Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd.u/s 3 of UGC Act 1956, Category A by MHRD
Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore-641 043, Tamil Nadu, India
Continuous Internal Assessment Test II – October 2024

SemesterIII

Class: II PG **Time: 2 Hours**

Branch: Mathematics **Max. Marks: 60**

23MMAC13 - Topology

Course Outcomes:
CO1: Understand the properties of various topologies on a general set.
CO2: Construct continuous functions in topological spaces.
CO3: Analyze the relation between metric spaces and topological spaces.
CO4: Relate the concept of continuity to connected space.
CO5: Demonstrate the properties of compact spaces.

PART-A

Circle the correct answer **6x1=6**

1. Which of the following is a metric? CO3K1

(i) $d(x,y)=|x-y|$ ii) $d(x,y)=\sqrt{x^2+y^2}$ iii) $d(x,y)=\max\{|x|,|y|\}$

a. (i) only b. (i) and (ii) only d.(iii) only d) (i), (ii) and (iii)

2. The diameter of a subset A of a metric space (X, d) is CO3K1

a. Sup{d(x, y)/ x, y ∈ A} b. Inf{d(x, y)/x, y ∈ A}

c. Max{d(x, y)/ x, y ∈ A} d. Mim{d(x, y)/ x, y ∈ A}

3. Which of the following is a necessary condition for a space to be connected? CO4K1

a. It must be compact

b. It cannot be partitioned into two non empty disjoint open sets

c.It can be partitioned into two non empty open sets

d.It cannot be partitioned into two non emptyopen sets

4. A finite Cartesian product of a connected spaces is CO4K2

a.connected b.compact c. disconnected d. open

5. The real line R is CO5K1

a. not connected b. not compact c. compact d.sequentially compact

6. Which of the following is locally compact but not compact? CO5K1

a.R^n b.R^ω c. [0, 1] d. The set of all rational numbers Q

PART-B
Answer ALL questions **3x6=18**

7. a. Prove that for a metric space X with metric d and a function $\bar{d}:X\times X\to$ Rdefined by $\bar{d}(x,y)=\min\{d(x,y),1\}$, $\bar{d}$ is a metric that induces the same topology as d. CO3K4

(or)

7. b. State and prove the sequence lemma. CO3K4

8. a. Prove that for a subspace Y of X, a separation of Y is a pair of disjoint nonempty sets A and B whose union is Y, neither of which contains a limit point of the other. CO4K4

(or)

8. b. State and prove Intermediate value theorem. CO4K3

9. a. Prove that the image of a compact space under a continuous map is compact. CO5K4

(or)

9. b. Prove that a Hausdorff space X is locally compact if and only if for a given x in X, and given a neighborhood U of x, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$. CO5K3

PART-C

Answer ALL questions **3x12=36**

10. a. Prove that the topologies on R^n induced by the Euclidean metric d and the square metric ρ are the same as the product topology on R^n. CO3K3

(or)

10. b. State and prove Uniform limit theorem. CO3K4

11. a. (i) Prove that the union of a collection of connected subspaces of X that have a point in common is connected CO4K4

(ii) State and prove Intermediate value theorem.

(or)

11. b. If L is a linear continuum in the order topology, then prove that L is connected, and so are intervals and rays in L. CO4K4

12. a. State and prove tube lemma. CO5K3

(or)

12. b. Prove that if X is a nonempty compact Hausdorff space and if X has no isolated points, then X is countable. CO5K4

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