Design of Single Sampling Plans

Gayathri, K

(14PMA004)

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore-641 043

In Partial Fulfilment of the Requirements for the Degree of

Master of Science in Mathematics

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CONTENTS

INTRODUCTION

In this present scenario of globalization, liberalization and privatization the need for maintaining and improving quality standards is gaining momentum. Due to increasing competitions manufacturing products of good quality becomes a prime concern to market any product to domestic or international level. India too become quality conscious and it is necessary to keep continuous watch over the quality of the goods.

A product with a history of consistent good quality requires less inspection than one which has no history or history of erratic quality. Accordingly it is a good practice to include inspection depending on level of quality. Producer wants to ensure himself that the manufactured goods are according to the specification and do not contain a large number of defectives and the consumer is anxious not to be paying for substandard merchandise. This brings often the inspection procedure to be agreed to by both manufacturer and consumer so as not to unfairly penalize either. Such procedures are referred to as acceptance sampling.

Acceptance sampling plans pioneered by Dodge and Romig (1959) are widely used in industries to make a decision whether to accept or reject lot. With the introduction of modern quality management systems such as ISO 9000, acceptance sampling plans are used in goods inwards, in-process and final inspection stages of the production process. Two major areas of acceptance sampling plan are attribute sampling plan and variables sampling plan. The most popularly used sampling plan for both attribute and variable inspection is the single sampling plan.

Attribute sampling plans represent the most common statistical application used to test the effectiveness and the rate of compliance with established criteria. The result of these plans provide a statistical basis to conclude whether the system are functioning as intended, reflecting either control compliance or noncompliance. In lot-by-lot inspection by attributes, each item of the sample drawn from a lot of manufactured items is classified or nonconforming and the decision of acceptance / rejection of the lot depends upon the number of non-conforming item in the sample. Sampling inspection by attributes is appropriate if the decision shall be made based only on the quality of the lot that is defined by the number of non-conforming items in the lot.

Acceptance sampling by variable is often used if quality characteristic is measured on a continuous scale following a certain type of stable statistical distribution. The basic theoretical nature of acceptance sampling by variable often assumes the underling distribution of individual measurement to be normal. Sampling inspection by variables is based on the values of a quality characteristic on a continuous scale. The measures namely sample mean and sample standard deviation are used as basis in deriving variable sampling plans when the population standard deviation is known as well as unknown.

Attribute single sampling plan saves documentation time, because it needs to record the number inspected, the number of nonconforming units scene but do not need to record the actual measurements of each and every unit sampled and inspected. The procedure involved in the above scenario applies statistical principles to specify the requirements of how many units to be inspected and how acceptance or rejection decision shall be made. For selecting single sampling plan one has to choose its parameters relating to the standard quality level with reference to which the plan should operate and the degree of sharpness of inspection around that level.

These circumstances motivated the researcher to study designing methods of attribute and variable single sampling acceptance plans indexed with various quality indices and the analysis of their performance.

SYNOPSIS

This dissertation attempts to present a comprehensive idea on the development of attribute single sampling plan and variable single sampling plan.

The content of the dissertation is divided into three chapters.

Basic concepts present important terms, definitions and probability distributions pertaining to the designing of single sampling plan of attribute and variable type.

Chapter I includes the designing of attribute single sampling plan using two specified points on the OC curve.

The designing of variable single sampling plan for two specified points on the OC curve is given in chapter II.

In chapter III, the sensitivity of sample size, acceptance number and fixed ratio of sample size to the acceptance number on operating characteristic function is analysed for attribute and variable single sampling plans.

The plans having same AQL, LQL and IQL are also discussed in case attribute single sampling plan. The effect of change of parameters on the OC function of variable single sampling plan is evaluated.

The result and recommendations are given in Summary and Conclusion.

Referenced materials are listed at the end under the title Bibliography.

Review of Literature

In the field of statistical quality control acceptance sampling plans are classified as acceptance sampling plans by attributes and acceptance sampling plans by variables. A number of articles appeared on acceptance sampling plans. A brief review of pertinent literature on attribute and variable single sampling plans are presented.

Dodge and Romig (1929) designed single sampling attribute plans based on LTPD with minimum amount of inspection for making decision on submitted lot. They developed sampling plans for known value of LQL with probabilities 0.10 with minimum amount of total inspection. Dodge – Romig (1959) provided separate tables for the selection of attribute single sampling plans having lot tolerance percent defective protection and also average outgoing quality limit protection for the lots.

