CHAPTER - V

CHAPTER V

INTUITIONISTIC FUZZY ANALYTIC HIERARCHY PROCESS METHOD

The decision-making procedure, AHP is widely used in many fields. In this classic AHP model, the comparison values over different criteria that are provided by the decision maker are represented by crisp numbers within the 1–9 scale. Due to this AHP has inability to adequately tackle the inherent uncertainty and vagueness. To overcome this issue, the fuzzy set theory was introduced to AHP, and then, the traditional AHP was extended to the Fuzzy AHP (FAHP), where each pairwise comparison judgment is represented as a fuzzy number that is described by a membership function.

Although the FAHP can do better than the classic AHP in capturing and representing a human's appraisal of ambiguity for a comprehensive multi-criteria decision-making problem, the single membership function of fuzzy set also limits the application of the FAHP because it cannot be used to express the support and objection evidences simultaneously. In this situation Intuitionistic Fuzzy Set comes to place. An intuitionistic fuzzy set has many advantages in handling vagueness and uncertainty over a fuzzy set. The Intuitionistic Fuzzy Set (IFS) is characterized by a membership function, a non-membership function, and a hesitancy function, is suitable to model these situations. Thus the Intuitionistic Fuzzy Analytic Hierarchy Process (IFAHP) can be used to handle more complex problems, where the decision maker has some uncertainty in assigning preference values to the objects considered.

The algorithm of the AHP and the FAHP involve three phases: 1) decomposition, 2) pairwise comparison, and 3) synthesis of priorities. To extend the conventional AHP and the FAHP to intuitionistic fuzzy circumstances, we investigate these phases in the context of IFS one by one.

1) Decomposition

The decomposition principle calls for structuring the hierarchy to capture the basic elements of the problem. The IFAHP is used to handle a variety of complex and comprehensive multi criteria decision-making problems which have several alternatives to implement and quite a lot of criteria are used to check and evaluate the implementation. Thus to apply the IFAHP, initially, the comprehensive problem needs to be structured into different hierarchical levels with regards to the properties or attributes of the problem considered. An affinity diagram, a tree diagram, or cluster analysis can be applied to construct the hierarchy. For simplifying the presentation, we suppose that $A = \{A_1, A_2, \ldots, A_n\}$ is the finite set of n alternatives, and $C = \{C_1, C_2, \ldots, C_m\}$ is the set of criteria with which the elements of A are compared in the hierarchical structure.

2) Comparative Judgments With Intuitionistic Fuzzy Values

After decomposing the complex multi criteria decision making problem into different levels, we need to set up a preference relation to carry out pairwise comparisons of the relative importance of the elements in a level with respect to the elements in the level immediately above it. In order to represent the relative importance between the pairwise compared elements, Saaty [55] gave the definition of the scale of pairwise comparison as follows.

Definition 5.1 [56]

Let $A = \{A_1, A_2, \ldots, A_n\}$ be a finite set of alternatives, and $C = \{C_1, C_2, \ldots, C_m\}$ be a set of criteria to compare the alternatives. The ***scale of pairwise comparison*** for the criteria $C_j \in C$ $(j = 1, 2, \ldots, m)$ is a mapping P_{C_j}, which assigns to every pair $(A_i, A_k) \in A \times A$ a positive real number $P_{C_j}(A_i, A_k) = a_{ik}$ that denotes the relative intensity with which an individual perceives the criterion $C_j \in C$ in an element $A_i \in A$ in relation to the other $A_k \in A$.

In IFAHP, this scale of the pairwise comparison can be represented by an intuitionistic fuzzy set.

3) Intuitionistic Preference Relation

If all the pairwise comparison judgments are represented by IFVs, an intuitionistic preference relation can be defined as follows.

