

## Intuitionistic Fuzzy Almost Generalized $\beta$ Closed Mappings

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### ABSTRACT:

In this paper we introduce intuitionistic fuzzy almost generalized  $\beta$  closed mappings and intuitionistic fuzzy almost generalized  $\beta$  open mappings. We investigate some of their properties. Also we provide the relation between intuitionistic fuzzy almost generalized  $\beta$  closed mappings and other intuitionistic fuzzy closed mappings.

**Key words and phrases :** Intuitionistic fuzzy topology, intuitionistic fuzzy generalized  $\beta$   $T_{1/2}$  space, intuitionistic fuzzy almost generalized  $\beta$  closed mappings.

### 1. Introduction

The notion of intuitionistic fuzzy sets is introduced by Atanassov [1]. Using this notion, Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. D. Jayanthi [4] introduced the notion of intuitionistic fuzzy generalized  $\beta$  closed mappings and intuitionistic fuzzy generalized  $\beta$  open mappings. In this paper we introduce intuitionistic fuzzy almost generalized  $\beta$  closed mappings. We investigate some of its properties. Also we provide the relation between an intuitionistic fuzzy almost generalized  $\beta$  closed mapping and other intuitionistic fuzzy closed mappings.

### 2. Preliminaries

**Definition 2.1:** [1] An intuitionistic fuzzy set (IFS)  $A$  in  $X$  is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$$

where the functions  $\mu_A: X \rightarrow [0,1]$  and  $\nu_A: X \rightarrow [0,1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set  $A$ , respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ .

**Definition 2.2:** [1] Let  $A$  and  $B$  be IFSs of the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$ .

Then

- (a)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$
- (b)  $A = B$  if and only if  $A \subseteq B$  and  $B \subseteq A$
- (c)  $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- (d)  $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- (e)  $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation  $A = \langle x, \mu_A, \nu_A \rangle$  instead of  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ .

The intuitionistic fuzzy sets  $0_{\sim} = \{\langle x, 0, 1 \rangle / x \in X\}$  and  $1_{\sim} = \{\langle x, 1, 0 \rangle / x \in X\}$  are respectively the empty set and the whole set of  $X$ .

**Definition 2.3:** [2] Let  $\alpha, \beta \in [0, 1]$  with  $\alpha + \beta \leq 1$ . An intuitionistic fuzzy point (IFP), written as  $p_{(\alpha, \beta)}$ , is defined to be an IFS of  $X$  given by

$$p_{(\alpha, \beta)}(x) = (\alpha, \beta) \quad \text{if } x = p,$$

$$= (0, 1) \quad \text{otherwise.}$$

**Definition 2.4:** [2] An intuitionistic fuzzy topology (IFT) on  $X$  is a family  $\tau$  of IFSs in  $X$  satisfying the following axioms.

- (i)  $0_{\sim}, 1_{\sim} \in \tau$
- (ii)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$
- (iii)  $\cup G_i \in \tau$  for any family  $\{G_i / i \in J\} \subseteq \tau$ .

In this case the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS) and any IFS in  $\tau$  is known as an intuitionistic fuzzy open set (IFOS) in  $X$ . The complement  $A^c$  of an IFOS  $A$  in IFTS  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS) in  $X$ .

**Definition 2.5:**[2] Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in  $X$ . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{G / G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$$

$$\text{cl}(A) = \cap \{K / K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$$

**Definition 2.6:**[3] An IFS  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS  $(X, \tau)$  is said to be an

- (i) *intuitionistic fuzzy regular closed set* (IFRCS) if  $\text{cl}(\text{int}(A)) = A$
- (ii) *intuitionistic fuzzy semi closed set* (IFSCS) if  $\text{int}(\text{cl}(A)) \subseteq A$
- (iii) *intuitionistic fuzzy pre closed set* (IFPCS) if  $\text{cl}(\text{int}(A)) \subseteq A$
- (iv) *intuitionistic fuzzy  $\alpha$  closed set* (IF $\alpha$ CS) if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$
- (v) *intuitionistic fuzzy  $\beta$  closed set* (IF $\beta$ CS) if  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$

The respective complements of the above IFCSs are called their respective IFOSs.