Peach and Littauer (1946) presented tables of percentage for designing of single sampling plan when the fraction defective is not greater than 0.10 using Poisson approximation. Grubbs (1949) presented designing sampling plan using beta approximation to binomial distribution. Cameron (1952) prepared tables for computing the operating characteristic of single sampling plans and for constructing single sampling plans for desired operating ratio.

Hamaker (1959) has explained the modification in designing of single sampling plan for finite lot sizes. MIL-STD-105D (1963) presents the sampling procedures and tables for inspection by attributes.

Guenther (1972) introduced variable sampling plan when the underlying distribution is normal. Guenther (1977) introduced variable sampling plan when the quality characteristic obeys an exponential distribution and gave procedure for finding a variable sampling plan which meets the given specification (p_1,1-α) and (p_2, β).

Hald (1981) presented optimal single sampling plans of desired strength and derived theorems related to the designing of single sampling plans with suitable examples to maintain quality of desired standards.

Chakraborty (1989) presented the procedure to determine single sampling attribute plans with given indifference quality level and the slope at that point.

Chakraborty (1990) gave solution methods for designing single sampling plans by assuming the incoming quality AQL and LQL as random variables.

Govindaraju (1991a), Balamurali and Kalyansundaram (1997) discussed a special type of double sampling plan which has the same OC curve as the single sampling plan with fractional acceptance number introduced by Hamaker (1950).

McWilliams et al. (2001) provided a method of finding exact single sample acceptance sampling plan. Ferrell and Chhoker (2002) presented a sequence of models that addressed 100% inspection and single sampling with and without inspector error when a Taguchi loss function is used to describe the cost associated with any deviation between the actual value of producers quality characteristics and its target value.

Festervand et al. (2001) presented the application of the attribute plans to field of the Marketing of industrial real estate with application of Taguchi Loss functions. Kratschmer (2005) gave a frame work for sampling inspection by attributes which are based on soft quality standards. Cavone (2009) presented a procedure based on closed form equations to design single sampling plans for isolated lots.

TzerHsu (2008) derived an economic model to determine the optimal single sampling plans that minimizes the total cost while satisfying both the producer's and consumer's quality and risk requirements.

Bowker and Goode (1952) presented the method of designing variable sampling plan. Resnikoff and Liebermann (1955) developed variable sampling plans for normal population indexed by producer's risk and presented matching plans relating to variance and range.

Owen (1966) introduced one – sided variable sampling plan indexed by AQL and LTPD for the normal population with unknown variance. Owen (1967) developed variable sampling plans with two specification limits. Owen (1969) enlightened on the summary of recent work on variable acceptance sampling plans based on normal distribution.

Hamaker (1979) presented the methods of adjusting variable and attributes plans having nearly identical OC curves for known sigma plans by using normal approximation. The relative efficiency of variable single sampling plans are discussed.

Morita (2007) presented rectifying sampling inspection by variables for assuring average outgoing surplus quality loss limit indexed by Taguchi's loss function. Kulfa (2010) presented calculation of the operating characteristic values using non-central t-distribution. Also the economic characteristics of the exact LTPD plans for the inspection by variables and attributes are derived. The impact of input parameters values on resulting sampling plan and its economic efficiency are discussed. Kasprikova (2012) presented LTPD and AOQL plans for acceptance sampling inspection by variable.

Campbell (1922) prepared probability curves showing Poisson's exponential summation. Catherine (1941) constructed table of percentage points of χ^2 distribution. Guenther (1969) derived tables of distributions of hypergeometric, binomial and Poisson to obtain sampling plans.

Vijayaraghavan et al.(2005) presented the discussion on choosing the prior distribution for lot fraction nonconforming. Vijayaraghavan et al. (2008) presented a design methodology and outlined a procedure for selection of Gamma-Poisson single sampling plans by attributes and constructed tables for application by attributes.

Rajagopal et al. (2009) presented selection of Bayesian single sampling plans. Loganathan et al. (2010) discussed designing of the single sampling plans by variables using predictive distribution. In this paper, using the normal approximation for the process distribution, single sampling plans by variables are derived assuming conjugate priors for the process parameters.

Dodge (1969) presented the evolution of acceptance sampling plans. Gupta and Kapoor (1976) provided basic ideas, methods and the uses of acceptance sampling in quality control. Duncan (1986) proposed quality control methods to suit industrial purpose. Montgomery (2004) outlined the basic theory and application of quality control techniques to analyses quality in manufacturing industries.

