t Closed Sets in Intuitionistic Fuzzy Topological Spaces

By

Yuvashri B

(20PMA021)

Supervisor

Dr. S. M. Sudha

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore-641 043

In Partial Fulfilment of the Requirement for the Degree of

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N. Balamani 18-05-2022

Signature of the Head of the Department

18/5/22

Signature of the Supervisor

DECLARATION

I do hereby declare that the thesis entitled **" t Closed Sets in Intuitionistic Fuzzy Topological Spaces"** submitted to the Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore for the award of the degree of **MASTER OF SCIENCE** in Mathematics is a record of original work done by me during the period from December 2021 to May 2022 under the guidance and supervision of **Dr. S. M. SUDHA**, Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and it has not previously formed the basis for the award of any degree, diploma, fellowship or associateship or any other similar title of any candidate of any university.

B. Yuvashri.

Signature of the Candidate

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CONTENT

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CHAPTER 1

INTRODUCTION

TOPOLOGY:

Topology is a part of mathematics that studies the properties that are preserved under continuous deformation of a space. In 1817, Bolzano associated the concept of convergence with subsets of real numbers which are bounded, infinite and removed its association with a sequence of numbers. Through this idea of convergence as a generalization, the concept of topology was introduced.

Topology developed as a field of study out of geometry and set theory, through analysis of such concepts as space, dimension, and transformation. By the middle of the 20th century, topology had become an important area of study within Mathematics. Topology has many subfields like point-set topology, algebraic topology and geometric topology. The sets in the topology τ for a set X are defined as open. A set is defined as closed if its complement with respect to X is open. There are other equivalent ways to define a topology for a set besides open sets and closed sets.

The main aim of general topology has been to investigate and compare different classes of topological spaces. This goal continues interesting problems and results, which provide their importance from their relevance with respect to this primary goal and from the requirement of applications.

Modern topology is based on the study of set theory investigated by Cantor in 19th century in Euclidean space. Cantor studied point sets as a part of his study of Fourier series. He introduced the concept of derived sets, i.e., the set of limit points in 1872. Riesz suggested a new axiomatic approach to topology. His approach was on the notion of limit points. Hausdorff in 1914 was first topologist who defined the term 'Topological Space' and studied many concepts including the concept of Hausdroff Space.

One of the applications of topology was the seven bridges problem solved by Euler of Konigsberg. The problem was that how to find a route on which one can cross all seven bridges exactly once. The result was not based on their distances from one another nor on the lengths of the bridges but only on the connectively of the bridges.

In pure mathematics, general topology was first branch to which fuzzy sets have been used. It can be seen from literature that Chang C. L. [16] applied the first grafting notion of a fuzzy set on to general topology. He defined the concept that is called as 'Chang Fuzzy Space' and investigated useful topological notions for such spaces. In 1983, β open sets and β continuous mappings are introduced by Abd El Monsef et. al [1] in the topological space. Then, Andrijevic D [2,3] introduced semi pre open sets and b-open sets in topological space.

FUZZY TOPOLOGY:

Fuzzy concept was introduced by Zadeh L A [49] in 1965 to represent data and information possessing non-statistical uncertainties. It was specifically designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems. The publication in fuzzy subset theory by Zadeh L A [49] in 1965 and by Goguen J A [21] in 1967 show the intention of the authors to generalize the classical set. Starting from 1985, this theory was further generalized into many valued logics with the efforts of Lukasiwicz, Gottwald, Post, Godel and so on.

Fuzzy Set Theory is once a generalization as well as extension of Crisp Set Theory. Thus, the basic theme and ideas of crisp set theory will be reflected in fuzzy set theory also, though in an extended way. The idea of fuzzy set is welcomed because of handful uncertainties and vagueness which cantorian set could not address. A crisp set on a universal set U is defined by its characteristic function from U to {0,1}and a fuzzy set on a domain U is defined by its membership function from U to [0,1]. Fuzzy theory holds that many things in life are matters of degree. For example, the universe of discourse of the linguistic variable speed might have the range between 0 and 220 km/hr and may include such fuzzy subsets as very slow, slow, medium, fast and very fast.

In 1968, Chang C L [16] became the pioneer to present the idea of fuzzy topology. Fundamentally, he replaced the classical notion of open sets by a notion called fuzzy open sets. However, later in 1976, Lowen [31] found that many of the well-known results in general topology cannot be obtained if one follows Chang's definition of fuzzy topology. So, he redefined fuzzy topology by including constant fuzzy sets in it. This implies that in Chang's definition of fuzzy topology one more axiom was included.

Then, Chang C L [16] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like Warren R H [47], Azad K K [12], Balasubramanian G and Sundaram P [13, 14], Malghan S R and Benchalli S S [35], Mukherjee M N and Ghosh B [37], Anjan Mukherjee [5], Goguen J A [21] and many others have contributed to the development of fuzzy topological spaces.

Further, in fuzzy topological space, Singal M K and Niti Rajvanshi [42] introduced fuzzy α sets, Thakur S S and Singh S [46] have introduced fuzzy semi preopen and Hanafy I M [23] introduced fuzzy γ-open sets and fuzzy γ-continuity.

INTUITIONISTIC FUZZY TOPOLOGY:

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was first introduced by Atanassov K [6] in 1986. Further Atanassov K [7,8,9,11] introduced many concepts in intuitionistic fuzzy sets. Intuitionistic fuzzy set has been found to be more efficient in dealing with vagueness and ambiguity. It is characterized by a membership function ($\mu_p(x)$) and a non-membership function ($\nu_p(x)$) with their sum being less than or equal to one ($0 \leq \mu_p(x) + \nu_p(x) \leq 1$). This relaxes the enforced duality $\nu_p(x) = 1- \mu_p(x)$ from fuzzy set theory. Intuitionistic fuzzy set allows one to address the positive and negative side of an imprecise concept separately.

Intuitionistic fuzzy sets can be more precisely expressed. For example, the fact that the temperature of a patient changes, and other symptoms are not quite clear. There is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object. Intuitionistic fuzzy set is useful in decision making problems, particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services, etc. Recently various applications of intuitionistic fuzzy set like artificial intelligence, intuitionistic fuzzy expert systems, intuitionistic fuzzy neural networks, intuitionistic fuzzy decision making, intuitionistic fuzzy machine learning, intuitionistic fuzzy semantic representations etc., have appeared.

In 1995, Coker D [19] introduced the concept of intuitionistic fuzzy point and some notes on intuitionistic fuzzy set. Then, using the notion of intuitionistic fuzzy sets, Coker D [18] in 1997 has constructed the basic concepts of intuitionistic fuzzy topological spaces. After giving the fundamental definitions and the necessary examples he introduced the definitions of intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, intuitionistic fuzzy connectedness and obtained several preservation properties and some characterizations concerning intuitionistic fuzzy connectedness.

Lee S J and Lee E P [29] introduced some categories in intuitionistic fuzzy topological spaces in the year 2000. Later in 2001, Atanassov K [10] introduced four operators in intuitionistic fuzzy topological spaces. Then, Lupianez F G [32, 33, 34] and Kul Hur et.al [28] developed various concepts in intuitionistic fuzzy topological spaces. Bhattacharjee P and Bhaumil R N [15] introduced pre semi closed set and Jayanthi D [25] founded the relation between semi pre closed set and beta closed set in intuitionistic fuzzy topological spaces. Rajarajeshwari P and Senthil Kumar L [37], Santhi R and Jayanthi D [40] have introduced generalized pre closed sets and semi pre connectedness respectively in intuitionistic fuzzy topological space.

In this thesis, intuitionistic fuzzy t closed sets and intuitionistic fuzzy t open sets in intuitionistic fuzzy topological spaces are introduced. After giving the fundamental definitions, we have studied the relations between the new class of set with the other existing intuitionistic fuzzy closed sets and intuitionistic fuzzy open sets. Also, we have discussed some properties and acquired some theorems.

This thesis starts with the introduction and the review of literature. Chapter one includes some background materials which are useful for the present study. It throws the insight to the introduction of fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy closed sets and intuitionistic fuzzy open sets.

In Chapter 2, section 2.1 deals with the introduction of intuitionistic fuzzy t closed sets with suitable examples and the set is compared with other existing sets like intuitionistic fuzzy closed set, intuitionistic fuzzy regular closed set, intuitionistic fuzzy semi closed set, intuitionistic fuzzy pre closed set, intuitionistic fuzzy α closed set, intuitionistic fuzzy β closed set, intuitionistic fuzzy γ closed set and intuitionistic fuzzy semi pre closed set and the interrelation is shown with a diagram. The union and intersection of two intuitionistic fuzzy t closed sets where examined. The section 2.2 deals with the definition of intuitionistic fuzzy t open sets with suitable examples and the set is compared with other existing intuitionistic fuzzy open sets, and the interrelation is shown with a diagram. Union and intersection of two intuitionistic fuzzy t open sets and few theorems were derived.

REVIEW OF LITERATURE

A review of literature of recent developments on generalized notions of closed and open sets in topological spaces, fuzzy topological spaces and intuitionistic fuzzy topological spaces are discussed here.

In my thesis work a new type of closed set called t closed sets in intuitionistic fuzzy topological spaces is introduced. their basic properties, preservation propositions, interrelation are established with necessary counter examples.

Some of the research articles which I refer for the thesis are given below.

1. SEMI-OPEN SETS AND SEMI-CONTINUITY IN TOPOLOGICAL SPACES

[Levine, N, 1963]

In this article, the authors introduced a new class of sets called semi-open sets. He also introduced semi continuity and studied their properties.

2. ON SEMI-PRE-OPEN SETS

[Andrijevic, D., 1986]

This article is about the introduction of semi-preopen sets and the relationship between various topological sets is established and their fundamental properties were also checked.

3. A DECOMPOSITION OF CONTINUITY

[Hatir, E., Noiri, T. and Yuksel, S., 1996]

In this article, the authors introduced the notions of t sets and C-continuity and obtained another decomposition of continuity.

4. FUZZY SETS

[Zadeh, L. A., 1965]

In this study, the author introduces a new set called fuzzy sets, which are defined by a membership function that provides a grade of membership to each item ranging from zero to one. With regard to fuzzy sets, the author has also offered the ideas of inclusion, union, intersection, complement, and so on.