Definition 5.2 [78]

An ***Intuitionistic Preference Relation*** R on the set $X = \{x_1, x_2, \dots, x_n\}$ is represented by a matrix $R = (r_{ik})_{n \times n}$, where $r_{ik} =< (x_i, x_k), \mu(x_i, x_k), \nu(x_i, x_k) >$ for all $i, k = 1,2 \dots, n$. For convenience, we let $r_{ik} = (\mu_{ik}, \nu_{ik})$, where μ_{ik} denotes the degree to which the object x_i is preferred to the object x_k , ν_{ik} indicates the degree to which the object x_i is not preferred to the object x_k, and $\pi(x_i, x_k) = 1 - \mu(x_i, x_k) - \nu(x_i, x_k)$ is interpreted as an indeterminacy degree or a hesitancy degree, with the condition

$$\mu_{ik}, \nu_{ik} \in [0,1], \;\; \mu_{ik} + \nu_{ik} \le 1, \;\; \mu_{ik} = \nu_{ki}, \;\; \mu_{ki} = \nu_{ik}$$

$$\mu_{ii} = \nu_{ii} = 0.5, \;\; \pi_{ik} = 1 - \mu_{ik} - \nu_{ik} \quad \forall\, i, k = 1,2, \dots, n$$

Let $r_{ik} = (\mu_{ik}, \nu_{ik})$ and $r_{tl} = (\mu_{tl}, \nu_{tl})$ be two IFVs in R. Then:

i. $r_{ik} \oplus r_{tl} = (\mu_{ik} + \mu_{tl} - \mu_{ik}\mu_{tl}, \nu_{ik}\nu_{tl})$

ii. $r_{ik} \otimes r_{tl} = (\mu_{ik}\mu_{tl}, \nu_{ik} + \nu_{tl} - \nu_{ik}\nu_{tl})$

iii. $\lambda\, r_{ik} = \left(1 - (1 - \mu_{ik})^{\lambda}, {\nu_{ik}}^{\lambda}\right)$, $\lambda > 0$

iv. ${r_{ik}}^{\lambda} = \left({\mu_{ik}}^{\lambda}, 1 - (1 - \nu_{ik})^{\lambda}\right)$, $\lambda > 0$

The Intuitionistic preference relation R may have several properties:

Let $R = (r_{ik})_{n \times n}$ be an intuitionistic preference relation, where $r_{ik} = (\mu_{ik}, \nu_{ik})$ $i, k = 1,2 \dots, n$. By using the comparison law of IFVs, we have:

i. If $r_{it} \oplus r_{tk} \ge r_{ik}$ for all $i, t, k = 1,2 \dots, n$, then we say R satisfies the triangle condition;

ii. If $r_{it} \ge (0.5, 0.5)$, $r_{tk} \ge (0.5, 0.5) \Rightarrow r_{ik} \ge (0.5, 0.5)$, for all $i, t, k = 1,2 \dots, n$ then we say R satisfies the weak transitivity property;

iii. If $r_{ik} \ge \min\{r_{it}, r_{tk}\}$, for all $i, t, k = 1,2 \dots, n$, then we say R satisfies the max–min transitivity property;

iv. If $r_{ik} \ge \max\{r_{it}, r_{tk}\}$, for all $i, t, k = 1,2 \dots, n$, then we say R satisfies the max–max transitivity property;

v. If $r_{it} \ge (0.5,0.5)$, $r_{tk} \ge (0.5,0.5) \Rightarrow r_{ik} \ge \min\{r_{it}, r_{tk}\}$ for all $i, t, k = 1,2 \dots, n$, then we say R satisfies the restricted max–min transitivity property;

vi. If $r_{it} \geq (0.5,0.5)$, $r_{tk} \geq (0.5,0.5) \Rightarrow r_{ik} \geq \max\{r_{it}, r_{tk}\}$ for all $i, t, k = 1,2 \dots, n$, then we say R satisfies the restricted max–max transitivity property.

4) Consistency Checking

In the IFAHP, in order to get a reasonable solution, before deriving the priorities of the alternatives and criteria, we need to check whether the intuitionistic preference relation is consistent or not. Consistency is an important topic in preference relations and the lack of consistency of preference relations may lead to misleading solutions.

Multiplicative consistency is a very important property of preference relations. Here we employ this property to discover whether an interval fuzzy preference relation is consistent or not, as well as to derive the priority vector of a consistent interval fuzzy preference relation.