Schilling (1982) gave the review of the entire research work on variable sampling plan for proportion non – conforming relating to single and double specification limits for known and unknown sigma cases. Govindaraju (1990) presented and tables for the selection of variable single sampling plans indexed by AQL and AOQL.

Jacobson (1949) constructed the Nomograph for determination of variables inspection plans for fraction defectives. Nelson (1981) presented Nomograph for determining variable sampling plans for the desired degree of discrimination. Extensive tables of variables single sampling plan are given in military standard MIL-STD-414(1957). This table categorize five levels of inspection along with their OC curves.

Basic Concepts

Basic concepts section includes the terms and definitions, basic distributions and their approximations which are essential to prepare this dissertation.

Inspection

Inspection is the process of measuring, examining, testing or otherwise comparing the unit of product with the stated requirements.

100% Inspection

The inspection of every unit of product for the defects listed for an inspection station is 100% inspection.

Sampling inspection

The inspection of selected units for the units for the defects concerned from a given lot is sampling inspection.

Conforming unit

A unit, which meets the acceptance criteria established for the characteristic being considered, is a conforming unit.

Nonconforming unit

A unit which does not meet the acceptance criteria established for the characteristic being considered is a non-conforming unit.

Inspection by Attributes

Inspection based on certain characteristics of a unit or product is inspection by attributes. After inspection units are classified as conforming or non-conforming to the specified requirements.

Inspection by Variables

Inspection based on measurements of a unit or product is inspection by variables. After inspection measurements of the units of the sample are compared with specification limits.

Acceptance Sampling

A methodology that deals with procedures by which decisions to accept or not to accept are based on the results of the inspection of sample is acceptance sampling.

According to Dodge (1969), the major areas of acceptance sampling are

i. Lot-by-lot sampling by the method of attributes in which each unit in a sample is inspected on a go-non-go basis for one or more characteristics
ii. Lot-by-lot sampling by the method of variable, in which each unit in a sample is measured for a single characteristic, such as weight or strength.
iii. Continuous sampling of a flow of units in which each unit is inspected by the method of attributes.
iv. Special purpose plans including chain sampling, skip-lot sampling, small sampling plans, etc..

Acceptance Sampling Plan

A specified plan that states the sample size or sizes and the associated acceptance and non-acceptance criteria is known as acceptance sampling plan.

Quality Indices

Quality indices adopted for designing of single sampling plans in this dissertation are presented below

Acceptable Quality Level (AQL)

The maximum percentage or proportion of variant units in a lot or batch that for the purpose of acceptance sampling, can be considered satisfactory as a process average.

Limiting Quality Level (LQL)

The percentage or proportion of variant units in batch or lot for which, for the purpose of acceptance sampling plan consumer wishes the probability of acceptance to be restricted to a specific low value.

Indifference Quality Level (IQL)

The percentage or proportion of variant units in batch or lot for which, for the purpose of acceptance sampling plan consumer and producer wish the probability of acceptance to be 0.5 .

In this dissertation, the AQL and LQL are taken as an indices for designing sampling plans at which the probability of acceptance are respectively greater than or equal to (1-α) and less than or equal to β.

Risks involved in Acceptance Sampling

Producer's Risk(α)

For a given sampling plan, the probability of not accepting a lot of quality at which it has designated numerical value representing a level which it is generally desired to accept.

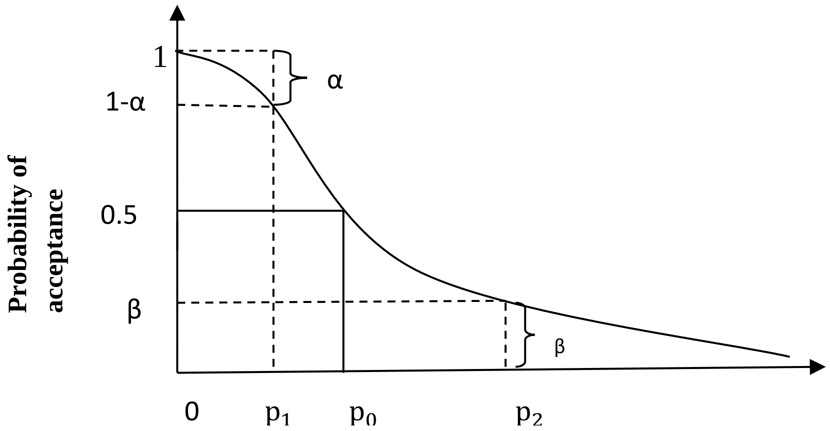
Consumer's Risk (β)

For a given sampling plan, the probability of acceptance of a lot of quality at which it has a designated numerical value representing a level which it is seldom desired to accept.