5. FUZZY PRE-OPEN SETS AND FUZZY PRE-SEPARATION AXIOMS

[Singal, M. K. and Niti Prakash., 1991]

In this article, the concept of preopen sets has been generalized to the fuzzy setting. The authors also introduced fuzzy separation axioms and investigated it with the help of fuzzy preopen sets.

6. FUZZY TOPOLOGICAL SPACES

[Chang, C. L., 1968]

The author discussed fuzzy topological space in this paper. The generalisation of generic topological spaces is the idea to be understood. Fuzzy open sets, fuzzy closed sets, fuzzy neighbourhoods, and fuzzy continuity are some of the core ideas presented.

7. A NOTE ON FUZZY SEMI OPEN SETS IN FUZZY TOPOLOGICAL SPACES

[Ganguly, S. and Saha, S., 1986]

In this article, the authors introduced the concepts of semi-T_i ($i = 0, 1, 2$) spaces and semi-R_i ($i = 0, 1$) spaces. They investigated some fuzzy topological properties under the newly introduced concepts. They also introduced and investigated some mappings involving semi-open sets on fuzzy topological spaces.

8. INTERIOR AND CLOSURE OF FUZZY OPEN SETS IN A FUZZY TOPOLOGICAL TM-SYSTEM

[Annalakshmi, M. and Chandramouleeswaran, M., 2015]

In this article, the author introduced the interior and closure of fuzzy open sets in a fuzzy topological TM-system. The author also introduced semi open and semi closed of fuzzy open sets in a fuzzy topological TM-system.

9. INTUITIONISTIC FUZZY SETS

[Atanassov, K., 1986]

In this article, the author defined intuitionistic sets and intuitionistic points and obtained their fundamental properties.

10. A NOTE ON INTUITIONISTIC SETS AND INTUITIONISTIC POINTS

[Coker, D., 1996]

In this article, the author introduces intuitionistic fuzzy topological spaces and the concepts of intuitionistic fuzzy interior and intuitionistic fuzzy closure are presented, followed by a discussion of several key features of these concepts. In addition, the concept of intuitionistic fuzzy continuity is presented.

11. AN INTRODUCTION TO INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Coker, D., 1997]

In this article, the author introduces intuitionistic fuzzy topological spaces and the concepts of intuitionistic fuzzy interior and intuitionistic fuzzy closure are presented, followed by a discussion of several key features of these concepts. In addition, the concept of intuitionistic fuzzy continuity is presented.

12. REGULAR GENERALIZED CLOSED SET IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

[Thakur, S. S. and Rekha Chaturvedi, 2006]

In this paper, the authors have discussed and studied the concept of intuitionistic fuzzy generalized continuous mappings in intuitionistic fuzzy topological spaces. They have analysed some of their properties and obtained some interesting theorems.

13. NOWHERE DENSE SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Thakur, S. S. and Dhavaseelan, R., 2015]

In this article, the authors introduced the concepts of intuitionistic fuzzy nowhere dense sets and their characterizations are studied.

14. ON INTUITIONISTIC FUZZY β GENERALIZED CLOSED SETS

[Saranya, M. and Jayanthi, D., 2016]

The concept of intuitionistic fuzzy β generalised closed sets is discussed in this article. The authors looked at some of their characteristics and came up with some intriguing theorems. Also highlighted is the link between this new class of sets and several previously existing sets.

15. ON INTUITIONISTIC FUZZY γ* GENERALIZED CLOSED SETS

[Riya, V. M. and Jayanthi, D., 2017]

In this article, the author introduced a new class of closed sets namely intuitionistic fuzzy γ* generalized closed set. The author discussed the inter relation between this set and other intuitionistic fuzzy sets.

16. β**GENERALIZED CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Sudha, S. M. and Jayanthi, D., 2020]

The author presented the concept of intuitionistic fuzzy β** generalised closed sets, looked at some of their characteristics, and came up with several characterization theorems in this article.

CHAPTER 1

PREMILINARIES

Definition 1.1: [6] An **intuitionistic fuzzy sets** (IFS) P is an object having the form

$$P= \{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$$

where the functions $\mu_P : X \to [0,1]$ and $\nu_P : X \to [0,1]$ denote the degree of membership and the degree of non-membership of each element x$\in X$ to the set P, respectively, $0 \leq \mu_P(x) + \nu_P(x) \leq 1$ for each x $\in X$. Denoted by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set P in X is simply denoted by P = $\langle x, \mu_P(x), \nu_P(x)\rangle$ instead of denoting P = $\{\langle x, \mu_P, \nu_P\rangle : x \in X\}$.

Definition 1.2: [6] Let P and Q be two IFSs of the form

$$P= \{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$$

and

$$Q= \{\langle x, \mu_Q(x), \nu_Q(x)\rangle : x \in X\}.$$

Then,

i. P $\subseteq Q$ if and only if $\mu_P(x) \leq \mu_Q(x)$ *and* $\nu_P(x) \geq \nu_Q(x)$ for all $x \in X$,
ii. P = Q if and only if P$\subseteq$Q and P$\supseteq$Q,
iii. $P^c = \{\langle x, \nu_P(x), \mu_P(x)\rangle : x \in X\}$,
iv. P$\cup$Q = $\{\langle x, \mu_P(x) \vee \mu_Q(x), \nu_P(x) \wedge \nu_Q(x)\rangle : x \in X\}$
v. P$\cap$Q = $\{\langle x, \mu_P(x) \wedge \mu_Q(x), \nu_P(x) \vee \nu_Q(x)\rangle : x \in X\}$

The intuitionistic fuzzy sets $0_\sim = \langle x, 0,1\rangle$ *and* $1_\sim = \langle x, 1,0\rangle$ are respectively the empty set and the whole set of X.

Definition 1.3: [18] An **intuitionistic fuzzy topology** (IFT) on X is a family τ of IFSs in X satisfying the followings:

i. $0_\sim, 1_\sim \in \tau$,

ii. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

iii. $\cup G_i \in \tau$ for any family $\{G_i : i \in I\} \in \tau$.

In this case the pair (X, τ) is called the **intuitionistic fuzzy topological space** (IFTS) and any IFS in τ is known as an **intuitionistic fuzzy open set** (IFOS) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an **intuitionistic fuzzy closed set** (IFCS) in X.

Definition 1.4: [45] Two IFSs P and Q are said to be **q-coincident** (P_qQ) if and only if there exists an element x ∈ X such that $\mu_P(x) > \nu_Q(x)$ or $\nu_P(x) > \mu_Q(x)$.

Definition 1.5: [45] Two IFSs P and Q are said to be **not q-coincident** ($P_{\bar{q}}Q$) if and only if P ⊆ Q^c.

Definition 1.6: [19] An **intuitionistic fuzzy point** (IFP), written as $p_{(\alpha,\beta)}$ is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases}$$

An IFP $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_P$ and $\beta \geq \nu_P$.

Definition 1.7: [18] Let P be an IFS in an IFTS (X, τ). Then the **interior** and **closure** of a P are defined as

$$\text{int}(P) = \cup\{G \,/\, G \text{ is an IFOS in X and } G \subseteq P\}$$

$$\text{cl}(P) = \cap\{H \,/\, H \text{ is an IFOS in X and } P \subseteq H\}$$

Note that for any IFS P in (X, τ), we have $\text{cl}(P^c) = (\text{int}(P))^c$ and $\text{int}(P^c) = (\text{cl}(P))^c$.

Definition 1.8: [39] An IFS P in (X, τ) is an **IFQ-set** if int(cl(P)) = cl(int(P))

Definition 1.9: [44] An IFS P in (X, τ) is called an **intuitionistic fuzzy nowhere dense set** if there exists no IFOS U such that U ⊆ cl (P). That is int(cl(P)) = $0_\sim$.

Proposition 1.10: [44] Let P be an IFS in X. If P is an **intuitionistic fuzzy nowhere dense set** in X, then int(P) = $0_\sim$.

Definition 1.11: [27] An IFS P = $\langle x, \mu_P, \nu_P \rangle$ in an IFTS (X, τ) is said to be an

i. **intuitionistic fuzzy regular closed set** (IFRCS) if cl(int(P)) = P
ii. **intuitionistic fuzzy pre-closed set** (IFPCS) if cl(int(P)) ⊆ P
iii. **intuitionistic fuzzy semi-closed set** (IFSCS) if int(cl(P)) ⊆ P
iv. **intuitionistic fuzzy α-closed set** (IFαCS) if cl(int(cl(P))) ⊆ P

v. **intuitionistic fuzzy β-closed set** (IfβCS) if int(cl(int(P))) ⊆ P

Definition 1.12: [27] An IFS P = $\langle x, \mu_P, \nu_P \rangle$ in an IFTS (X, τ) is said to be an

i. **intuitionistic fuzzy regular open set** (IFROS) if int(cl(P)) = P
ii. **intuitionistic fuzzy pre-open set** (IFPOS) if P ⊆ int(cl(P))
iii. **intuitionistic fuzzy semi-open set** (IFSOS) if P ⊆ cl(int(P))
iv. **intuitionistic fuzzy α-open set** (IFαOS) if P ⊆ int(cl(int(P)))
v. **intuitionistic fuzzy β-open set** (IfβOS) if P ⊆ cl(int(cl(P)))

Definition 1.13: [24] An IFS P = $\langle x, \mu_P, \nu_P \rangle$ in an IFTS (X, τ) is said to be

i. **intuitionistic fuzzy γ-closed set** (IFγCS) if int(cl(P)) ∩ cl(int(P)) ⊆ P
ii. **intuitionistic fuzzy γ-open set** (IFγOS) if P ⊆ cl(int(P)) ∪ int(cl(P))

Definition 1.14: [48] An IFS P = $\langle x, \mu_P, \nu_P \rangle$ in an IFTS (X, τ) is said to be an

i. **intuitionistic fuzzy semi-pre closed set** (IFSPCS) if there exists an intuitionistic fuzzy pre-closed set Q such that int(Q) ⊆ P ⊆ Q.
ii. **intuitionistic fuzzy semi-pre-open set** (IFSPOS) if there exists an intuitionistic fuzzy pre-open set Q such that Q ⊆ P ⊆ cl(Q).