**Definition 2.7:**[2] Let  $A$  be an IFS in an IFTS  $(X, \tau)$ . Then the  $\beta$  interior and the  $\beta$  closure of  $A$  are defined by

$$\beta\text{int}(A) = \cup \{G / G \text{ is an IF}\beta\text{OS in } X \text{ and } G \subseteq A\}.$$

$$\beta\text{cl}(A) = \cap \{K / K \text{ is an IF}\beta\text{CS in } X \text{ and } A \subseteq K\}.$$

**Definition 2.8:**[5] An IFS  $A$  in an IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy generalized  $\beta$  closed set (IFG $\beta$ CS) if  $\beta\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS in  $(X, \tau)$ . The complement  $A^c$ , is called an intuitionistic fuzzy generalized  $\beta$  open set (IFG $\beta$ OS) in  $X$ .

**Definition 2.9:**[3] Let  $p_{(\alpha, \beta)}$  be an IFP of  $(X, \tau)$ . An IFS  $A$  of  $X$  is called an intuitionistic fuzzy neighborhood (IFN) of  $p_{(\alpha, \beta)}$  if there exists an IFOS  $B$  in  $X$  such that  $p_{(\alpha, \beta)} \in B \subseteq A$ .

**Definition 2.10:**[5] If every IFG $\beta$ CS in  $(X, \tau)$  is an IF $\beta$ CS in  $(X, \tau)$ , then the space can be called as an intuitionistic fuzzy  $\beta$   $T_{1/2}$  space (IF $\beta T_{1/2}$  space).

**Definition 2.11:**[3] A map  $f: X \rightarrow Y$  is called an intuitionistic fuzzy closed mapping (IFCM) if  $f(A)$  is an IFCS in  $Y$  for each IFCS  $A$  in  $X$ .

**Definition 2.12:**[3] A map  $f: X \rightarrow Y$  is called an

(i) intuitionistic fuzzy semi open mapping (IFSOM) if  $f(A)$  is an IFSOS in  $Y$  for each IFOS  $A$  in  $X$ .

(ii) intuitionistic fuzzy  $\alpha$  open mapping (IF $\alpha$ OM) if  $f(A)$  is an IF $\alpha$ OS in  $Y$  for each IFOS  $A$  in  $X$ .

(iii) intuitionistic fuzzy preopen mapping (IFPOM) if  $f(A)$  is an IFPOS in  $Y$  for each IFOS  $A$  in  $X$ .

(iv) intuitionistic fuzzy  $\beta$  open mapping (IF $\beta$ OM) if  $f(A)$  is an IF $\beta$ OS in  $Y$  for each IFOS  $A$  in  $X$ .

**Definition 2.13:** [3] The intuitionistic fuzzy semi closure and the intuitionistic fuzzy  $\alpha$  closure of an IFS  $A$  in an IFTS  $(X, \tau)$  are defined by

$$scl(A) = \bigcap \{ K / K \text{ is an IFSCS in } X \text{ and } A \subseteq K \}.$$

$$\alpha cl(A) = \bigcap \{ K / K \text{ is an IF}\alpha\text{CS in } X \text{ and } A \subseteq K \}.$$

**Definition 2.14:**[5] Let  $p_{(\alpha, \beta)}$  be an IFP in  $(X, \tau)$ . An IFS  $A$  of  $X$  is called an intuitionistic fuzzy  $\beta$  neighborhood (IF $\beta$ N) of  $p_{(\alpha, \beta)}$  if there is an IF $\beta$ OS  $B$  in  $X$  such that  $p_{(\alpha, \beta)} \in B \subseteq A$ .

**Definition 2.15:** [4] A mapping  $f: X \rightarrow Y$  is said to be an intuitionistic fuzzy generalized  $\beta$  closed mapping (IFG $\beta$ CM) if  $f(A)$  is an IFG $\beta$ CS in  $Y$  for every IFCS  $A$  in  $X$ .