The definition of multiplicative consistent intuitionistic preference relation as follows:

Definition 5.3 [78]

An intuitionistic preference relation $R = (r_{ik})_{n\times n}$ with $r_{ik} = (\mu_{ik}, \nu_{ik})$, $(i, k = 1,2 \dots, n)$ is ***multiplicative consistent*** if

$$\mu_{ik} = \begin{cases} 0 \ , & if\ (\mu_{it}, \mu_{tk}) \in \{(0,1),(1,0)\} \\ \dfrac{\mu_{it}\mu_{tk}}{\mu_{it}\mu_{tk} + (1-\mu_{it})(1-\mu_{tk})} \ , & otherwise \\ & i \leq t \leq k \end{cases} \qquad (20)$$

$$\nu_{ik} = \begin{cases} 0 \ , & if\ (\nu_{it}, \nu_{tk}) \in \{(0,1),(1,0)\} \\ \dfrac{\nu_{it}\nu_{tk}}{\nu_{it}\nu_{tk} + (1-\nu_{it})(1-\nu_{tk})} \ , & otherwise \\ & i \leq t \leq k \end{cases} \qquad (21)$$

Note that if $(\mu_{it}, \mu_{tk}) \in \{(0,1),(1,0)\}$, which means $(\mu_{it}, \mu_{tk}) = (0,1)$ or $(\mu_{it}, \mu_{tk}) = (1,0)$ or both of them hold, the denominator in (20) will be equal to 0, i.e., $\mu_{it}\mu_{tk} + (1-\mu_{it})(1-\mu_{tk}) = 0$; thus, $\mu_{tk} = \frac{\mu_{it}\mu_{tk}}{\mu_{it}\mu_{tk}+(1-\mu_{it})(1-\mu_{tk})}$ makes no sense. Hence, we let $\mu_{tk} = 0$ when $(\mu_{it}, \mu_{tk}) \in \{(0,1),(1,0)\}$. Similarly, in (21), we let $\nu_{ik} = 0$ when $(\nu_{it}, \nu_{tk}) \in \{(0,1),(1,0)\}$.

An algorithm to construct a perfect multiplicative consistent intuitionistic preference relation $\bar{R} = (\overline{r_{ik}})_{n\times n}$.

Algorithm I:

Step 1: For $k > i + 1$, let $\overline{r_{ik}} = (\overline{\mu_{ik}}, \overline{\nu_{ik}})$, where

$$\overline{\mu_{ik}} = \frac{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \mu_{it}\mu_{tk}}}{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \mu_{it}\mu_{tk}} + \sqrt[k-i-1]{\prod_{t=i+1}^{k-1}(1-\mu_{it})(1-\mu_{tk})}} \quad ; \quad k > i+1$$

$$\overline{\nu_{ik}} = \frac{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \nu_{it}\,\nu_{tk}}}{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \nu_{it}\,\nu_{tk}} + \sqrt[k-i-1]{\prod_{t=i+1}^{k-1}(1-\nu_{it})(1-\nu_{tk})}} \quad ; \quad k > i+1$$

Step 2: For $k = i + 1$, let $\overline{r_{ik}} = r_{ik}$.

Step 3: For $k < i$, let $\overline{r_{ik}} = (\overline{\mu_{ki}}, \overline{\nu_{ki}})$.

Thus we update less than half of the elements in the original intuitionistic preference relation to construct the perfect multiplicative consistent intuitionistic preference relation $\bar{R} = (\overline{r_{ik}})_{n\times n}$ for R.

Definition 5.4 [78]

Let R be an intuitionistic preference relation, then we call R an ***acceptable multiplicative consistent intuitionistic preference relation***, if

$$d(R, \bar{R}) < \tau$$

where $d(R, \bar{R})$ is the distance measure between the given intuitionistic preference relation R and its corresponding perfect multiplicative consistent intuitionistic preference relation $\bar{R}$, which can be calculated by

$$d(R,\bar{R}) = \frac{1}{2(n-1)(n-2)}\sum_{i=1}^{n}\sum_{k=1}^{n}(|\overline{\mu_{ik}} - \mu_{ik}| + |\overline{\nu_{ik}} - \nu_{ik}| + |\overline{\pi_{ik}} - \pi_{ik}|) \quad (22)$$

and τ is the consistency threshold.