Result 1.15: In an intuitionistic fuzzy topological space (X, τ). We have the following:

i. Every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set in X. **[22]**
ii. Every intuitionistic fuzzy closed set is an intuitionistic fuzzy α-closed set in X. **[22]**
iii. Every intuitionistic fuzzy α-closed set is both an intuitionistic fuzzy semi-closed set and intuitionistic fuzzy pre-closed set in X. **[22]**
iv. Every intuitionistic fuzzy semi-closed set is an intuitionistic fuzzy semi-pre-closed set in X. **[48]**
v. Every intuitionistic fuzzy pre-closed set is an intuitionistic fuzzy semi-pre-closed set in X. **[48]**

Result 1.16: In an intuitionistic fuzzy topological space (X, τ). We have the following:

i. Every intuitionistic fuzzy regular open set is an intuitionistic fuzzy open set in X. **[22]**

ii. Every intuitionistic fuzzy open set is an intuitionistic fuzzy α-open set in X. **[22]**

iii. Every intuitionistic fuzzy α-open set is both an intuitionistic fuzzy semi-open set and intuitionistic fuzzy pre-open set in X. **[22]**

iv. Every intuitionistic fuzzy semi-open set is an intuitionistic fuzzy semi-pre-open set in X. **[48]**

v. Every intuitionistic fuzzy pre-open set is an intuitionistic fuzzy semi-pre-open set in X. **[48]**

Definition 1.17: [18] Let (X, τ) be an intuitionistic fuzzy topological space and P, Q be intuitionistic fuzzy sets in X. Then the following properties hold:

i. $int(P) \subseteq P$

ii. $P \subseteq cl(P)$

iii. $P \subseteq Q \Rightarrow int(P) \subseteq int(Q)$

iv. $P \subseteq Q \Rightarrow cl(P) \subseteq cl(Q)$

v. $int(int(P)) = int(P)$

vi. $cl(cl(P)) = cl(P)$

vii. $int(P \cap Q) = int(P) \cap int(Q)$

viii. $cl(P \cup Q) = cl(P) \cup cl(Q)$

ix. $int(1_\sim) = 1_\sim$

x. $cl(0_\sim) = 0_\sim$

Result 1.18: [4] Let (X, τ) be an intuitionistic fuzzy topological space and P, Q be intuitionistic fuzzy sets in X. Then the following properties hold:

i. $int(P \cup Q) \supseteq int(P) \cup int(Q)$

ii. $cl(P \cap Q) \subseteq cl(P) \cap cl(Q)$

CHAPTER 2

CHAPTER 2

2.1 t Closed Sets in Intuitionistic Fuzzy Topological Spaces

Here we have defined the introduction of intuitionistic fuzzy t closed sets with suitable examples and the set is compared with other existing sets like intuitionistic fuzzy closed set, intuitionistic fuzzy regular closed set, intuitionistic fuzzy semi closed set, intuitionistic fuzzy pre closed set, intuitionistic fuzzy α closed set, intuitionistic fuzzy β closed set, intuitionistic fuzzy γ closed set and intuitionistic fuzzy semi pre closed set and the interrelation is shown with a diagram. The union and intersection of two intuitionistic fuzzy t closed sets were examined and few theorems are derived.

Definition 2.1.1: An IFS P= $\langle x, \mu_P, \nu_P\rangle$ is an IFTS (X, τ) is said to be an **intuitionistic fuzzy t closed sets** (IFtCS) if

$$cl(int(P)) = cl(P)$$

whenever P ⊆ U and U is an IFOS in (X, τ).

Example 2.1.2: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b), (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.3_b), (0.4_a, 0.7_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set Q = $\langle x, (0.8_a, 0.3_b), (0.2_a, 0.7_b)\rangle$ in X, then Q is an intuitionistic fuzzy t closed set as,

$$cl(int(Q)) = cl(P_2)$$
$$= 1_\sim$$
$$cl(Q) = 1_\sim$$

Hence cl(int(Q)) = cl(Q) and Q is an intuitionistic fuzzy t closed set in X.

Proposition 2.1.3: Every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy t closed set in (X, τ) But the reverse is not true in general.

Proof: Let X be an intuitionistic fuzzy topological space and P be an intuitionistic fuzzy regular closed set in (X, τ). Let P ⊆ U is an intuitionistic fuzzy open set.

$$cl(int(P)) = P \qquad (1)$$

Since every intuitionistic fuzzy regular closed set is intuitionistic fuzzy closed set, we have

$$\text{cl(P)} = \text{P} \qquad (2)$$

Now, consider

$$\text{cl(int(P))} = \text{P} \qquad \text{[From (1)]}$$

$$= \text{cl(P)} \qquad \text{[From (2)]}$$

Hence, cl(int(P)) = cl(P)

And therefore, A is an intuitionistic fuzzy t closed set in (X, τ).

Example 2.1.4: Let X = {a,b}, τ = {$0_{\sim}$, P_1, P_2, $1_{\sim}$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b),\ (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.3_b),\ (0.4_a, 0.7_b)\rangle$, then (X, τ) is a fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.8_a, 0.3_b),\ (0.2_a, 0.7_b)\rangle$ in X, then Q is an intuitionistic fuzzy t closed set as,

$$\text{cl(int(Q))} = \text{cl}(P_2)$$

$$= 1_{\sim}$$

$$\text{cl(Q)} = 1_{\sim}$$

hence, cl(int(Q)) = cl(Q)

But Q is not an intuitionistic fuzzy regular closed set in X as,

$$\text{cl(int(Q))} = \text{cl}(P_2)$$

$$= 1_{\sim}$$

$$\neq Q$$

Remark 2.1.5: Every intuitionistic fuzzy closed set in X is independent of intuitionistic fuzzy t closed set in X. This can be clearly shown from the following examples.

Example 2.1.6: Let X = {a,b}, τ = {$0_{\sim}$, P_1, P_2, $1_{\sim}$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b), (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.3_b),\ (0.4_a, 0.7_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set Q = $\langle x, (0.8_a, 0.3_b), \ (0.2_a, 0.7_b)\rangle$ in X, then Q is an intuitionistic fuzzy t closed set as,

$$\mathrm{cl(int(Q))} = \mathrm{cl}(P_2) = 1_\sim$$

$$\mathrm{cl(Q)} = 1_\sim$$

hence, cl(int(Q)) = cl(Q)

But Q is not intuitionistic fuzzy closed set in X as,

$$\mathrm{cl(Q)} = 1_\sim \neq Q$$

Example 2.1.7: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b), \ (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set Q = $\langle x, (0.4_a, 0.4_b), \ (0.6_a, 0.6_b)\rangle$ in X,

Here Q is intuitionistic fuzzy closed set in X as,

$$\mathrm{cl(Q)} = P_2^c = Q$$

But Q is not intuitionistic fuzzy t closed set in X as,

$$\mathrm{cl(int(Q))} = \mathrm{cl}(0_\sim) = 0_\sim$$

$$\mathrm{cl(Q)} = P_2^c$$

hence, cl(int(Q)) $\neq$ cl(Q)

Remark 2.1.8: Every intuitionistic fuzzy semi-closed set in X is independent of intuitionistic fuzzy t closed set in X. This can be clearly shown from the following examples.

Example 2.1.9: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b), \ (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.3_b), \ (0.4_a, 0.7_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set Q = $\langle x, (0.8_a, 0.3_b), (0.2_a, 0.7_b)\rangle$ in X, then Q is an intuitionistic fuzzy t closed set as,

$$cl(int(Q)) = cl(P_2)$$
$$= 1_\sim$$
$$cl(Q) = 1_\sim$$

hence, cl(int(Q)) = cl(Q)

But Q is not intuitionistic fuzzy semi-closed set in X as,

$$int(cl(Q)) = int(1_\sim)$$

$$= 1_\sim$$

$$\nsubseteq Q$$

Example 2.1.10: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set Q = $\langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$ in X,

Here Q is intuitionistic fuzzy closed set in X as,

$$int(cl(Q)) = int(P_2^c)$$

$$= 0_\sim$$

$$\subseteq Q$$

But Q is not intuitionistic fuzzy t closed set in X as,

$$cl(int(Q)) = cl(0_\sim)$$

$$= 0_\sim$$

$$cl(Q) = P_2^c$$

hence, cl(int(Q)) $\neq$ cl(Q)

Remark 2.1.11: Every intuitionistic fuzzy pre-closed set in X is independent of intuitionistic fuzzy t closed set in X. This can be clearly shown from the following examples.

Example 2.1.12: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b),\ (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.3_b),\ (0.4_a, 0.7_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.8_a, 0.3_b),\ (0.2_a, 0.7_b)\rangle$ in X, then Q is an intuitionistic fuzzy t closed set as,

$$cl(int(Q)) = cl(P_2)$$

$$= 1_\sim$$

$$cl(Q) = 1_\sim$$

hence, cl(int(Q)) = cl(Q)

But Q is not intuitionistic fuzzy pre-closed set in X as,

$$cl(int(Q)) = cl(P_2)$$

$$= 1_\sim$$

$$\nsubseteq Q$$

Example 2.1.13: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$, then (X, τ) is a fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ in X,

Here Q is intuitionistic fuzzy closed set in X as,

$$cl(int(Q)) = cl(0_\sim)$$

$$= 0_\sim$$

$$\subseteq Q$$

But Q is not intuitionistic fuzzy t closed set in X as,

$$cl(int(Q)) = cl(0_\sim)$$

$= 0_\sim$

$cl(Q) = P_2^c$

hence, $cl(int(Q)) \neq cl(Q)$

Remark 2.1.14: Every intuitionistic fuzzy α-closed set in X is independent of intuitionistic fuzzy t closed set in X. This can be clearly shown from the following examples.