**Definition 2.16:** [4] A mapping  $f: X \rightarrow Y$  is said to be an intuitionistic fuzzy M-generalized  $\beta$  closed mapping (IFMG $\beta$ CM) if  $f(A)$  is an IFG $\beta$ CS in  $Y$  for every IFG $\beta$ CS  $A$  in  $X$ .

**Definition 2.17:** [5] An IFS  $A$  is said to be an intuitionistic fuzzy dense (IFD) in another IFS  $B$  in an IFTS  $(X, \tau)$ , if  $cl(A) = B$ .

**Definition 2.18:** [6] A mapping  $f: X \rightarrow Y$  is said to be an intuitionistic fuzzy generalized  $\beta$  continuous mapping (IFG $\beta$ cts.M) if  $f^{-1}(A)$  is an IFG $\beta$ CS in  $X$  for every IFCS  $A$  in  $Y$ .

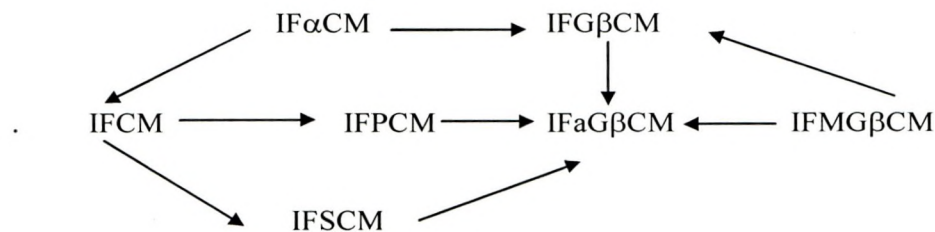
### 3. Intuitionistic fuzzy almost generalized $\beta$ closed mappings and intuitionistic fuzzy almost generalized $\beta$ open mappings.

In this section we introduce intuitionistic fuzzy almost generalized  $\beta$  closed mappings and intuitionistic fuzzy almost generalized  $\beta$  open mappings. We study some of their properties

**Definition 3.1:** A map  $f: X \rightarrow Y$  is called an intuitionistic fuzzy almost generalized  $\beta$  closed mapping (IFaG $\beta$ CM) if  $f(A)$  is an IFG $\beta$ CS in  $Y$  for each IFRCS  $A$  in  $X$ .

**Example 3.2:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$ ,  $G_2 = \langle y, (0.2, 0.3), (0.8, 0.7) \rangle$ . Then  $\tau = \{0_-, G_1, 1_-\}$  and  $\sigma = \{0_-, G_2, 1_-\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an IFaG $\beta$ CM.

The relation between various types of intuitionistic fuzzy closedness is given in the following diagram.



The reverse implications are not true in general in the above diagram. This can be seen from the following examples.

**Example 3.3:** In Example 3.2,  $f$  is an IFaG $\beta$ CM but not an IFCM, since  $G_1^c = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$  is an IFCS in  $X$  but  $f(G_1^c) = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$  is not an IFCS in  $Y$ .

**Example 3.4:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$ ,  $G_2 = \langle y, (0.5, 0.4), (0.2, 0.3) \rangle$ . Then  $\tau = \{0_-, G_1, 1_-\}$  and  $\sigma = \{0_-, G_2, 1_-\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: X \rightarrow Y$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an IFaG $\beta$ CM but not an IFSCM, since  $G_1^c = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$  is an IFCS in  $X$  but  $f(G_1^c) = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$  is not an IFSCS in  $Y$ , since  $\text{int}(\text{cl}(f(G_1^c))) = 1_- \not\subset f(G_1^c)$ .

**Example 3.5:** In Example 3.4  $f$  is an IFaG $\beta$ CM but not an IF $\alpha$ CM, since  $G_1^c = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$  is an IFCS in  $X$  but  $f(G_1^c) = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$  is not an IF $\alpha$ CS in  $Y$ , since  $\text{cl}(\text{int}(\text{cl}(f(G_1^c)))) = 1_- \not\subset f(G_1^c)$ .