Performance Measures

i. Operating Characteristic (OC) Curve

Associated with each sampling plan is an Operating Characteristic curve which portrays the performance of the sampling plan against good and poor quality. Operating characteristic curve shows graphically the interrelationship of risk, associated probabilities and quality of a given sampling plan. The form of Operating Characteristic curve is presented in the following figure. The rectangle with dotted lines represents the ideal OC curve.



AQL RQL

Process average

ii. Average Sample Number (ASN)

The average number of units inspected per lot used for making decisions (acceptance or non-acceptance) is a function of the incoming lot quality p. A plot of ASN against p is called the ASN curve. For Single Sampling plan ASN = n.

iii. Average Total Inspection(ATI)

The expected number of items inspected per lot to arrive at a decision in an acceptance-rectification sampling inspection plan calling for 100% inspection of the rejected lot is called average of total inspection (ATI). It is a function of the quality p.

$$ATI = n\, P_a(p) + N\,(1 - P_a(p))$$

where $P_a(p)$ is probability acceptance of the lot of quality p on the basis of the sampling inspection.

Average Outgoing Quality(AOQ)

The expected quality of outgoing product following the use of acceptance sampling plan for a given value of incoming product quality. A plot of AOQ against p is called Average Outgoing Quality curve.

For rectifying inspection single sampling plan calling for 100% inspection of the rejected lots, the AOQ values given by the formula,

$$\text{AOQ} = p(N - n)P_a(p) / N$$

where N is lot size, n is sample size and P_a(p) is probability of acceptance of lot.

Probability Distribution

A probability distribution is a mathematical model that relates the value of the variable with the probability of occurrence of that value in the population. Several probability distribution arise frequently relating to real life situation in statistical quality control. Hypergeometric distribution, the binomial distribution, the Poisson distributon and Normal distribution are widely used.

Hypergeometric Distribution

A random variable, X is said to have hypergeometric distribution, if its probability mass function is

$$p(x) = \frac{C_x^{N_p} C_{n-x}^{N_q}}{C_n^N}$$

where N- lot size, N>0

p- proportion defective in the lot, p=0, 1/N, 2/n,…….,1

q- proportion effective in the lot , q=1-p

n- sample size , n=1, 2, 3…., N

mean – np

variance – npq (N-n) / (N-1)

A recursion formula to obtain successive values of the Hypergeometric probability is

$$p(x+1) = \frac{(n-x)(N_p - x)}{(x+1)(N_q + x - n + 1)} p(x)$$

The Hypergeometric distribution is fundamental to acceptance sampling. Hypergeometric model is exact for designing the probability of acceptance for isolated lots. It is applicable when sampling an attribute characteristic from a finite lot without replacement.

Binomial Distribution

A random variable is said to have the binomial distribution if the probability mass function is

$$p(x) = C_x^n p^x q^{n-x}, x = 0,1,2, \dots, n$$

where sample size $n, n > 0$

proportion defective p , $0 \leq p \leq 1$

proportion effective q , $q = 1$

mean = np

variance = npq

The successive probabilities can be calculated recursively using the formula

$$p(x+1) = \frac{(n-x)p}{(x+1)q} p(x)$$

This model is exact for the case of nonconforming units whenever $n/N \leq 0.10$,where n and N are respectively the sample and lot size.

Poisson Distribution

A random variable is said to have Poisson distribution , if the probability mass function is

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!} , x = 0,1,2,3, \dots\dots \infty$$

Mean number of defective , where $\lambda > 0$

Mean = λ

Variance = λ

The successive values of the Poisson probabilities can be calculated with the recursion formula

$$p(x+1) = \frac{\lambda}{x+1} p(x)$$

The Poisson distribution is used in calculating the characteristics of sampling plans which specify a given number of defects per unit such as the number of defects rivets in a aircraft wing or the number of stones allowed in a piece of glass of a given size. This model is exact for the case of non- conformities when $/N \leq 0.01$, n is large p is small such than np is finite.