Example 2.1.15: Let X = {a,b}, $\tau = \{0_\sim, P_1, P_2, 1_\sim\}$ be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b),\ (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.3_b),\ (0.4_a, 0.7_b)\rangle$, then (X, τ) is a fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.8_a, 0.3_b),\ (0.2_a, 0.7_b)\rangle$ in X, then Q is an intuitionistic fuzzy t closed set as,

$$cl(int(Q)) = cl(P_2)$$

$$= 1_\sim$$

$$cl(Q) = 1_\sim$$

hence, $cl(int(Q)) = cl(Q)$

But Q is not intuitionistic fuzzy α-closed set in X as,

$$cl(int(cl(Q))) = cl(int(1_\sim))$$

$$= 1_\sim \nsubseteq Q$$

Example 2.1.16: Let X = {a,b}, $\tau = \{0_\sim, P_1, P_2, 1_\sim\}$ be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ in X,

Here Q is intuitionistic fuzzy α-closed set in X as,

$$cl(int(cl(Q))) = cl(int(P_2^c))$$

$$= cl(0_\sim)$$

$$= 0_\sim$$

$$\subseteq Q$$

But Q is not intuitionistic fuzzy t closed set in X as,

$$\mathrm{cl}(\mathrm{int}(Q)) = \mathrm{cl}(0_\sim)$$

$$= 0_\sim$$

$$\mathrm{cl}(Q) = P_2^c$$

hence, $\mathrm{cl}(\mathrm{int}(Q)) \neq \mathrm{cl}(Q)$

Remark 2.1.17: Every intuitionistic fuzzy β-closed set in X is independent of intuitionistic fuzzy t closed set in X. This can be clearly shown from the following examples.

Example 2.1.18: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b),\ (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.3_b),\ (0.4_a, 0.7_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.8_a, 0.3_b),\ (0.2_a, 0.7_b)\rangle$ in X, then Q is an intuitionistic fuzzy t closed set as,

$$\mathrm{cl}(\mathrm{int}(Q)) = \mathrm{cl}(P_2)$$

$$= 1_\sim$$

$$\mathrm{cl}(Q) = 1_\sim$$

hence, $\mathrm{cl}(\mathrm{int}(Q)) = \mathrm{cl}(Q)$

But Q is not intuitionistic fuzzy β-closed set in X as,

$$\mathrm{int}(\mathrm{cl}(\mathrm{int}(Q))) = \mathrm{int}(\mathrm{cl}(P_2))$$

$$= \mathrm{int}(1_\sim)$$

$$= 1_\sim$$

$$\nsubseteq Q$$

Example 2.1.19: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ in X,

Here Q is intuitionistic fuzzy β-closed set in X as,

$$\text{int}(\text{cl}(\text{int}(Q))) = \text{int}(\text{cl}(0_\sim))$$
$$= \text{int}(0_\sim)$$
$$= 0_\sim$$
$$\subseteq Q$$

But Q is not intuitionistic fuzzy t closed set in X as,

$$\text{cl}(\text{int}(Q)) = \text{cl}(0_\sim)$$
$$= 0_\sim$$
$$\text{cl}(Q) = P_2^c$$

hence, cl(int(Q)) $\neq$ cl(Q)

Remark 2.1.20: Every intuitionistic fuzzy γ-closed set in X is independent of intuitionistic fuzzy t closed set in X. This can be clearly shown from the following examples.

Example 2.1.21: Let X = {a,b}, $\tau = \{0_\sim, P_1, P_2, 1_\sim\}$ be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b), (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.3_b), (0.4_a, 0.7_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.8_a, 0.3_b), (0.2_a, 0.7_b)\rangle$ in X, then Q is an intuitionistic fuzzy t closed set as,

$$\text{cl}(\text{int}(Q)) = \text{cl}(P_2)$$
$$= 1_\sim$$
$$\text{cl}(Q) = 1_\sim$$

hence, cl(int(Q)) = cl(Q)

But Q is not intuitionistic fuzzy γ-closed set in X as,

$$\text{int}(\text{cl}(Q)) \cap \text{cl}(\text{int}(Q)) = \text{int}(1_\sim) \cap \text{cl}(P_2)$$
$$= 1_\sim \cap 1_\sim$$
$$= 1_\sim \nsubseteq Q$$

Example 2.1.22: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$ in X,

Here Q is intuitionistic fuzzy γ-closed set in X as,

$$\begin{aligned} int(cl(Q)) \cap cl(int(Q)) &= int(P_2^c) \cap cl(0_\sim) \\ &= 0_\sim \cap 0_\sim \\ &= 0_\sim \\ &\subseteq Q \end{aligned}$$

But Q is not intuitionistic fuzzy t closed set in X as,

$$\begin{aligned} cl(int(Q)) &= cl(0_\sim) \\ &= 0_\sim \\ cl(Q) &= P_2^c \end{aligned}$$

hence, $cl(int(Q)) \neq cl(Q)$

Remark 2.1.23: Every intuitionistic fuzzy semi-pre closed set in X is independent of intuitionistic fuzzy t closed set in X. This can be clearly shown from the following examples.

Example 2.1.24: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b), (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.3_b), (0.4_a, 0.7_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q_1 = \langle x, (0.8_a, 0.3_b), (0.2_a, 0.7_b)\rangle$ and $Q_2 = \langle x, (0.7_a, 0.3_b), (0.3_a, 0.7_b)\rangle$ in X, then Q_1 and Q_2 are intuitionistic fuzzy t closed sets as,

$$\begin{aligned} cl(int(Q_1)) &= cl(P_2) \\ &= 1_\sim \\ cl(Q_1) &= 1_\sim \end{aligned}$$

hence, $cl(int(Q_1)) = cl(Q_1)$

$$cl(int(Q_2)) = cl(P_2)$$
$$= 1_\sim$$
$$cl(Q_2) = 1_\sim$$

hence, $cl(int(Q_2)) = cl(Q_2)$

But Q_1 is not an intuitionistic fuzzy semi-pre closed set, since, Q_2 is not an intuitionistic fuzzy pre-closed set as,

$$cl(int(Q_2)) = cl(P_2)$$
$$= 1_\sim$$
$$\nsubseteq Q_2$$

Example 2.1.25: Let X = {a,b}, $\tau = \{0_\sim, P_1, P_2, 1_\sim\}$ be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q_1 = \langle x, (0.3_a, 0.3_b), (0.7_a, 0.7_b)\rangle$ and $Q_2 = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$ in X,

Here Q_1 is an intuitionistic fuzzy semi-pre closed set in X as,

$$cl(int(Q_2) = cl(0_\sim)$$
$$= 0_\sim$$
$$\subseteq Q_2$$

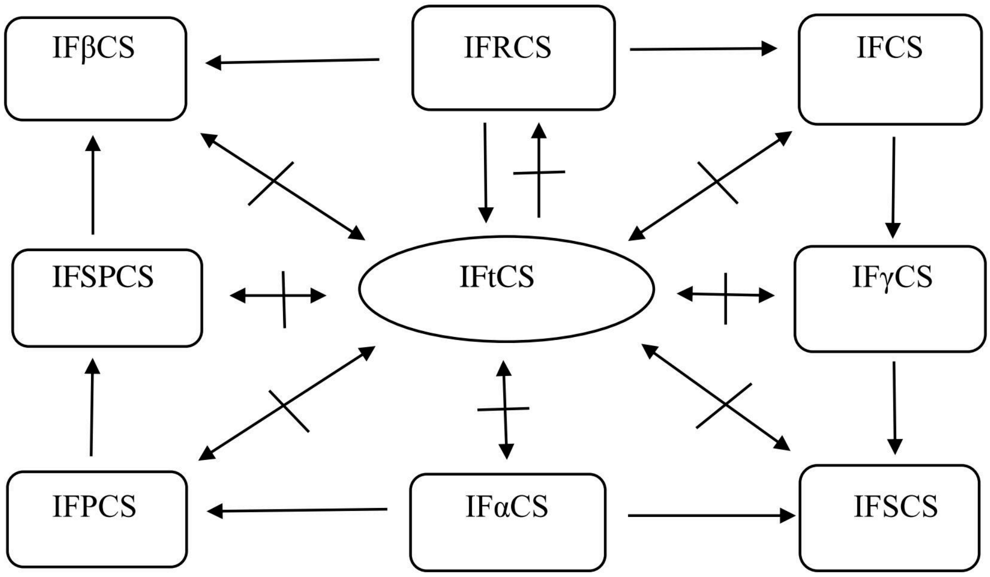
hence, Q_2 is an intuitionistic fuzzy pre-closed set such that $int(Q_2) \subseteq Q_1 \subseteq Q_2$

But Q_1 is not intuitionistic fuzzy t closed set in X as,

$$cl(int(Q_1)) = cl(0_\sim)$$
$$= 0_\sim$$
$$cl(Q_1) = P_2^c$$

hence, $cl(int(Q_1)) \neq cl(Q_1)$

In the following diagram we have provided relation between various types of intuitionistic fuzzy closedness.



Proposition 2.1.26: Let P and Q be any two t closed sets in (X, τ), then P ∪ Q is also an intuitionistic fuzzy t closed set in (X, τ).

Proof: Given P and Q be any two intuitionistic fuzzy t closed sets in (X, τ), then by definition,

$$cl(int(P)) = cl(P) \qquad (1)$$

and

$$cl(int(Q)) = cl(Q) \qquad (2)$$

consider, $$cl(int(P \cup Q) \subseteq cl(cl(P \cup Q))$$

$$\subseteq cl(P \cup Q) \qquad (3)$$

now,

$$cl(int(P \cup Q) \supseteq cl(int(P) \cup int(Q))$$

$$= cl(int(P)) \cup cl(int(Q))$$

$$= cl(P) \cup cl(Q) \quad \text{[from (1) \& (2)]}$$

$$= cl(P \cup Q)$$

we have,

$$cl(int(P \cup Q) \supseteq cl(P \cup Q) \qquad (4)$$

from (3) & (4), we have

$$cl(int(P \cup Q) = cl(P \cup Q)$$

Therefore, $P \cup Q$ is an intuitionistic fuzzy t closed set in (X, τ).

Proposition 2.1.27: Let P and Q be any two t closed sets in (X, τ), then $P \cap Q$ is also an intuitionistic fuzzy t closed set in (X, τ).

Proof: Given P and Q be any two intuitionistic fuzzy t closed sets in (X, τ), then by definition,

$$cl(int(P)) = cl(P) \qquad (1)$$

and

$$cl(int(Q)) = cl(Q) \qquad (2)$$

consider,

$$\begin{aligned} cl(int(P \cap Q)) &= cl(int(P) \cap int(Q)) \\ &\subseteq cl(int(P)) \cap cl(int(Q)) \\ &= cl(P) \cap cl(Q) \qquad \text{[from (1) and (2)]} \\ &\supseteq cl(P \cap Q) \end{aligned}$$

hence, $cl(int(P \cap Q)) = cl(P \cap Q)$

Therefore, $P \cap Q$ is an intuitionistic fuzzy t closed set in (X, τ).