**Example 3.6:** In Example 3.2  $f$  is an IFaG $\beta$ CM but not an IFPCM, since  $G_1^c = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$  is an IFCS in  $X$  but  $f(G_1^c) = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$  is not an IFPCS in  $Y$ , since  $\text{cl}(\text{int}(f(G_1^c))) = G_2^c \not\subset f(G_1^c)$ .

**Example 3.7:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.2, 0.2), (0.4, 0.4) \rangle$ ,  $G_2 = \langle x, (0.2, 0), (0.5, 0.4) \rangle$ ,  $G_3 = \langle y, (0.5, 0.6), (0.2, 0) \rangle$  and  $G_4 = \langle x, (0.4, 0.1), (0.2, 0.1) \rangle$ . Then  $\tau = \{0_-, G_1, G_2, 1_-\}$  and  $\sigma = \{0_-, G_3, G_4, 1_-\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an IFaG $\beta$ CM but

not an IFG $\beta$ CM, since  $G_2^c = \langle x, (0.5, 0.4), (0.2, 0) \rangle$  is an IFCS in  $X$  but  $f(G_2^c) = \langle y, (0.5, 0.4), (0.2, 0) \rangle$  is not an IFG $\beta$ CS in  $Y$ , since  $f(G_2^c) \subseteq G_3$  but  $\beta cl(f(G_2^c)) = 1 \sim \notin G_3$ .

**Example 3.8:** In Example 3.7  $f$  is an IFaG $\beta$ CM but not an IFMG $\beta$ CM, since  $A = \langle x, (0.4, 0.2), (0.2, 0) \rangle$  is an IFG $\beta$ CS in  $X$  but  $f(A) = \langle y, (0.4, 0.2), (0.2, 0) \rangle$  is not an IFG $\beta$ CS in  $Y$ , since  $f(A) \subseteq G_3$  but  $\beta cl(f(A)) = 1 \sim \notin G_3$ .

**Definition 3.9:** A map  $f: X \rightarrow Y$  is called an intuitionistic fuzzy almost generalized  $\beta$  open mapping (IFaG $\beta$ OM) if  $f(A)$  is an IFG $\beta$ OS in  $Y$  for each IFROS  $A$  in  $X$ . An IFaG $\beta$ OM is an IFaG $\beta$ CM if it is a bijective mapping.

**Theorem 3.10:** Let  $p_{(\alpha, \beta)}$  be an IFP in  $X$ . A bijective mapping  $f: X \rightarrow Y$  is an IFaG $\beta$ CM if for every IFOS  $A$  in  $X$  with  $f^{-1}(p_{(\alpha, \beta)}) \in A$ , there exists an IFOS  $B$  in  $Y$  with  $p_{(\alpha, \beta)} \in B$  such that  $f(A)$  is IFD in  $B$ .

**Proof:** Let  $A$  be an IFROS in  $X$ . Then  $A$  is an IFOS in  $X$ . Let  $f^{-1}(p_{(\alpha, \beta)}) \in A$ , then there exists an IFOS  $B$  in  $Y$  such that  $p_{(\alpha, \beta)} \in B$  and  $cl(f(A)) = B$ . Since  $B$  is an IFOS,  $cl(f(A)) = B$  is also an IFOS in  $Y$ . Therefore  $int(cl(f(A))) = cl(f(A))$ . Now  $f(A) \subseteq cl(f(A)) = int(cl(f(A))) \subseteq cl(int(cl(f(A))))$ . This implies  $f(A)$  is an IF $\beta$ OS in  $Y$  and hence an IFG $\beta$ OS in  $Y$ . Thus  $f$  is an IFaG $\beta$ CM.