Consider that Saaty [55] derived a consistency ratio from the maximum eigenvalue of a multiplicative preference relation and a randomization process, and pointed out that the multiplicative preference relation is of acceptable consistency if its consistency ratio is less than 0.1. Without loss of generality, we also let $\tau = 0.1$ as the consistency threshold.

For any inconsistent intuitionistic preference relation $R = (r_{ik})_{n\times n}$, we can use Algorithm I to transform it into its corresponding perfect multiplicative consistent intuitionistic preference relation $\bar{R} = (\overline{r_{ik}})_{n\times n}$. Meanwhile, the deviation $d(R,\bar{R})$ between the initial intuitionistic preference relation and the transformed one can be calculated by using (22). If the deviation $d(R,\bar{R})$ is too large, then we may think that the transformed intuitionistic preference relation $\bar{R}$ can not represent the initial preferences of the decision maker. It is desirable that the modified intuitionistic preference relation should not only have acceptable multiplicative consistency but maintain the original preference information of the decision maker as much as possible as well. Hence, it is proper to fuse the initial intuitionistic preference relation R and its corresponding perfect multiplicative consistent intuitionistic preference relation $\overline{R}$ into a new intuitionistic preference relation, where each $\tilde{R} = (\widetilde{r_{ik}})_{n\times n}$ element is defined as

$$\widetilde{\mu_{ik}} = \frac{(\mu_{ik})^{1-\sigma}\,(\overline{\mu_{ik}})^{\sigma}}{(\mu_{ik})^{1-\sigma}(\overline{\mu_{ik}})^{\sigma} + (1-\mu_{ik})^{1-\sigma}(1-\overline{\mu_{ik}})^{\sigma}} \qquad i,k = 1,2,\ldots,n$$

$$\widetilde{\nu_{ik}} = \frac{(\nu_{ik})^{1-\sigma}\,(\overline{\nu_{ik}})^{\sigma}}{(\nu_{ik})^{1-\sigma}(\overline{\nu_{ik}})^{\sigma} + (1-\nu_{ik})^{1-\sigma}(1-\overline{\nu_{ik}})^{\sigma}} \qquad i,k = 1,2,\ldots,n$$

where σ is a controlling parameter that is determined by the decision maker, the smaller the value of σ, the closer $\tilde{R}$ is to R. Especially if $\sigma = 0$, $\tilde{R} = R$; if $\sigma = 1$, $\tilde{R} = \bar{R}$. Obviously, $\tilde{R}$ is also an intuitionistic preference relation. Generally, the fused intuitionistic preference relation $\tilde{R}$ contains not only the preference information of the initial intuitionistic preference relation R but the preference information of its corresponding perfect multiplicative consistent intuitionistic preference relation $\bar{R}$ as well. The controlling parameter σ also represents the preference of the decision maker to some extent.

Based on the aforementioned analysis, an automatic algorithm to repair the inconsistent intuitionistic preference relation can be developed.

Algorithm II

Step 1: Suppose that p is the number of iterations. Let $p = 1$, and construct the perfect multiplicative consistent intuitionistic preference relation $\bar{R}$ from $R^{(p)}$ by Algorithm I.

Step 2: Calculate the distance $d\left(\bar{R}, R^{(p)}\right)$ between R and $R^{(p)}$, where

$$d\left(\bar{R}, R^{(p)}\right) = \frac{1}{2(n-1)(n-2)} \sum_{i=1}^{n} \sum_{k=1}^{n} \left(\left| \overline{\mu_{ik}} - \mu_{ik}{}^{(p)} \right| + \left| \overline{\nu_{ik}} - \nu_{ik}{}^{(p)} \right| + \left| \overline{\pi_{ik}} - \pi_{ik}{}^{(p)} \right| \right)$$

If $d\left(\bar{R}, R^{(p)}\right) < \tau$, then output $R^{(p)}$; otherwise, go to the next step.