Normal Distribution

A random variable is said to follows the normal distribution, if its probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)} \qquad -\infty < x < \infty$$

mean = μ

variance = σ^2

Chi-square Distribution

A random variable is said to have the chi-square distribution with degree of freedom n if its probability density function is given by

$$f(x) = \begin{cases} \frac{x^{(n/2)-1} e^{\frac{-x}{2}}}{2^{n/2}\Gamma(n/2)}, & x > 0 \\ 0, & othrewise \end{cases}$$

mean = n

variance = 2n

CHAPTER I

Design of Attribute Single Sampling Plan

This chapter presents the operating procedure of single sampling plan and the designing of single sampling plan when two specified points $(AQL, 1-\alpha), (LQL, \beta)$ on the operating characteristic curve are given. Selection of plans to the given specifications and method of construction of tables are also explained.

Probability of acceptance is computed using χ^2 distribution by utilising the fact that the approximation to the partial sums of the Poisson distribution can be computed in terms of chi-square distribution.

Operating procedure

Operating procedure of single sampling plan (N, n, c) has the following steps.

Step1: A random sample of size n is selected from the submitted lot of N items.

Step2: Each item in the sample is then classified as either defective (or) non-defective and d be the number of defectives.

Step3: Accept the lot if $d \leq c$.

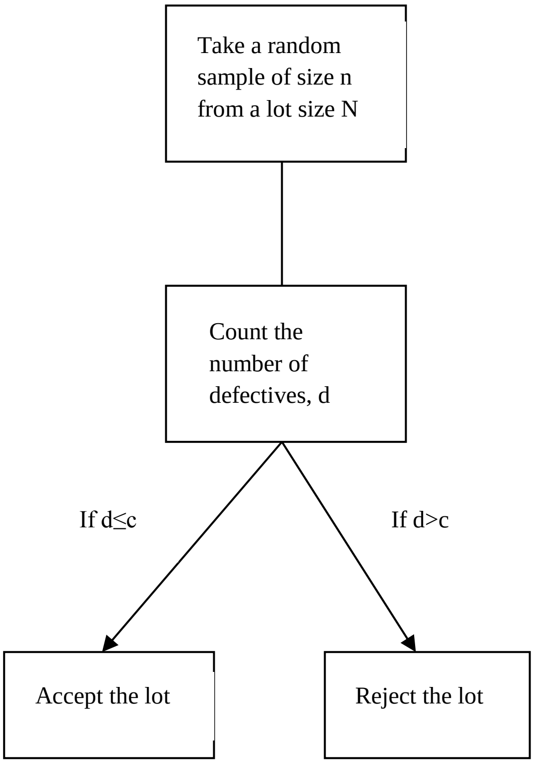
Step4: Reject the lot if $d > c$

where N is a lot size, n is sampling size and c is the acceptance number. This indicates that single sampling plan is identified effectively with only two parameters n and c.

Operating procedure is given in the following flow chart 1.1.

Designing method

Designing of a sampling plan enables one to find the parameters of sampling plan for the given specification related to quality indices and the corresponding risks of producer and consumer. Designing of a sampling plan means estimation of parameters n and c through specification of quality indices and their risks.



Flow chart 1.1

Operating procedure of Attribute single sampling plan(n,c)

The probability of acceptance based on Poisson distribution for a single sampling plan with parameters n and c is

$$P_a(p) = \sum_{r=o}^{c} e^{-np} \frac{(np)^r}{r!}\ ,\ \text{where p is the process average.}$$

The designing of the sampling plan for specified specifications depends on the following two requirement constraints.

$$P_a(p_1) = \sum_{r=o}^{c} e^{-np_1} \frac{(np_1)}{r!} \geq 1 - \alpha \qquad (1.1)$$

$$P_a(p_2) = \sum_{r=o}^{c} e^{-np_2} \frac{(np_2)}{r!} \leq \beta \qquad (1.2)$$

where $p_1$is Acceptance Quality Level, p_2 is Lot Quality Level, α is producer's risk and β is consumer's risk

Using the concept of approximation that the sum of Poisson probabilities can be computed in terms of chi-square distribution one derives

$$np_1 = \frac{1}{2}\chi^2{}_{1-\alpha,(2c+2)} \tag{1.3}$$

$$np_2 = \frac{1}{2}\chi^2{}_{\beta,(2c+2)} \tag{1.4}$$

For various values of $(p_1, 1-\alpha)$ and (p_2, β) values for np_1 and np_2 may be computed by using chi-square probability table.