**Theorem 3.11:** Let  $f: X \rightarrow Y$  be a bijective mapping where  $Y$  is an IF $\beta T_{1/2}$  space. Then the following are equivalent

- (i)  $f$  is an IFaG $\beta$ CM.
- (ii)  $\beta cl(f(A)) \subseteq f(cl(A))$  for every IF $\beta$ OS  $A$  in  $X$
- (iii)  $\beta cl(f(A)) \subseteq f(cl(A))$  for every IFSOS  $A$  in  $X$
- (iv)  $f(A) \subseteq \beta int(f(int(cl(A))))$  for every IFPOS  $A$  in  $X$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $A$  be an IF $\beta$ OS in  $X$ . Then  $cl(A)$  is an IFRCS in  $X$ . By hypothesis  $f(A)$  is an IFG $\beta$ CS in  $Y$  and hence is an IF $\beta$ CS in  $Y$ , since  $Y$  is an IF $\beta T_{1/2}$  space. This implies  $\beta cl(f(cl(A))) = f(cl(A))$ . Now  $\beta cl(f(A)) \subseteq \beta cl(f(cl(A))) = f(cl(A))$ . Thus  $\beta cl(f(A)) \subseteq f(cl(A))$ .

(ii)  $\Rightarrow$  (iii) Since every IFSOS is an IF $\beta$ OS, the proof directly follows.

(iii)  $\Rightarrow$  (i) Let  $A$  be an IFRCS in  $X$ . Then  $A = cl(int(A))$ . Therefore  $A$  is an IFSOS in  $X$ . By hypothesis,  $\beta cl(f(A)) \subseteq f(cl(A)) = f(A) \subseteq \beta cl(f(A))$ . Hence  $f(A)$  is an IF $\beta$ CS and hence is an IFG $\beta$ CS in  $Y$ . Thus  $f$  is an IFaG $\beta$ CM.

(i)  $\Rightarrow$  (iv) Let  $A$  be an IFPOS in  $X$ . Then  $A \subseteq int(cl(A))$ . Since  $int(cl(A))$  is an IFROS in  $X$ , by hypothesis,  $f(int(cl(A)))$  is an IFG $\beta$ OS in  $Y$ . Since  $Y$  is an IF $\beta T_{1/2}$  space,  $f(int(cl(A)))$  is an IF $\beta$ OS in  $Y$ . Therefore  $f(A) \subseteq f(int(cl(A))) \subseteq \beta int(f(int(cl(A))))$ .

(iv)  $\Rightarrow$  (i) Let  $A$  be an IFROS in  $X$ . Then  $A$  is an IFPOS in  $X$ . By hypothesis,  $f(A) \subseteq \beta int(f(int(cl(A)))) = \beta int(f(A)) \subseteq f(A)$ . This implies  $f(A)$  is an IF $\beta$ OS in  $Y$  and hence is an IFG $\beta$ OS in  $Y$ . Therefore  $f$  is an IFaG $\beta$ CM.

**Theorem 3.12:** Let  $f: X \rightarrow Y$  be a map. Then  $f$  is an IFaG $\beta$ CM if for each IFP  $p_{(\alpha, \beta)} \in Y$  and for each IF $\beta$ OS  $B$  in  $X$  such that  $f^{-1}(p_{(\alpha, \beta)}) \in B$ ,  $\beta cl(f(B))$  is an IF $\beta$ N of  $p_{(\alpha, \beta)} \in Y$ .

**Proof:** Let  $p_{(\alpha, \beta)} \in Y$  and let  $A$  be an IFROS in  $X$ . Then  $A$  is an IF $\beta$ OS in  $X$ . By hypothesis  $f^{-1}(p_{(\alpha, \beta)}) \in A$ , that is  $p_{(\alpha, \beta)} \in f(A)$  in  $Y$  and  $\beta cl(f(A))$  is an IF $\beta$ N of  $p_{(\alpha, \beta)}$

in  $Y$ . Therefore there exists an IF $\beta$ OS  $B$  in  $Y$  such that  $p_{(\alpha, \beta)} \in B \subseteq \beta\text{cl}(f(A))$ . We have  $p_{(\alpha, \beta)} \in f(A) \subseteq \beta\text{cl}(f(A))$ . Now  $B = \cup \{ p_{(\alpha, \beta)} / p_{(\alpha, \beta)} \in B \} = f(A)$ . Therefore  $f(A)$  is an IF $\beta$ OS in  $Y$  and hence an IFG $\beta$ OS in  $Y$ . Thus  $f$  is an IFaG $\beta$ OM. By Theorem  $f$  is an IFaG $\beta$ CM.

