) \mid x \in X\}$, where the functions $M_A(x) : X \rightarrow \text{Int}([0,1])$, such that $x \rightarrow M_A(x) = [M_A^-(x), M_A^+(x)]$. $M_A^-(x)$ and $M_A^+(x)$ are the lower bound and the upper bound, respectively, of the interval $M_A(x)$.

Definition: 1.10 [35]

Let $\tilde{a}$ be a fuzzy set and its values will be located between 0 and 1. It is a ***triangular fuzzy number*** $\tilde{a}$ can be defined by a triplet $\tilde{a} = (a, b, c)$ shown in Fig. 1. The membership function $\mu_{\tilde{a}}$ is defined as

$$\mu_{\tilde{a}} = \begin{cases} 0 & \text{if} \quad x < a \\ \dfrac{x-a}{b-a} & \text{if} \quad a \le x < b \\ \dfrac{c-x}{c-b} & \text{if} \quad b \le x \le c \\ 0 & \text{if} \quad x > c \end{cases}$$

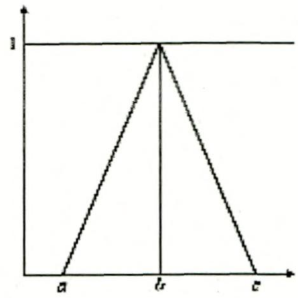


Fig. 1

Definition: 1.11 [35]

Let $\tilde{m} = (m_1, m_2, m_3)$ and $\tilde{n} = (n_1, n_2, n_3)$ be two triangular fuzzy numbers. If $m = n$, then $m_1 = n_1$, $m_2 = n_2$ and $m_3 = n_3$.

Definition: 1.12 [35]

If $\tilde{n}$ is a triangular fuzzy number and $\tilde{n}_l^{\alpha} > 0$ and $\tilde{n}_u^{\alpha} \geq 1$ for $\alpha \in [0,1]$, then $\tilde{n}$ is called a ***normalized positive triangular fuzzy number***.

Definition: 1.13 [10]

A linguistic variable is a variable whose values are linguistic terms [69]. The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions. For example, "weight" is a linguistic variable and its values are very low, low, medium, high, very high, etc. These linguistic values can also be represented by fuzzy numbers. It can also be represented as triangular fuzzy numbers as

Very low (VL)	(0, 0.1, 0.3)
Low (L)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)

Definition: 1.14 [58]

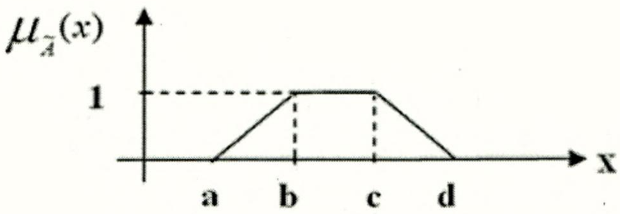
Let $\tilde{a} = (a_1, b_1, c_1)$ and $\tilde{b} = (a_2, b_2, c_2)$ be two triangular fuzzy numbers. The vertex method for calculating distance between them is

$$d(\tilde{a},\tilde{b}) = \left[\frac{1}{3}((a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2)\right]^{0.5}$$

Definition: 1.15 [3]

A Fuzzy Number $\tilde{m} = (a, b, c, d)$ is said to be Trapezoidal Fuzzy Number if its membership function is given by,

$$\mu_{\tilde{m}}(x) = \begin{cases} 0 & \text{for} \quad x < a \\ \dfrac{x-a}{b-a} & \text{for} \quad a \le x < b \\ 1 & \text{for} \quad b < x < c \\ \dfrac{d-x}{d-c} & \text{for} \quad c \le x < d \\ 0 & \text{for} \quad x > d \end{cases}$$



where $a \le b \le c \le d$.

Definition: 1.16 [3]

If $\tilde{m} = (a_1, b_1, c_1, d_1)$ and $\tilde{n} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, then the distance between them is calculated as

$$d(\tilde{n},\tilde{m}) = \frac{(a_2 + b_2 + c_2 + d_2) - (a_1 + b_1 + c_1 + d_1)}{4}$$

Algebraic Operations:[12,46]

Let $\tilde{a} = (a_1, b_1, c_1)$ and $\tilde{b} = (a_2, b_2, c_2)$ be two triangular fuzzy numbers.

i) Triangular Fuzzy Addition (+):

$$\tilde{a}\,(+)\,\tilde{b} = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

ii) Triangular Fuzzy subtraction, (-) :

$$\tilde{a}\ (-)\ \tilde{b} = (a_1 + c_2, b_1 + b_2, c_1 + a_2)$$

iii) Triangular Fuzzy multiplication, (x) :

$$k\ (x)\ \tilde{b} = (ka_2, kb_2, kc_2), k \in R , k \geq 0,$$

$$\tilde{a}\ (x)\ \tilde{b} \cong (a_1a_2, b_1b_2, c_1c_2)\ a_1 \geq 0, a_2 \geq 0$$

iv) Triangular Fuzzy division, (/) :

$$\tilde{a}^{-1} = (a, b, c)^{-1} \cong (1/c, 1/b, 1/a), a > 0,$$

$$\tilde{a}\ (/)\ \tilde{b} \cong (a_1/c_2, b_1/b_2, c_1/a_2), a_1 \geq 0, a_2 > 0.$$

Let $\tilde{a} = (a_1, b_1, c_1, d_1)$ and $\tilde{b} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers

i) Trapezoidal Fuzzy Addition (+):

$$\tilde{a}\ (+)\ \tilde{b} = (a_1+ a_2, b_1+ b_2, c_1+ c_2 , d_1+ d_2)$$

ii) Trapezoidal Fuzzy Multiplication (x):

$$\tilde{a}\ (x)\ \tilde{b} = (a_1 x\ a_2, b_1 x\ b_2, c_1 x\ c_2 , d_1 x\ d_2)$$