Table 1.1 is constructed for np values corresponding to probability of acceptance p_a(= 0.995, 0.99, 0.95,............., 0.001) and acceptance number c(= 1, 2, 3,.........., 30) using equations (1.1),(1.2) and (1.3),(1.4).

For α (= 0.005, 0.05, 0.01, 0.10) and β (= 0.10, 0.001) and c (= 1,2,3,........30)
Table 1.2 is constructed for operating ratio p_2/p_1 by using Table 1.1.

Selection of plans

Method of selecting plans for the specifications of

i. Sample size and one point on the operating characteristic curve and
ii. Two points on the operating characteristic curve are given

Selection of plan for given n and a point on the OC curve

Table 1.1 can be used to derive attribute single sampling plan. For some fraction defective p it is desired that the probability acceptance be $P_a(\mathrm{p})$, the value of np, which is either less than or equal to np, is located in the column headed by the specified $P_a(\mathrm{p})$ and the corresponding c determines the sampling plan to be used.

Let us suppose that the manufacturer wants to have the product of lots to be accepted at least 90% of the time which has a quality of p = 0.05 and the sample size is assumed to be 50. The consumer wants to make decision of the submitted lot. Then for the stated requirements the consumer obtains the plan from the table as follows.

The sample size is set at $n = 50$ and it is desired to accept material with fraction defective $p = 0.05$,90 percent of the time the parameter c is to be determined by scanning the column headed for $P_a(\mathrm{p}) = 0.90$ from Table 1.1 the value of c just less than or equal to $np = 50(0.05) = 2.5$ is 4. Since corresponding to the nearest value is 2.43252 against the value $c = 4$. Therefore the required attribute single sampling plan is (50, 5).

Selection of plan for two specified points on the OC curve

To construct sampling plan for given $(p_1, 1-\alpha)$, (p_2, β) the ratio p_2/p_1 is to be calculated. The value which is greater than or equal to the desired ratio is located in the appropriate column of Table 1.2 corresponding to this ratio the values of np_2 and c can be selected. The sample size is determined by dividing np_2 by p_2 and acceptance number c is read off directly from the table.

Suppose that the sampling plan parameters (n, c) to be obtained which will accept material with fraction defective $p_1 = 0.02$, 90 percent of the time and accept material with fraction defective $p_2 = 0.036$ only 10 percent of the time. Compute p_2/p_1= 1.8 and refer Table 1.2 for α = 0.10 and β = 0.10 select the value of the ratio p_2/p_1 in the column for α = 0.10, β = 0.10 equal to or just greater than 1.8. The located value is 1.81078 which has associated with it a value of np_2 = 24.75629 and value of c =18. The sample size for the desired plan is taken as

$n = np_2/p_1$ =688 and the corresponding acceptance number is c =18.

Thus the required attributes single sampling plan (for the specified requirements) using this plan, the buyer may make decision on the submitted lots is (688,18).

Sensitivity of Sample Size n and acceptance number c on the OC curve

In order to analyses the sensitivity of sample size and acceptance number on sampling plans the operating characteristic values are used. The OC values corresponding to the plans (200,2),(200,4) and (100,2) are obtained from Table 1.1 and presented in Table 1.3.

Fig.1.1 shows the operating characteristic curves of the sampling plans (200,2), (200,3) and (100,2) by taking p on x-axis and $p_a(p)$ on the y-axis.

The curves indicate that the increase in acceptance number increases the probability of acceptance and increase in sample size decreases the probability of acceptance.

TABLE 1.3 OC values of Attribute Single Sampling plans

(200,2)		(200,4)		(100,2)	
p	$P_a(p)$	p	$P_a(p)$	p	$P_a(p)$
0.0016	0.995	0.0054	0.995	0.0067	0.995
0.0015	0.990	0.0064	0.990	0.0082	0.990
0.0041	0.950	0.0094	0.950	0.0137	0.950
0.0055	0.900	0.0123	0.900	0.0175	0.900
0.0134	0.500	0.0234	0.500	0.0340	0.500
0.0266	0.100	0.0399	0.100	0.0675	0.100
0.0314	0.050	0.0457	0.050	0.0780	0.050
0.0420	0.01	0.0580	0.010	0.1010	0.010
0.0464	0.005	0.0629	0.005	0.1080	0.005
0.0561	0.001	0.0739	0.001	0.1300	0.001