**Theorem 3.13:** Let  $f: X \rightarrow Y$  be a mapping where  $Y$  is an IF $\beta$ T $_{1/2}$  space. Then the following are equivalent.

- (i)  $f$  is an IFaG $\beta$ OM
- (ii) for each IFP  $p_{(\alpha, \beta)}$  in  $Y$  and each IFROS  $B$  in  $X$  such that  $f^{-1}(p_{(\alpha, \beta)}) \in B$ ,  $\text{cl}(f(\text{cl}(B)))$  is an IF $\beta$ N of  $p_{(\alpha, \beta)}$  in  $Y$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $p_{(\alpha, \beta)} \in Y$  and let  $B$  be an IFROS in  $X$  such that  $f^{-1}(p_{(\alpha, \beta)}) \in B$ . That is  $p_{(\alpha, \beta)} \in f(B)$ . By hypothesis,  $f(B)$  is an IFG $\beta$ OS in  $Y$ . Since  $Y$  is an IF $\beta$ T $_{1/2}$  space,  $f(B)$  is an IF $\beta$ OS in  $Y$ . Now  $p_{(\alpha, \beta)} \in f(B) \subseteq f(\text{cl}(B)) \subseteq \text{cl}(f(\text{cl}(B)))$ . Hence  $\text{cl}(f(\text{cl}(B)))$  is an IF $\beta$ N of  $p_{(\alpha, \beta)}$  in  $Y$ .

(ii)  $\Rightarrow$  (i) Let  $B$  be an IFOS in  $X$ . Then  $f^{-1}(p_{(\alpha, \beta)}) \in B$ . This implies  $p_{(\alpha, \beta)} \in f(B)$ . By hypothesis,  $\text{cl}(f(\text{cl}(B)))$  is an IF $\beta$ N of  $p_{(\alpha, \beta)}$ . Therefore there exists an IF $\beta$ OS  $A$  in  $Y$  such that  $p_{(\alpha, \beta)} \in A \subseteq \text{cl}(f(\text{cl}(B)))$ . Now  $A = \cup \{ p_{(\alpha, \beta)} / p_{(\alpha, \beta)} \in A \} = f(B)$ . Therefore  $f(B)$  is an IF $\beta$ OS and hence an IFG $\beta$ OS in  $Y$ . Thus  $f$  is an IFaG $\beta$ OM.

**Theorem 3.14:** The following are equivalent for a mapping  $f: X \rightarrow Y$  where  $y$  is an IF $\beta$ T $_{1/2}$  space.

- (i)  $f$  is an IFaG $\beta$ CM
- (ii)  $\beta\text{cl}(f(A)) \subseteq f(\alpha\text{cl}(A))$  for every IF $\beta$ OS  $A$  in  $X$
- (iii)  $\beta\text{cl}(f(A)) \subseteq f(\alpha\text{cl}(A))$  for every IFSOS  $A$  in  $X$
- (iv)  $f(A) \subseteq \beta\text{int}(f(\text{scl}(A)))$  for every IFPOS  $A$  in  $X$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $A$  be an IF $\beta$ OS in  $X$ . Then  $\text{cl}(A)$  is an IFRC $\beta$ S in  $X$ . By hypothesis  $f(A)$  is an IFG $\beta$ CS in  $Y$  and hence is an IF $\beta$ CS in  $Y$ , since  $Y$  is an IF $\beta$ T $_{1/2}$  space. This implies  $\beta\text{cl}(f(\text{cl}(A))) = f(\text{cl}(A))$ . Now  $\beta\text{cl}(f(A)) \subseteq \beta\text{cl}(f(\text{cl}(A))) = f(\text{cl}(A))$ . Since  $\text{cl}(A)$  is an IFRC $\beta$ S,  $\text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(A)$ . Therefore  $\beta\text{cl}(f(A)) \subseteq f(\text{cl}(A)) = f(\text{cl}(\text{int}(\text{cl}(A)))) \subseteq f(A \cup \text{cl}(\text{int}(\text{cl}(A)))) \subseteq f(\alpha\text{cl}(A))$ . Hence  $\beta\text{cl}(f(A)) \subseteq f(\alpha\text{cl}(A))$ .