Proposition 2.1.28: If P is both an intuitionistic fuzzy closed set and an intuitionistic fuzzy t closed set in (X, τ), then P is an intuitionistic fuzzy regular closed set in (X, τ).

Proof: Let P be an intuitionistic fuzzy closed in (X, τ). Then,

$$cl(P) = P \qquad (1)$$

Given P is an intuitionistic fuzzy t closed set in (X, τ) , then

$$\begin{aligned} cl(int(P)) &= cl(P) \\ &= P \qquad \text{[from(1)]} \end{aligned}$$

Therefore, $cl(int(P)) = P$ and hence P is an intuitionistic fuzzy regular closed in (X, τ).

Proposition 2.1.29: If an intuitionistic fuzzy closed set P is both an intuitionistic fuzzy pre-closed set and an intuitionistic fuzzy semi-open set in (X, τ), then P is an intuitionistic fuzzy t closed in (X, τ).

Proof: Let P be an intuitionistic fuzzy closed set in (X, τ). Then,

$$cl(P) = P \qquad (1)$$

Given P is an intuitionistic fuzzy pre-closed set in (X, τ). Then,

$$cl(int(P)) \subseteq P$$

$$= cl(P) \quad [from(1)]$$

Therefore, $$cl(int(P)) \subseteq cl(P) \qquad (2)$$

Also given P is an intuitionistic fuzzy semi-open in (X, τ). Then,

$$P \subseteq cl(int(P))$$

$$cl(P) \subseteq cl(int(P)) \quad [from\ (1)] \qquad (3)$$

from (2) and (3), cl(int(P)) = cl(P)

Therefore, P is an intuitionistic fuzzy t closed set in (X, τ).

Proposition 2.1.30: If P is both an intuitionistic fuzzy semi-closed set and an intuitionistic fuzzy t closed set in (X, τ), then P is an intuitionistic fuzzy β-closed set in (X, τ).

Proof: Let P be both an intuitionistic fuzzy semi-closed set and an intuitionistic fuzzy t closed set in (X, τ). Then,

$$int(cl(P)) \subseteq P \qquad (1)$$

and $$cl(int(P)) = cl(P) \qquad (2)$$

Now consider, $$int(cl(int(P))) = int(cl(P)) \quad [from\ (2)]$$

$$\subseteq P \quad [from\ (1)]$$

Therefore, int(cl(int(P))) ⊆ P and hence P is an intuitionistic fuzzy β-closed set in (X, τ).

Proposition 2.1.31: If P is both an intuitionistic fuzzy closed set and an intuitionistic fuzzy t closed set in (X, τ), then P is an intuitionistic fuzzy pre-closed set in (X, τ).

Proof: Let P be an intuitionistic fuzzy closed set in (X, τ). Then,

$$cl(P) = P \qquad (1)$$

Given P is an intuitionistic fuzzy t closed set in (X, τ). Then,

$$\begin{aligned} cl(int(P)) &= cl(P) \\ &\subseteq cl(P) \\ &= P \quad [from(1)] \end{aligned}$$

Therefore, cl(int(P)) ⊆ P and hence P is an intuitionistic fuzzy pre-closed set in (X, τ).

Proposition 2.1.32: If P is an intuitionistic fuzzy t closed set in (X, τ), then P is an intuitionistic γ-open set in (X, τ). The converse may not be true in general.

Proof: Let P be an intuitionistic fuzzy t closed set in (X, τ). Then,

$$cl(int(P)) = cl(P) \qquad (1)$$

Now consider,

$$\begin{aligned} int(cl(P)) \cup cl(int(P)) &= int(cl(P)) \cup cl(P) \quad [from(1)] \\ &= cl(P) \\ &\supseteq P \end{aligned}$$

Therefore, P ⊆ int(cl(P)) ∪ cl(int(P)) and hence P is an intuitionistic γ-open set in (X, τ).

Example 2.1.33: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.4_b), (0.2_a, 0.6_b)\rangle$, $P_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$, then (X, τ) is a fuzzy topological space.

Now, consider the intuitionistic fuzzy set $Q = \langle x, (0.4_a, 0.1_b), (0.6_a, 0.9_b)\rangle$ in X, then Q is an intuitionistic fuzzy γ-open set as,

$$\begin{aligned} int(cl(Q)) \cup cl(int(Q)) &= int(P_2^c) \cup cl(0_\sim) \\ &= P_2 \cup 0_\sim \\ &= P_2 \supseteq Q \end{aligned}$$

But Q is not intuitionistic fuzzy t closed set in X as,

$$\mathrm{cl}(\mathrm{int}(Q)) = \mathrm{cl}(0_\sim)$$

$$= 0_\sim$$

$$\mathrm{cl}(Q) = P_2^c$$

hence, $\mathrm{cl}(\mathrm{int}(Q)) \neq \mathrm{cl}(Q)$.

Proposition 2.1.34: If P is both an intuitionistic fuzzy Q-set and an intuitionistic fuzzy t closed set in (X, τ), then P is an intuitionistic fuzzy pre-open set in (X, τ).

Proof: Let P be an intuitionistic fuzzy Q-set. Then,

$$\mathrm{int}(\mathrm{cl}(P)) = \mathrm{cl}(\mathrm{int}(P)) \qquad (1)$$

Given P is an intuitionistic fuzzy t closed set in (X, τ). Then,

$$\mathrm{cl}(\mathrm{int}(P)) = \mathrm{cl}(P)$$

$$\mathrm{int}(\mathrm{cl}(P) = \mathrm{cl}(P) \qquad [\text{from}(1)]$$

$$\mathrm{int}(\mathrm{cl}(P)) \supseteq P$$

Therefore, $P \subseteq \mathrm{int}(\mathrm{cl}(P))$ and hence P is an intuitionistic fuzzy pre-open set in (X, τ).

Proposition 2.1.35: For an intuitionistic fuzzy closed set P in (X, τ), the following are equivalent:

i. $P \subseteq \mathrm{cl}(\mathrm{int}(P))$
ii. P is an intuitionistic fuzzy t closed set
iii. P is an intuitionistic fuzzy regular closed set

Proof: (i) $\Rightarrow$ (ii) Let $P \subseteq \mathrm{cl}(\mathrm{int}(P))$

$$\mathrm{cl}(P) \subseteq \mathrm{cl}(\mathrm{cl}(\mathrm{int}(P)))$$

$$\mathrm{cl}(P) \subseteq \mathrm{cl}(\mathrm{int}(P)) \qquad (1)$$

Now consider, $\mathrm{int}(P) \subseteq P$

$$\mathrm{cl}(\mathrm{int}(P)) \subseteq \mathrm{cl}(P) \qquad (2)$$

from (1) and (2), $\mathrm{cl}(\mathrm{int}(p)) = \mathrm{cl}(P)$

Hence, P is an intuitionistic fuzzy t closed set in (X, τ).

(ii) ⇒ (iii) Let P be an intuitionistic fuzzy t closed set in (X, τ). By Proposition 2.1.28, the proof is obvious. Hence, P is an intuitionistic fuzzy regular closed set in (X, τ).

(iii) ⇒ (i) Let P be an intuitionistic fuzzy regular closed set in (X, τ). Then,

$$P = cl(int(P))$$

$$P \subseteq cl(int(P))$$

Hence, (i) is proved.

Proposition 2.1.36: If P is an intuitionistic fuzzy clopen set in (X, τ), then the following conditions are equivalent:

i. P is an intuitionistic fuzzy t closed set
ii. P is an intuitionistic fuzzy Q-set

Proof: Let P be an intuitionistic fuzzy clopen set in (X, τ). Then,

$$cl(P) = P \text{ and } int(P) = P \quad (1)$$

(i) ⇒ (ii) Let P be an intuitionistic fuzzy t closed set in (X, τ). Then,

$$cl(int(P)) = cl(P) \quad (2)$$

$$cl(int(P)) = P \quad [\text{from}(1)] \quad (3)$$

$$int(cl(int(P))) = int(P) \; [\text{from}(1)]$$

$$int(cl(P)) = P \;\; [\text{from}(1) \text{ and } (2)] \quad (4\,)$$

From (2) and (3), cl(int(P)) = int(cl(P)).

Hence, P is an intuitionistic fuzzy Q-set in (X, τ).

(ii) ⇒ (i) Let P be an intuitionistic fuzzy Q-set in (X, τ). Then,

$$cl(int(P)) = int(cl(P))$$

$$= int(P) \; [\text{ from } (1)]$$

$$= P = cl(P) \; [\text{from } (1)]$$

Therefore, cl(int(P)) = cl(P)

Hence, P is an intuitionistic fuzzy t closed set in (X, τ).

Proposition 2.1.37: If P is both an intuitionistic fuzzy nowhere dense set and an intuitionistic fuzzy t closed set in (X, τ), then P is an intuitionistic fuzzy γ-closed set in (X, τ).

Proof: Let P be an intuitionistic fuzzy nowhere dense set in (X, τ). Then,

$$int(cl(P)) = 0_{\sim} \qquad (1)$$

Given P is an intuitionistic fuzzy t closed set in (X, τ). Then,

$$cl(int(P)) = cl(P) \qquad (2)$$

Now consider, $int(cl(P)) \cap cl(int(P)) = 0_{\sim} \cap cl(P)$ [from (1) and (2)]

$$= 0_{\sim}$$

$$\subseteq P$$

Therefore, $int(cl(P)) \cap cl(int(P)) \subseteq P$ and hence P is an intuitionistic fuzzy γ-closed set in (X, τ).

Proposition 2.1.38: If an intuitionistic fuzzy open set P is an intuitionistic fuzzy t closed set in (X, τ), then P is an intuitionistic fuzzy Q-set in (X, τ).

Proof: Let P is an intuitionistic fuzzy open set in (X, τ). Then,

$$int(P) = P \qquad (1)$$

Given P is an intuitionistic fuzzy t closed set in (X, τ). Then,

$$cl(int(P)) = cl(P)$$

$$int(cl(int(P))) = int(cl(P))$$

$$cl(int(P)) = int(cl(P)) \qquad \text{[from(1)]}$$

Therefore, P is an intuitionistic fuzzy Q-set (X, τ).

Proposition 2.1.39: If P is an intuitionistic fuzzy regular closed set and Q is an intuitionistic fuzzy t closed set, then $P \cap Q$ is an intuitionistic fuzzy regular closed set in (X, τ).