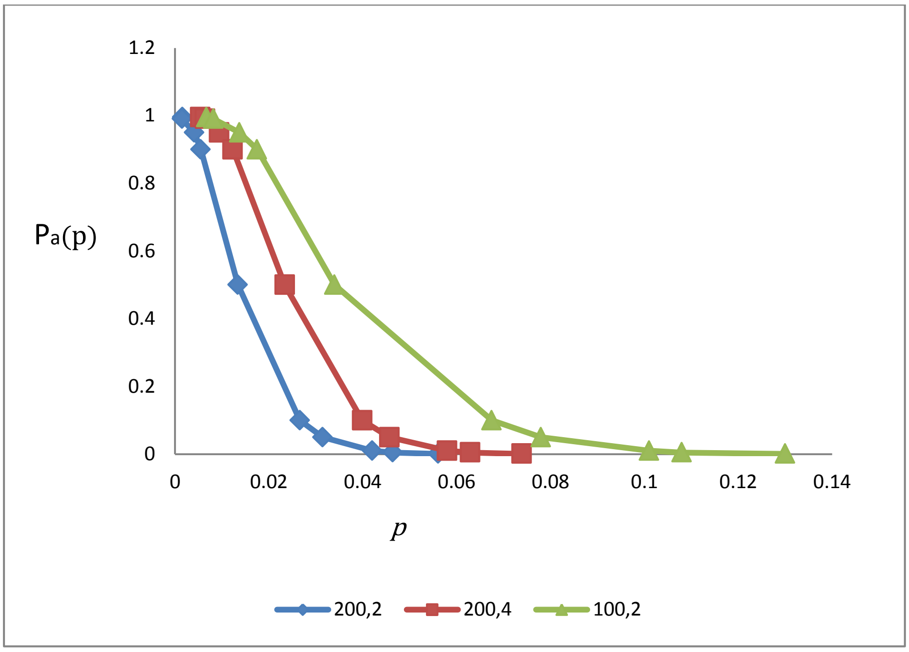


Fig. 1.1 OC values of Attribute Single Sampling plans (200, 2), (200,4) and (100,2).

Conclusion

In case of Attribute Single Sampling plan (n , c) one may see that

i. increases in c probabilities of acceptance increases
ii. increases in n probabilities of acceptance decreases

Construction of tables

By using expression (1.1) and (1.2) the values of np are computed for various values of α and β and the acceptance number c. The Table 1.1 gives np for probability acceptance corresponding to 0.995, 0.99, 0.95, 0.90, 0.05, 0.10, 0.05, 0.005, 0.001 for various acceptance number c = 0, 1, 2, 3, ..., 30.

Table 1.2 is constructed to derive sampling plans of desired point on operating characteristic curve. The two points through which the operating characteristic curve requires to pass are the fraction defective p_1 for which the probabilities of acceptance is greater than or equal to 1 –α and p_2 for which the probabilities of acceptance is less than or equal to β. Table 1.2 gives values of the operating ratio p_2 / p_1 and np_2 corresponding to various combination of α [0.005, 0.001, 0.05, 0.10] and β [0.001, 0.10] for various values of c.

CHAPTER II

Designing of Variable Single Sample Plan

This chapter presents the assumption, procedure and the designing of variable sampling plan for desired points on the OC curve when the standard deviation of the underling normal population is known as well as unknown. Table are prepared to enable the user in selecting plans for desired discrimination.

Assumptions

The underlying assumptions to derive variable sampling plan are

i. Only single quality characteristic measurable on continuous scale is taken into consideration
ii. the measurements of the items in the lots are distributed according to normal with mean μ and the standard deviation σ
iii. The purpose of inspection is to control the fraction defective in future production.
iv. Under these assumptions the fraction defective in a lot is dyined by

$$\mathrm{p} = \int_{\mathrm{k_p}}^{\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\mathrm{u}^2}\, \mathrm{du}$$

where K_p is a quantity to be determined from normal probability table.

Specification Limit

Consider a variable sampling plan to control the lot or process fraction nonconforming. Since the quality characteristic in a variable, the quality of product is specified by specification limits. Specification limits are of two types: A single specification limit implies only one boundary value for acceptability. A product is confirming if the measurement, $X \leq \mathrm{U}$ for an upper specification limit (USL), or $\mathrm{X} \geq \mathrm{L}$ for a lower specification limit (LSL). A double specification limit implies two boundary values. A product conforms to the specification if the measurement, X is such that $L \leq X \leq U$.