Step 3: Construct the fused intuitionistic preference relation $\tilde{R}^{(p)} = \left(\widetilde{r_{ik}}^{(p)}\right)_{n\times n}$, $\left(\widetilde{r_{ik}}^{(p)} = \left(\widetilde{\mu_{ik}}^{(p)}, \widetilde{\nu_{ik}}^{(p)}\right)\right)$ by using

$$\widetilde{\mu_{ik}}^{(p)} = \frac{\left(\mu_{ik}{}^{(p)}\right)^{1-\sigma} \left(\overline{\mu_{ik}}\right)^{\sigma}}{\left(\mu_{ik}{}^{(p)}\right)^{1-\sigma} \left(\overline{\mu_{ik}}\right)^{\sigma} + \left(1-\mu_{ik}{}^{(p)}\right)^{1-\sigma} \left(1-\overline{\mu_{ik}}\right)^{\sigma}} \qquad i,k = 1,2,\dots,n$$

$$\widetilde{\nu_{ik}}^{(p)} = \frac{\left(\nu_{ik}{}^{(p)}\right)^{1-\sigma} \left(\overline{\nu_{ik}}\right)^{\sigma}}{\left(\nu_{ik}{}^{(p)}\right)^{1-\sigma} \left(\overline{\nu_{ik}}\right)^{\sigma} + \left(1-\nu_{ik}{}^{(p)}\right)^{1-\sigma} \left(1-\overline{\nu_{ik}}\right)^{\sigma}} \qquad i,k = 1,2,\dots,n$$

where σ is a controlling parameter determined by the decision maker: The smaller the value of σ, the closer $\tilde{R}^{(p)}$ is to $R^{(p)}$. Let $R^{(p+1)} = \tilde{R}^{(p)}$, i.e., $\mu_{ik}{}^{(p+1)} = \widetilde{\mu_{ik}}^{(p)}$ and $\nu_{ik}{}^{(p+1)} = \widetilde{\nu_{ik}}^{(p)}$. Let $p = p + 1$, and then, go to Step 2.

Through this algorithm, we can improve the consistency level of any intuitionistic preference relation automatically without losing much original information. Comparing this algorithm with the interactive method, our procedure can save a lot of time for the decision maker. It shows many advantages in helping the decision maker to reach a quick decision. This procedure is convergent, and the derived intuitionistic preference relation has weak transitivity.

5) Priority Method

A fundamental scale stored in the preference relation does not directly give a scale of priorities. A scale of priorities is, per Saaty's [56] idea, an n-dimensional vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ obtained from the multiplicative preference relation, and ω_i is a weight which accurately represents the relative dominance of the alternative A_i among the alternatives in A. Here we intended to find a method to generate the global or composite priorities of the elements at the lowest level of the hierarchy.

Considering an intuitionistic preference relation $R = (r_{ik})_{n\times n}$, where $r_{ik} = (\mu_{ik}, \nu_{ik})$, since $\mu_{ik}, \nu_{ik} \in [0,1]$, $\mu_{ik} + \nu_{ik} \leq 1$, we have $\mu_{ik} \leq 1 - \nu_{ik}$. Then, we can transform the pair (μ_{ik}, ν_{ik}) into the interval $[\mu_{ik}, 1 - \nu_{ik}]$. Hence, the intuitionistic preference relation $R = ((\mu_{ik}, \nu_{ik}))_{n\times n}$ can be transformed into an interval-valued preference relation $R' = ([\mu_{ik}, 1 - \nu_{ik}])_{n\times n}$. If we want to derive the priority vector of the intuitionistic preference relation $R = (r_{ik})_{n\times n}$, we can accomplish it via analyzing the interval-valued preference relation $R' = (r_{ik}')_{n\times n} = ([\mu_{ik}, 1 - \nu_{ik}])_{n\times n}$. Based on the operational laws of intervals, we develop a new normalizing rank summation method to derive the priority weights as follows:

$$\omega_i = \frac{\sum_{k=1}^{n} r_{ik}'}{\sum_{i=1}^{n}\sum_{k=1}^{n} r_{ik}'}$$

$$= \frac{\sum_{k=1}^{n}[\mu_{ik}, 1 - \nu_{ik}]}{\sum_{i=1}^{n}\sum_{k=1}^{n}[\mu_{ik}, 1 - \nu_{ik}]}$$

$$= \frac{[\sum_{k=1}^{n}\mu_{ik}, \sum_{k=1}^{n}(1 - \nu_{ik})]}{[\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}, \sum_{i=1}^{n}\sum_{k=1}^{n}(1 - \nu_{ik})]}$$

$$= \left[\frac{\sum_{k=1}^{n}\mu_{ik}}{\sum_{i=1}^{n}\sum_{k=1}^{n}(1 - \nu_{ik})}, \frac{\sum_{k=1}^{n}(1 - \nu_{ik})}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}\right] \qquad i = 1,2,\dots,n$$