Proof: Let P be an intuitionistic fuzzy regular closed set and Q be an intuitionistic fuzzy t closed set. Then,

$$\mathrm{cl}(\mathrm{int}(P)) = P \qquad (1)$$

and
$$\mathrm{cl}(\mathrm{int}(Q)) = \mathrm{cl}(Q) \qquad (2)$$

Now consider, $P \cap Q \subseteq P \cap \mathrm{cl}(Q) = \mathrm{cl}(\mathrm{int}(P)) \cap \mathrm{cl}(\mathrm{int}(Q))$ [from (1) and (2)]

$$\supseteq \mathrm{cl}(\mathrm{int}(P) \cap \mathrm{int}(Q))$$

$$= \mathrm{cl}(\mathrm{int}(P \cap Q))$$

Therefore, $P \cap Q = \mathrm{cl}(\mathrm{int}(P \cap Q))$ and hence $P \cap Q$ is an intuitionistic fuzzy regular closed set in (X, τ).

Proposition 2.1.40: If P is an intuitionistic fuzzy regular closed set and Q is an intuitionistic fuzzy t closed set, then $P \cup Q$ is an intuitionistic fuzzy semi open set in (X, τ).

Proof: Let P be an intuitionistic fuzzy regular closed set and Q be an intuitionistic fuzzy t closed set. Then,

$$\mathrm{cl}(\mathrm{int}(P)) = P \qquad (1)$$

and
$$\mathrm{cl}(\mathrm{int}(Q)) = \mathrm{cl}(Q) \qquad (2)$$

Now consider, $P \cup Q \subseteq P \cup \mathrm{cl}(Q) = \mathrm{cl}(\mathrm{int}(P)) \cup \mathrm{cl}(\mathrm{int}(Q))$ [from (1) and (2)]

$$= \mathrm{cl}(\mathrm{int}(P) \cup \mathrm{int}(Q))$$

$$\subseteq \mathrm{cl}(\mathrm{int}(P \cup Q))$$

Therefore, $P \cup Q \subseteq \mathrm{cl}(\mathrm{int}(P \cup Q))$ and hence $P \cup Q$ is an intuitionistic fuzzy semi open set in (X, τ).

Proposition 2.1.41: If P is both an intuitionistic fuzzy semi-closed set and an intuitionistic fuzzy t closed set in (X, τ), then P is an intuitionistic fuzzy β-closed set in (X, τ).

Proof: Let P be an intuitionistic fuzzy semi-closed set. Then,

$$\mathrm{int}(\mathrm{cl}(P)) \subseteq P \qquad (1)$$

Given P is an intuitionistic fuzzy t closed set in (X, τ). Then,

$$cl(int(P)) = cl(P) \qquad (2)$$

Now consider, $int(cl(int(P))) = int(cl(P))$ [from(2)]

$$int(cl(int(P))) \subseteq P \qquad [from(1)]$$

Therefore, int(cl(int(P))) ⊆ P and hence P is an intuitionistic fuzzy β-closed set in (X, τ).

Proposition 2.1.42: If an intuitionistic fuzzy closed set P is both an intuitionistic fuzzy α-closed set and an intuitionistic fuzzy β-open set in (X, τ), then P is an intuitionistic fuzzy t closed in (X, τ).

Proof: Let P be an intuitionistic fuzzy closed set in (X, τ). Then,

$$cl(P) = P \qquad (1)$$

Given P is an intuitionistic fuzzy α-closed set in (X, τ). Then,

$$cl(int(cl(P))) \subseteq P$$

$$cl(int(P)) \subseteq P \qquad [from(1)]$$

$$cl(int(P)) = cl(P) \quad [from(1)]$$

Therefore, $cl(int(P)) \subseteq cl(P)$ (2)

Also given P is an intuitionistic fuzzy β-open in (X, τ). Then,

$$P \subseteq cl(int(cl(P)))$$

$$P \subseteq cl(int(P)) \qquad [from (1)]$$

$$cl(P) = cl(int(P)) \qquad [from (1)]$$

Therefore, $cl(P) \subseteq cl(int(P))$ (3)

From (2) and (3), cl(int(P)) = cl(P)

Therefore, P is an intuitionistic fuzzy t closed set in (X, τ).

Proposition 2.1.43: If P is both an intuitionistic fuzzy semi-closed set and an intuitionistic fuzzy t closed set in (X, τ), then P is an intuitionistic fuzzy γ-closed set in (X, τ).

Proof: Let P be both an intuitionistic fuzzy semi-closed set and an intuitionistic fuzzy t closed set in (X, τ). Then,

$$int(cl(P)) \subseteq P \qquad (1)$$

and

$$cl(int(P)) = cl(P) \qquad (2)$$

Now consider, $int(cl(P)) \cap cl(int(P)) \subseteq P \cap cl(int(P))$ [from(1)]

$= P \cap cl(P)$ [from(2)]

$= P$

Therefore, $int(cl(P)) \cap cl(int(P)) \subseteq P$ and hence P is an intuitionistic fuzzy γ-closed set in (X, τ).

2.2 t Open Sets in Intuitionistic Fuzzy Topological Spaces

Here we have defined the intuitionistic fuzzy t open sets with suitable examples and the set is compared with other existing sets like intuitionistic fuzzy open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy semi open set, intuitionistic fuzzy preopen set, intuitionistic fuzzy α open set, intuitionistic fuzzy β open set, intuitionistic fuzzy γ open set and intuitionistic fuzzy semi preopen set and the interrelation is shown with a diagram. The union and intersection of two intuitionistic fuzzy t open sets were examined and few theorems are derived.

Definition 2.2.1: An IFS P = $\langle x, \mu_P, \nu_P \rangle$ is an IFTS (X, τ) is said to be an **intuitionistic fuzzy t open sets** (IFyCS) if

$$\text{int}(\text{cl}(P)) = \text{int}(P)$$

whenever P ⊆ U and U is an IFOS in (X, τ).

Example 2.2.2: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.2_a, 0.6_b), (0.8_a, 0.4_b) \rangle$, $P_2 = \langle x, (0.4_a, 0.7_b), (0.6_a, 0.3_b) \rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set Q = $\langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b) \rangle$ in X is an intuitionistic fuzzy t open set.

Proposition2.2.3: Every intuitionistic fuzzy regular open set is an intuitionistic fuzzy t open set in (X, τ) But the reverse is not true in general.

Proof: Let X be an intuitionistic fuzzy topological space and P be an intuitionistic fuzzy regular open set in (X, τ). Let P ⊆ U is an intuitionistic fuzzy open set that is int(cl(P)) = P. Since every intuitionistic fuzzy regular open set is intuitionistic fuzzy open set, we have int(P) = P. Now, int(cl(P)) = P = int(P). Hence, int(cl(P)) = int(P), therefore A is an intuitionistic fuzzy t open set in (X, τ).

Example 2.2.4: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.2_a, 0.6_b), (0.8_a, 0.4_b) \rangle$, $P_2 = \langle x, (0.4_a, 0.7_b), (0.6_a, 0.3_b) \rangle$, then (X, τ) is a fuzzy topological space. Now, the intuitionistic fuzzy set Q = $\langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b) \rangle$ in X is an intuitionistic fuzzy t open set as, int(cl(Q)) = $0_\sim$ and int(Q) = $0_\sim$. But Q is not an intuitionistic fuzzy regular open set in X as, int(cl(Q)) = $0_\sim \neq$ Q.

Remark 2.2.5: Every intuitionistic fuzzy open set in X is independent of intuitionistic fuzzy t open set in X.

This can be clearly shown from the following examples.

Example 2.2.6: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where P_1 = $\langle x, (0.2_a, 0.6_b),\ (0.8_a, 0.4_b)\rangle$, P_2 = $\langle x, (0.4_a, 0.7_b),\ (0.6_a, 0.3_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set Q = $\langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ in X is an intuitionistic fuzzy t open set. But Q is not intuitionistic fuzzy open set in X as, int(Q) = $0_\sim \neq$ Q.

Example 2.2.7: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be intuitionistic fuzzy topology on X where P_1 = $\langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, P_2 = $\langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set Q = $\langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$ in X is an intuitionistic fuzzy open set in X, but Q is not intuitionistic fuzzy t open set in X as, int(cl(Q)) = $1_\sim$ and int(Q) = P_2. Hence, int(cl(Q)) $\neq$ int(Q).

Remark 2.2.8: Every intuitionistic fuzzy semi-open set in X is independent of intuitionistic fuzzy t open set in X.

This can be clearly shown from the following examples.

Example 2.2.9: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where P_1 = $\langle x, (0.2_a, 0.6_b),\ (0.8_a, 0.4_b)\rangle$, P_2 = $\langle x, (0.4_a, 0.7_b),\ (0.6_a, 0.3_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set Q = $\langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ in X is an intuitionistic fuzzy t open set as, int(cl(Q)) = $0_\sim$ and int(Q) = $0_\sim$. But Q is not intuitionistic fuzzy semi-open set in X as, cl(int(Q)) = $0_\sim \not\supseteq$ Q.

Example 2.2.10: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where P_1 = $\langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, P_2 = $\langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space.

Now, the intuitionistic fuzzy set Q = $\langle x, (0.7_a, 0.7_b),\ (0.3_a, 0.3_b)\rangle$ in X is an intuitionistic fuzzy semi-open set in X. But Q is not intuitionistic fuzzy t open set in X.

Remark 2.2.11: Every intuitionistic fuzzy pre-open set in X is independent of intuitionistic fuzzy t open set in X.

This can be clearly shown from the following examples.

Example 2.2.12: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} where $P_1 = \langle x, (0.2_a, 0.6_b), (0.8_a, 0.4_b)\rangle$, $P_2 = \langle x, (0.4_a, 0.7_b),\ (0.6_a, 0.3_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set $Q = \langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ in X is an intuitionistic fuzzy t open set, but Q is not intuitionistic fuzzy pre-open set in X.

Example 2.2.13: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set $Q = \langle x, (0.7_a, 0.7_b),\ (0.3_a, 0.3_b)\rangle$ in X is an intuitionistic fuzzy pre-open set in X, but Q is not intuitionistic fuzzy t open set in X.

Remark 2.2.14: Every intuitionistic fuzzy α-open set in X is independent of intuitionistic fuzzy t open set in X.

This can be clearly shown from the following examples.

