INTRODUCTION

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***"Life is good for only two things –
Discovering Mathematics and Teaching Mathematics".***

-Simeon Poisson

Soft set theory is one of the recent topics gaining significance in finding rational and logical solutions to various real life problems which involve uncertainty, impreciseness and vagueness.

Uncertainty is present in almost every sphere of our daily life. Traditional mathematical tools are not sufficient to handle all the practical problems in fields such as Medical Science, Social Science, Engineering, Economics etc, which involve various types of uncertainty. Zadeh (1965) was the first to come up with his remarkable theory of Fuzzy sets for dealing these types of uncertainties where conventional tools fail. His theory brought a grand paradigmatic change in mathematics.

Later, there are theories namely the Intuitionistic Fuzzy sets, Vague sets, Rough sets, Interval Mathematics etc, for handling uncertainty. Molodtsov (1999) initiated the theory of soft sets as a new mathematical tool for dealing uncertainty and Maji et al. (2001) initiated the concept of Fuzzy soft sets. Atanassov (1986) defined the concept of an Intuitionistic Fuzzy set which is more general than a Fuzzy set. Maji et al. (2001) introduced the concept of Intuitionistic Fuzzy soft set.

"Let U be the initial universe set and E be the set of parameters. For $A \subseteq E$, the pair (F, A) is called a **Soft Set** over U if F is a mapping from A to the set of all subsets of the set U. The pair (F, A) is called a **Fuzzy Soft Set** over U if F is a mapping from A to the set of all Fuzzy subsets of the set U. The pair (F, A) is called an **Intuitionistic Fuzzy Soft Set** over U if F is a mapping from A to the set of all Intuitionistic Fuzzy subsets of U."

It is well known that the matrix formulation of a mathematical formula gives extra facility to study the problem. Due to the presence of uncertainty in many mathematical formulations in different branches of science and technology, Fuzzy matrices were introduced by Thomason (1977). Fuzzy matrices play an important role in scientific development.

In 2011, Yong Yang and Chenli Ji proposed matrix representation of a Fuzzy soft set and successfully applied the proposed notion of Fuzzy soft matrices in certain decision making problems. Chetia and Das (2012) introduced Intuionistic Fuzzy soft matrices.

"Let $U = \{c_1, c_2 \ldots c_m\}$ be the universal set and $E = \{e_1, e_2, e_3 \ldots e_n\}$ be the set of parameters. For $A \subseteq E$, the Fuzzy soft set (F, A) can be represented in matrix form as $S_{m\times n} = [a_{ij}]_{m\times n}$ or simply by $S = [a_{ij}]$, $i = 1,2,\ldots,m$; $j = 1,2,\ldots,n$, where

$$a_{ij} = \begin{cases} \mu_j(c_i), & \text{if } e_j \in A \\ 0, & \text{if } e_j \notin A \end{cases}$$

Here $\mu_j(c_i)$ represents the membership of c_i in the Fuzzy set $F(e_j)$. The matrix $S_{m\times n}$ is called a **Fuzzy soft matrix**.

For $A \subseteq E$, the Intuitionistic Fuzzy soft set (F, A) can be represented in matrix form as $S_{mxn} = [a_{ij}]_{mxn}$ or $S = [a_{ij}]$, $i = 1, 2, 3,\ldots,m$, $j = 1, 2, 3,\ldots,n$, where

$$a_{ij} = \begin{cases} (\mu_j(c_i), \nu_j(c_i)) & \text{if } e_j \in A \\ (0,1) & \text{if } e_j \notin A \end{cases}$$

Here $\mu_j(c_i)$ represents the membership of c_i in the Intuitionistic Fuzzy set $F(e_j)$ and $\nu_j(c_i)$ represents the non-membership of c_i in the Intuitionistic Fuzzy set $F(e_j)$. The matrix $S_{m\times n}$ is called **Intuitionistic Fuzzy soft matrix**."

The concepts of soft relations and Fuzzy soft relations are applied in many decision making problems. In approximate reasoning, Fuzzy soft relations have shown to be of a primordial importance. Molodtsov (1999) gave the definition of Fuzzy soft relation.

"A Fuzzy soft relation may be defined as a soft set over the fuzzy power set of the Cartesian product of two crisp sets. If $P(X\otimes Y)$ is the Fuzzy power set; X and Y are two non-empty crisp sets of some Universal set and E is a set of parameters, then a function $R: E \rightarrow P(X\otimes Y)$ is called a fuzzy soft relation."

Majumder and Samanta (2010) introduced generalized Fuzzy soft sets. While generalizing the concept of Fuzzy soft sets, Majumder and Samanta (2010) considered the same set of parameters. But Manash Jyoti Borah et al. (2012) generalized the concept of Fuzzy soft sets by considering different sets of parameters and also they defined Generalized Fuzzy soft relations.