Then, we can transform each interval

$$\left[\frac{\sum_{k=1}^{n}\mu_{ik}}{\sum_{i=1}^{n}\sum_{k=1}^{n}(1-\nu_{ik})},\frac{\sum_{k=1}^{n}(1-\nu_{ik})}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}\right]$$

into a corresponding IFV

$$\left(\frac{\sum_{k=1}^{n}\mu_{ik}}{\sum_{i=1}^{n}\sum_{k=1}^{n}(1-\nu_{ik})},1-\frac{\sum_{k=1}^{n}(1-\nu_{ik})}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}\right)$$

Thus

$$\omega_i=\left(\frac{\sum_{k=1}^{n}\mu_{ik}}{\sum_{i=1}^{n}\sum_{k=1}^{n}(1-\nu_{ik})},1-\frac{\sum_{k=1}^{n}(1-\nu_{ik})}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}\right)\quad i=1,2,\dots,n \tag{23}$$

from which we get the priority vector $\omega=(\omega_1,\omega_2,\dots,\omega_n)$ of the intuitionistic preference relation $R=(r_{ik})_{n\times n}$, where each weight ω_i is an IFV. In fact, since $\mu_{ik}\leq 1-\nu_{ik}$ then, we get

$$\frac{\sum_{k=1}^{n}\mu_{ik}}{\sum_{i=1}^{n}\sum_{k=1}^{n}(1-\nu_{ik})}+1-\frac{\sum_{k=1}^{n}(1-\nu_{ik})}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}$$

$$=1+\frac{\sum_{k=1}^{n}\mu_{ik}}{\sum_{i=1}^{n}\sum_{k=1}^{n}(1-\nu_{ik})}-\frac{\sum_{k=1}^{n}(1-\nu_{ik})}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}$$

$$\leq 1+\frac{\sum_{k=1}^{n}\mu_{ik}}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}-\frac{\sum_{k=1}^{n}(1-\nu_{ik})}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}$$

$$=\frac{\sum_{k=1}^{n}\mu_{ik}-\sum_{k=1}^{n}(1-\nu_{ik})}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}$$

$$=1+\frac{\sum_{k=1}^{n}(\mu_{ik}-1+\nu_{ik})}{\sum_{i=1}^{n}\sum_{k=1}^{n}\mu_{ik}}\leq 1$$

The procedure of Intuitionistic Fuzzy Analytic Hierarchy Process, which is as follows.

Algorithm

Step 1: Identify the objective, criteria, sub criteria, and alternatives of the decision-making problem, and then, construct the hierarchy of the considered problem. Then, go to the next step.

Step 2: Determine the intuitionistic preference relations via the pairwise comparison between each criterion and subcriterion. Simultaneously, the alternatives are compared under each criterion or subcriterion, and then, the intuitionistic preference relations are constructed. The scale regarding the relative importance degrees is denoted as IFVs. Go to the next step.

Step 3: Check the consistency of each intuitionistic preference relation according to equation (22). If all of the intuitionistic preference relations are of acceptable consistency, go to Step 5; otherwise, goto Step 4.

Step 4: Repair the inconsistent intuitionistic preference relations according to Algorithm II (or return the inconsistent intuitionistic preference relations to the decision makers for reevaluation until they are acceptable). Then, go to the next step.

Step 5: Calculate the priority vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ of each intuitionistic preference relation through (23).

Step 6: Fuse all the weights from the lowest level to the highest level by the operations of IFVs, then rank the overall weights using the formula (1), and then, choose the best alternative.

Step 7: End