Example 2.2.15: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.2_a, 0.6_b),\ (0.8_a, 0.4_b)\rangle$, $P_2 = \langle x, (0.4_a, 0.7_b),\ (0.6_a, 0.3_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set $Q = \langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ in X is an intuitionistic fuzzy t open set, but Q is not intuitionistic fuzzy α-open set in X.

Example 2.2.16: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set $Q = \langle x, (0.7_a, 0.7_b),\ (0.3_a, 0.3_b)\rangle$ in X, is an intuitionistic fuzzy α-open set in X, but Q is not intuitionistic fuzzy t open set in X.

Remark 2.2.17: Every intuitionistic fuzzy β-open set in X is independent of intuitionistic fuzzy t open set in X.

This can be clearly shown from the following examples.

Example 2.2.18: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.2_a, 0.6_b),\ (0.8_a, 0.4_b)\rangle$, $P_2 = \langle x, (0.4_a, 0.7_b),\ (0.6_a, 0.3_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set

Q = $\langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ in X is an intuitionistic fuzzy t open set, but Q is not intuitionistic fuzzy β-open set in X.

Example 2.2.19: Let X = {a,b}, τ = {0~, P_1, P_2, 1~} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set Q = $\langle x, (0.7_a, 0.7_b),\ (0.3_a, 0.3_b)\rangle$ in X is an intuitionistic fuzzy β-open set in X, but Q is not intuitionistic fuzzy t open set in X.

Remark 2.2.20: Every intuitionistic fuzzy γ-open set in X is independent of intuitionistic fuzzy t open set in X.

This can be clearly shown from the following examples.

Example 2.2.21: Let X = {a,b}, τ = {0~, P_1, P_2, 1~} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.2_a, 0.6_b),\ (0.8_a, 0.4_b)\rangle$, $P_2 = \langle x, (0.4_a, 0.7_b),\ (0.6_a, 0.3_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set Q = $\langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ in X is an intuitionistic fuzzy t open set, but Q is not intuitionistic fuzzy γ-open set in X.

Example 2.2.22: Let X = {a,b}, τ = {0~, P_1, P_2, 1~} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b),\ (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set Q = $\langle x, (0.7_a, 0.7_b),\ (0.3_a, 0.3_b)\rangle$ in X is an intuitionistic fuzzy γ-open set in X, but Q is not intuitionistic fuzzy t open set in X.

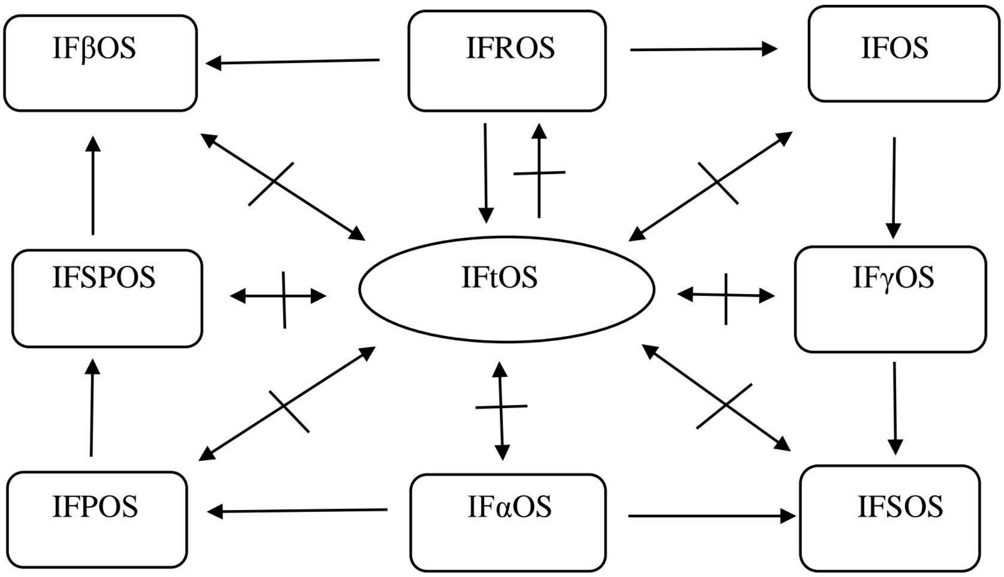
Remark 2.2.23: Every intuitionistic fuzzy semi-preopen set in X is independent of intuitionistic fuzzy t open set in X.

This can be clearly shown from the following examples.

Example 2.2.24: Let X = {a,b}, τ = {0~, P_1, P_2, 1~} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.2_a, 0.6_b),\ (0.8_a, 0.4_b)\rangle$, $P_2 = \langle x, (0.4_a, 0.7_b),\ (0.6_a, 0.3_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy sets $Q_1 = \langle x, (0.4_a, 0.4_b),\ (0.6_a, 0.6_b)\rangle$ and $Q_2 = \langle x, (0.4_a, 0.3_b),\ (0.6_a, 0.7_b)\rangle$ in X are intuitionistic fuzzy t open sets, but Q_1 is not an intuitionistic fuzzy semi-preopen set, since, Q_2 is not an intuitionistic fuzzy pre-open set.

Example 2.2.25: Let X = {a,b}, τ = {$0_\sim$, P_1, P_2, $1_\sim$} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$, then (X, τ) is an intuitionistic fuzzy topological space. Now, the intuitionistic fuzzy set $Q_1 = \langle x, (0.7_a, 0.7_b), (0.3_a, 0.3_b)\rangle$ is an intuitionistic fuzzy semi-preopen set in X. And $Q_2 = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$ in X is an intuitionistic fuzzy pre-open set such that $Q_2 \subseteq Q_1 \subseteq cl(Q_2)$. But Q_1 is not intuitionistic fuzzy t open set in X.

In the following diagram we have provided relation between various types of intuitionistic fuzzy openness.



Proposition 2.2.26: Let P and Q be any two t open sets in (X, τ), then P ∪ Q is also an intuitionistic fuzzy t open set in (X, τ).

Proof: Given P and Q be any two intuitionistic fuzzy t open sets in (X, τ), then by definition, int(cl(P)) = int(P) and int(cl(Q)) = int(Q). Consider, int(cl(P ∪ Q) = int(cl(P) ∪ cl(Q)) ⊇ int(cl(P)) ∪ int(cl(Q)) = int(P) ∪ int(Q) ⊆ int(P ∪ Q). Hence, int(cl(P ∪ Q) = int(P ∪ Q). Therefore, P ∪ Q is an intuitionistic fuzzy t open set in (X, τ).

Proposition 2.2.27: Let P and Q be any two t open sets in (X, τ), then P ∩ Q is also an intuitionistic fuzzy t open set in (X, τ).

Proof: Given P and Q be any two intuitionistic fuzzy t open sets in (X, τ), then by definition, int(cl(P)) = int(P) and int(cl(Q)) = int(Q). Consider, int(cl(P ∩ Q)) ⊆ int(cl(P) ∩ cl(Q)) = int(cl(P)) ∩ int(cl(Q)) = int(P) ∩ int(Q) = int(P ∩ Q). Hence, int(cl(P ∩ Q)) ⊆ int(P ∩ Q). Now, int(P ∩ Q) ⊆ cl(P ∩ Q) ⟹ int(int(P ∩ Q)) ⊆ int(cl(P ∩ Q)) ⟹ int(P ∩ Q) ⊆ int(cl(P ∩ Q)). Therefore, int(cl(P ∩ Q)) = int(P ∩ Q) which implies P ∩ Q is an intuitionistic fuzzy t open set in (X, τ).

Proposition 2.2.28: If P is both an intuitionistic fuzzy open set and an intuitionistic fuzzy t open set in (X, τ), then P is an intuitionistic fuzzy regular open set in (X, τ).

Proof: Let P be an intuitionistic fuzzy open in (X, τ). Then, int(P) = P. Given P is an intuitionistic fuzzy t open set in (X, τ), then int(cl(P)) = int(P) = P. Therefore, int(cl(P)) = P and hence P is an intuitionistic fuzzy regular open in (X, τ).

Proposition 2.2.29: If P is an intuitionistic fuzzy closed set in (X, τ), then P is an intuitionistic fuzzy t open set in (X, τ).

Proof: Let P be an intuitionistic fuzzy closed set in (X, τ). Then, cl(P) = P. Now consider, int(cl(P)) = int(P). Therefore, int(cl(P)) = int(P) and hence P is an intuitionistic fuzzy t open set in (X, τ).

Proposition 2.2.30: If an intuitionistic fuzzy open set P is both an intuitionistic fuzzy pre-open set and an intuitionistic fuzzy semi-closed set in (X, τ), then P is an intuitionistic fuzzy t open in (X, τ).

Proof: Let P be an intuitionistic fuzzy open set in (X, τ). Then, int(P) = P. Given P is an intuitionistic fuzzy pre-open set in (X, τ). Then, P ⊆ int(cl(P)) ⟹ int(P) ⊆ int(cl(P)). Also given P is an intuitionistic fuzzy semi-closed in (X, τ). Then, int(cl(P)) ⊆ P = int(P). Therefore, int(cl(P)) ⊆ int(P). Hence, int(cl(P)) = int(P) and P is an intuitionistic fuzzy t open set in (X, τ).

Proposition 2.2.31: If P is both an intuitionistic fuzzy semi-open set and an intuitionistic fuzzy t open set in (X, τ), then P is an intuitionistic fuzzy β-open set in (X, τ).

Proof: Let P be both an intuitionistic fuzzy semi-open set and an intuitionistic fuzzy t open set in (X, τ). Then, P ⊆ cl(int(P)) and int(cl(P)) = int(P). Now consider, (cl(int(cl(P))) = cl(int(P) ⊇ P. Therefore, P ⊆ (cl(int(cl(P))) and hence P is an intuitionistic fuzzy β-open set in (X, τ).

Proposition 2.2.32: If P is both an intuitionistic fuzzy open set and an intuitionistic fuzzy t open set in (X, τ), then P is an intuitionistic fuzzy pre-open set in (X, τ).

Proof: Let P be an intuitionistic fuzzy open set in (X, τ). Then, int(P) = P. Given P is an intuitionistic fuzzy t open set in (X, τ). Then, int(cl(P)) = int(P) ⊇ int(P) = P. Therefore, P ⊆ int(cl(P)) and hence P is an intuitionistic fuzzy pre-open set in (X, τ).