The concept of relations in Intuitionistic Fuzzy soft sets was introduced by Bivas Dinda and Samanta (2010).

Bivas Dinda et al. (2012) gave a new definition called Generalized Intuitionistic Fuzzy soft sets using the concepts of Generalized Fuzzy soft sets and Intuitionistic Fuzzy soft sets. Also they defined relations on Generalized Intuitionistic Fuzzy soft sets.

The main aim of this thesis is to study Fuzzy soft matrices and Fuzzy soft relations with applications to real life problems.

The plan of study is as follows :

1) Fuzzy Soft Matrices
2) Intuitionistic Fuzzy Soft Matrices
3) Fuzzy Soft Relations
4) Generalized Fuzzy Soft Relations
5) Intuitionistic Fuzzy Soft Relations
6) Generalized Intuitionistic Fuzzy Soft Relations

The First chapter deals with preliminary definitions and notations.

The Second chapter is devoted to the study of Fuzzy soft matrices. In this chapter some basic properties of Fuzzy soft matrices regarding

Fuzzy soft union, intersection, complement of a Fuzzy soft set, De Morgan's law etc., are studied with interesting examples. Also different products of Fuzzy soft matrices are defined and their properties are analysed. Finally two decision making methods one by using fuzzy soft "*" product and the other by using Fuzzy soft matrix decision function are illustrated with applications.

Intuitionistic Fuzzy soft matrices are studied in chapter III. Different types of Intuitionistic Fuzzy soft matrices are defined and their properties are studied. Five different types of products of Intuitionistic Fuzzy soft Matices are defined with examples. Finally, application of Intuitionistic Fuzzy soft matrices in decision making problems is illustrated.

Chapter IV deals with Fuzzy soft relations. In this chapter the concept of relations on Fuzzy soft sets introduced by Manash Jyoti Borah et al. (2012) are studied with their properties. Also the notions of symmetric, transitive, reflexive, irreflexive and equivalence Fuzzy soft relations are established. The important results are given in theorems 4.16, 4.17, 4.23, 4.24, 4.25, 4.27 and 4.28.

Chapter V is devoted to the study of Generalized Fuzzy soft relations introduced by Manash Jyoti Borah et al. (2012). In the first part of this chapter Generalized Fuzzy soft sets and their properties are studied with examples. In the second part of this paper the Generalized Fuzzy soft relations are studied. Important results proved regarding Generalized Fuzzy soft relations are given in theorems 5.24, 5.26, 5.27, 5.29 and 5.30.

In chapter VI, the concept of Intuitionistic Fuzzy Soft Relations is introduced and few of its algebraic properties are studied. The following are the important results proved in this chapter :

1) Composition of two Intuitionistic Fuzzy soft relations is an Intuitionistic Fuzzy Soft relation. (Theorem 6.11)
2) If R is a symmetric (respectively transitive, reflexive) Intuitionistic Fuzzy Soft relation then R^{-1} is also a symmetric (respectively transitive,

reflexive) Intuitionistic Fuzzy Soft relation. (Theorems 6.16, 6.20 and 6.22)

3) If R is a symmetric and transitive Intuitionistic Fuzzy soft relation then R is a reflexive Intuitionistic Fuzzy Soft relation. (Theorem 6.23)

Chapter VII deals with Generalized Intuitionistic Fuzzy soft relations. In this chapter relations on Generalized Intuitionistic Fuzzy soft sets and few of their algebraic properties are studied. Finally, an application of Generalized Intuitionistic Fuzzy soft relation in decision making problems is illustrated.

Chapter VIII deals with an application of Intuitionistic Fuzzy soft matrices. In this chapter the author made an attempt to develop a decision making model to analyse the impact of excessive television viewing by children.

Television, an effective medium of mass communication, plays a predominant role in the daily routine of the public. This indoor medium has multidimensional values. In addition to entertainment, matters relating to science and technology, social and cultural aspects, sports and games, economics and business, wild life and adventure and many more are telecasted. Viewing of such programs undoubtedly enriches wisdom of the people, especially the children. They are privileged to view and learn everything around the world. Many programs motivate and develop creativity among such younger generation.

It is quite natural that every coin has two sides. Though better exposure is the prime advantage, long time viewing is common in case of children. Such long time viewing of television programs by children not only causes health hazards to the children but also affects their overall performance in their academic activities. Many research studies in India and abroad bring to light the prevalence of excessive viewing of television by children and its ill – effects. So the author intended to identify the various problems encountered by the children (in the age group of 6 to 16 years), based on the opinion of the parents regarding the ill - effects of excessive viewing of television and analyze the same using intuitionistic fuzzy soft matrices.

It is worth mentioning that the author has sent an article entitled "Impact of Excessive Television Viewing by Children– An Analysis using Intuitionistic Fuzzy Soft Matrices" for publication in *InternationalJournal of Mathematical Sciences and Applications* and the same has been accepted for publication in the ensuing issue of the journal.