(ii)  $\Rightarrow$  (iii) Let  $A$  be an IFSOS in  $X$ . Since every IFSOS is an IF $\beta$ OS, the proof is obvious.

(iii)  $\Rightarrow$  (i) Let  $A$  be an IFRC $\beta$ S in  $X$ . Then  $A = \text{cl}(\text{int}(A))$ . Therefore  $A$  is an IFSOS in  $X$ . By hypothesis,  $\beta\text{cl}(f(A)) \subseteq f(\alpha\text{cl}(A)) \subseteq f(\text{cl}(A)) = f(A) \subseteq \beta\text{cl}(f(A))$ . That is  $\beta\text{cl}(f(A)) = f(A)$ . Hence  $f(A)$  is an IF $\beta$ CS and hence is an IFG $\beta$ CS in  $Y$ . Thus  $f$  is an IFaG $\beta$ CM.

(i)  $\Rightarrow$  (iv) Let  $A$  be an IFPOS in  $X$ . Then  $A \subseteq \text{int}(\text{cl}(A))$ . Since  $\text{int}(\text{cl}(A))$  is an IFROS in  $X$ , by hypothesis,  $f(\text{int}(\text{cl}(A)))$  is an IFG $\beta$ OS in  $Y$ . Since  $Y$  is an IF $\beta$ T $_{1/2}$  space,  $f(\text{int}(\text{cl}(A)))$  is an IF $\beta$ OS in  $Y$ . Therefore  $f(A) \subseteq f(\text{int}(\text{cl}(A))) \subseteq \beta\text{int}(f(\text{int}(\text{cl}(A)))) = \beta\text{int}(f(A \cup \text{int}(\text{cl}(A)))) = \beta\text{int}(f(\text{scl}(A)))$ . That is  $f(A) \subseteq \beta\text{int}(f(\text{scl}(A)))$ .

(iv)  $\Rightarrow$  (i) Let  $A$  be an IFROS in  $X$ . Then  $A$  is an IFPOS in  $X$ . By hypothesis,  $f(A) \subseteq \beta\text{int}(f(\text{scl}(A)))$ . This implies  $f(A) \subseteq \beta\text{int}(f(A \cup \text{int}(\text{cl}(A)))) \subseteq \beta\text{int}(f(A \cup A)) = \beta\text{int}(f(A)) \subseteq f(A)$ . Therefore  $f(A)$  is an IF $\beta$ OS in  $Y$  and hence an IFG $\beta$ OS in  $Y$ . Thus  $f$  is an IFaG $\beta$ CM.

Next we provide the characterization theorem for an IFaG $\beta$ OM.

**Theorem 3.15:** Let  $f: X \rightarrow Y$  be a bijective mapping. Then the following are equivalent.

- (i)  $f$  is an IFaG $\beta$ OM
- (ii)  $f$  is an IFaG $\beta$ CM
- (iii)  $f^{-1}$  is an IFaG $\beta$ Cts.M.

**Proof:** (i)  $\Leftrightarrow$  (ii) is obvious.

(ii)  $\Rightarrow$  (iii) Let  $A \subseteq X$  be an IFRCs. Then by hypothesis,  $f(A)$  is an IFG $\beta$ CS in  $Y$ . That is  $(f^{-1})^{-1}(A)$  is an IFG $\beta$ CS in  $Y$ . This implies  $f^{-1}$  is an IFaG $\beta$ Cts.M.

(iii)  $\Rightarrow$  (ii) Let  $A \subseteq X$  be an IFRCs. Then by hypothesis  $(f^{-1})^{-1}(A)$  is an IFG $\beta$ CS in  $Y$ . That is  $f(A)$  is an IFG $\beta$ CS in  $Y$ . Hence  $f$  is an IFaG $\beta$ CM.

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