Proposition 2.2.33: If P is both an intuitionistic fuzzy open set and an intuitionistic fuzzy t open set in (X, τ), then P is an intuitionistic γ-closed set in (X, τ). The converse may not be true in general.

Proof: Let P be both an intuitionistic fuzzy open set and an intuitionistic fuzzy t open set in (X, τ). Then, int(cl(P)) = int(P) and int(P) = P. Now consider, int(cl(P)) ∩cl(int(P)) = int(P) ∩ cl(int(P)) = int(P) ∩ c(P) = int(P) ⊆ P. Therefore, int(cl(P)) ∩ cl(int(P)) ⊆ P and hence P is an intuitionistic γ-closed set in (X, τ).

Example 2.2.34: Let X = {a,b}, τ = {0~, P_1, P_2, 1~} be an intuitionistic fuzzy topology on X where $P_1 = \langle x, (0.8_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $P_2 = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$, then (X, τ) is a fuzzy topological space. Now, the intuitionistic fuzzy set Q = $\langle x, (0.7_a, 0.7_b),\ (0.3_a, 0.3_b)\rangle$ in X is an intuitionistic fuzzy γ-closed set as, int(cl(Q)) ∩ cl(int(Q)) = int(1~) ∩ cl(P_2) = 0~ ⊆ Q. But Q is not an intuitionistic fuzzy open set in X as, int(Q) = P_2 ≠ Q. And Q is not an intuitionistic fuzzy t open set in X as, int(cl(Q)) = 1~ and int(Q) = P_2. Hence, int(cl(Q)) ≠ int(Q).

Proposition 2.2.35: If P is both an intuitionistic fuzzy Q-set and an intuitionistic fuzzy t open set in (X, τ), then P is an intuitionistic fuzzy pre-closed set in (X, τ).

Proof: Let P be an intuitionistic fuzzy Q-set. Then, int(cl(P)) = cl(int(P)). Given P is an intuitionistic fuzzy t open set in (X, τ). Then, int(cl(P)) = int(P) ⟹ cl(int(P)) = int(P) ⟹ cl(int(P)) ⊆P. Therefore, cl(int(P)) ⊆ P and hence P is an intuitionistic fuzzy pre-closed set in (X, τ).

Proposition 2.2.36: For an intuitionistic fuzzy open set P in (X, τ), the following are equivalent:

i. int(cl(P)) ⊆ P.
ii. P is an intuitionistic fuzzy t open set.
iii. P is an intuitionistic fuzzy regular open set.

Proof: (i) ⇒ (ii) Let $int(cl(P)) \subseteq P \Longrightarrow int(int(cl(P))) \subseteq int(P) \Longrightarrow int(cl(P)) \subseteq int(P)$. Now consider, $P \subseteq cl(P) \Longrightarrow int(P) \subseteq int(cl(P))$. Therefore, $int(cl(P)) = int(P)$. Hence, P is an intuitionistic fuzzy t open set in (X, τ).

(ii) ⇒ (iii) Let P is both an intuitionistic fuzzy open set and an intuitionistic fuzzy t open set in (X, τ). By Proposition 2.2.28, the proof is obvious. Hence, P is an intuitionistic fuzzy regular open set in (X, τ).

(iii) ⇒ (i) Let P is an intuitionistic fuzzy regular open set in (X, τ). Then, $int(cl(P)) = P \Longrightarrow int(cl(P)) \subseteq P$. Hence, (i) is proved.

Proposition 2.2.37: If P is an intuitionistic fuzzy clopen set in (X, τ), the following conditions are equivalent:

i. P is an intuitionistic fuzzy t open set
ii. P is an intuitionistic fuzzy Q-set

Proof: Let P be an intuitionistic fuzzy clopen set in (X, τ). Then, $cl(P) = P$ and $int(P) = P$.

(i) ⇒ (ii) Let P be an intuitionistic fuzzy t open set in (X, τ). Then, $int(cl(P)) = int(P) \Longrightarrow int(cl(P)) = P$ and $cl(int((cl\ P))) = cl(P) \Longrightarrow cl(int(P)) = P$. Therefore, $cl(int(P)) = int(cl(P))$. Hence, P is an intuitionistic fuzzy Q-set in (X, τ).

(ii) ⇒ (i) Let P be an intuitionistic fuzzy Q-set in (X, τ). Then, $cl(int(P)) = int(cl(P)) = int(P) = P = cl(P)$. Therefore, $cl(int(P)) = cl(P)$. Hence, P is an intuitionistic fuzzy t closed set in (X, τ).

Proposition 2.2.38: If P is both an intuitionistic fuzzy nowhere dense set and an intuitionistic fuzzy t open set in (X, τ), then P is an intuitionistic fuzzy γ-closed set in (X, τ).

Proof: Let P be an intuitionistic fuzzy nowhere dense set in (X, τ). Then, $int(P) = 0_{\sim}$. Given P is an intuitionistic fuzzy t open set in (X, τ). Then, $int(cl(P)) = int(P)$. Now consider, $int(cl(P)) \cap cl(int(P)) = int(P) \cap cl(0_{\sim}) = int(P) \cap 0_{\sim} = 0_{\sim} \subseteq P$. Therefore, $int(cl(P)) \cap cl(int(P)) \subseteq P$ and hence P is an intuitionistic fuzzy γ-closed set in (X, τ).

Proposition 2.2.39: If an intuitionistic fuzzy closed set P is an intuitionistic fuzzy t open set in (X, τ), then P is an intuitionistic fuzzy Q-set in (X, τ).

Proof: Let P is an intuitionistic fuzzy open set in (X, τ). Then, cl(P) = P. Given P is an intuitionistic fuzzy t open set in (X, τ). Then, int(cl(P)) = int(P) $\Longrightarrow$ cl(int(cl(P))) = cl(int(P)) $\Longrightarrow$ int(cl(P)) = cl(int(P)). Therefore, P is an intuitionistic fuzzy Q-set (X, τ).

Proposition 2.2.40: If P is an intuitionistic fuzzy regular open set and Q is an intuitionistic fuzzy t open set, then P ∩ Q is an intuitionistic fuzzy semi-closed set in (X, τ).

Proof: Let P be an intuitionistic fuzzy regular open set and Q be an intuitionistic fuzzy t open set. Then, int(cl(P)) = P and int(cl(Q)) = int(Q). Now consider, P ∩ Q ⊇ P ∩ int(Q) = int(cl(P)) ∩ int(cl(Q)) = int(cl(P) ∩ cl(Q)) ⊇ int(cl(P ∩ Q)). Therefore, int(cl(P ∩ Q)) ⊆ P ∩ Q and hence P ∩ Q is an intuitionistic fuzzy semi-closed set in (X, τ).

Proposition 2.2.41: If P is an intuitionistic fuzzy regular open set and Q is an intuitionistic fuzzy t open set, then P ∪ Q is also an intuitionistic fuzzy regular open set in (X, τ).

Proof: Let P be an intuitionistic fuzzy regular open set and Q be an intuitionistic fuzzy t open set. Then, int(cl(P)) = P and int(cl(Q)) = int(Q). Now consider, P ∪ Q ⊇ P ∪ int(Q) = int(cl(P)) ∪ int(cl(Q)) ⊆ int(cl(P) ∪ cl(Q)) = int(cl(P ∪ Q)). Therefore, int(cl(P ∪ Q)) = P ∪ Q and hence P ∪ Q is also an intuitionistic fuzzy regular open set in (X, τ).

Proposition 2.2.42: If P is both an intuitionistic fuzzy pre-closed set and an intuitionistic fuzzy t open set in (X, τ), then P is an intuitionistic fuzzy α-closed set in (X, τ).

Proof: Let P be an intuitionistic fuzzy pre-closed set. Then, cl(int(P)) ⊆ P. Given P is an intuitionistic fuzzy t open set in (X, τ). Then, int(cl(P)) = int(P). Now consider, cl(int(cl(P))) = cl(int(P)) $\Longrightarrow$ cl(int(cl(P))) ⊆ P. Therefore, cl(int(cl(P))) ⊆ P and hence P is an intuitionistic fuzzy α-closed set in (X, τ).

Proposition 2.2.43: If P is both an intuitionistic fuzzy semi-open set and an intuitionistic fuzzy t open set in (X, τ), then P is an intuitionistic fuzzy γ-closed set in (X, τ).

Proof: Let P be both an intuitionistic fuzzy semi-open set and an intuitionistic fuzzy t open set in (X, τ). Then, P ⊆ cl(int(P)) and int(cl(P)) = int(P). Now consider, int(cl(P)) ∩ cl(int(P)) ⊆ int(cl(P)) ∩ P = int(P) ∩ P = int(P) ⊆ P. Therefore, int(cl(P)) ∩ cl(int(P)) ⊆ P and hence P is an intuitionistic fuzzy γ-closed set in (X, τ).

SUMMARY AND CONCLUSION

SUMMARY AND CONCLUSION

In the literature it is observed that there are large number of ways to generalize the closed sets in intuitionistic fuzzy topological space.

The concept of "Intuitionistic fuzzy sets" was first published by Atanassov K in the year 1986. Using the notion of intuitionistic fuzzy sets, Coker D introduced the idea of intuitionistic fuzzy topological spaces in the year 1997.

In this thesis, we have introduced intuitionistic fuzzy t closed sets (intuitionistic fuzzy t open sets) and we made an attempt to compare intuitionistic fuzzy t closed sets and intuitionistic fuzzy t open sets in intuitionistic fuzzy topological spaces with some other existing intuitionistic fuzzy closed (open) sets, intuitionistic fuzzy regular closed (open) sets, intuitionistic fuzzy semi closed (open) sets, intuitionistic fuzzy pre closed (open) sets, intuitionistic fuzzy α closed (open) sets, intuitionistic fuzzy β closed (open) sets, intuitionistic fuzzy γ closed (open) sets, intuitionistic fuzzy semi pre closed (open) sets. Also, the union and intersection of intuitionistic fuzzy t closed(open) sets are examined.

The future research directions based on this research work may be extended as follows:

The notion of intuitionistic fuzzy t closed sets can be studied for continuous, connectedness, separation axioms, homeomorphisms, compactness in intuitionistic fuzzy topological spaces. It can also be extended to bitopological spaces, supra topological spaces and nano topological spaces.

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