Operating Procedure of Variable Single Sampling Plan

The application of variable single sampling plan consists of the following steps

Step 1: Take a random sample of size n and obtain the measurement of the quality characteristic from the submitted lot,

Step 2: Compute $\bar{x}$, the sample mean and s , the sample standard deviation

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}, \; s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Step 3: Acceptance criteria

If upper specification limit U is specified, then accept the unit for $X \leq U$

$\bar{x} + k\sigma \leq U$, accept the lot when σ is known.

$\bar{x} + ks \leq U$, accept the lot when σ is unknown.

If lower specification limit L is specified, then accept the unit for $X \geq L$

$\bar{X} - k\sigma \geq L$, accept the lot when σ is known.

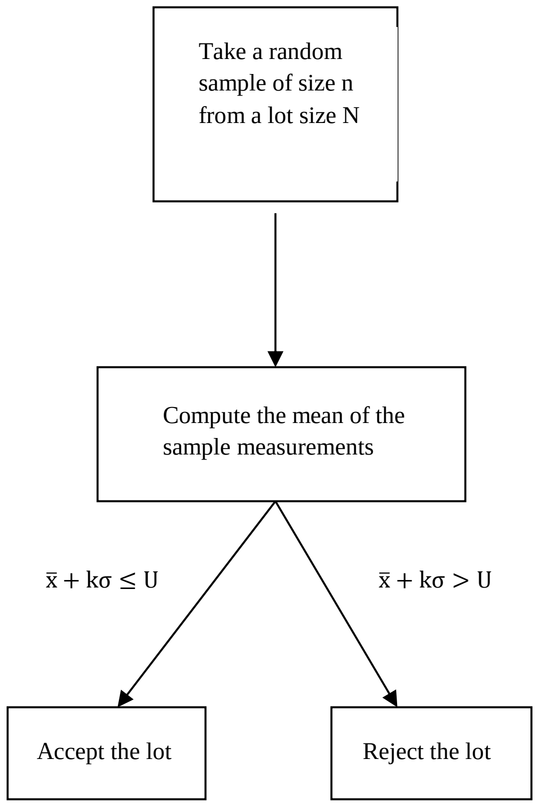
$\bar{x} - ks \geq L$, accept the lot when σ is unknown.

If double specification limits U and L are specified, then accept the unit for $L \leq X \leq U$

$L + k\sigma \leq \bar{x} \leq U - k\sigma$, accept the lot when σ is known.

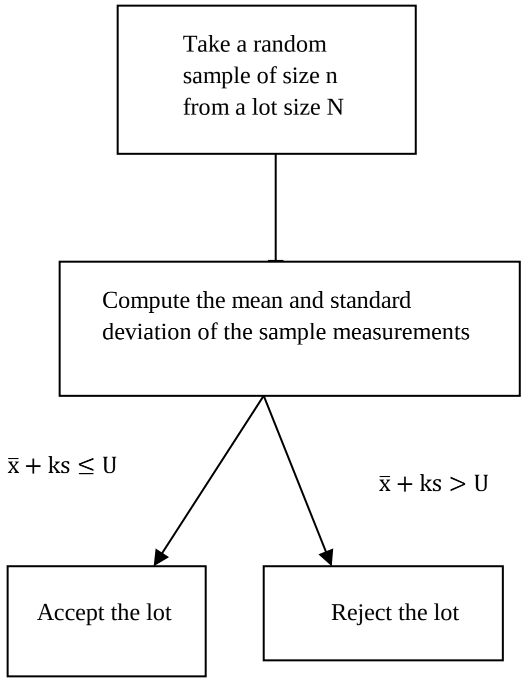
$L + ks \leq \bar{x} \leq U - ks$, accept the lot when σ is unknown.

Only the designing of plans with upper specification limit is provided in this dissertation. Operating procedures of variable single sampling plan with known sigma and unknown sigma are given in the following flow charts (2.1) and (2.2).



Flow chart 2.1

Operating Procedure of Variable single sampling plan (n,k) when σ is know



Flow chart 2.2

Operating procedure of Variable single sampling plan (n,k) when σ is unknown

OC Function of Known σ Variable Single Sampling Plan

It is assumed that the individual measurements of the submitted lot follow normal distribution with mean μ and standard deviation σ. Products which are defectives, have the quality characteristic$X > U$. The proportional area to the right of the value U of the normal curve is equal to p which is shown in following fig 2.1

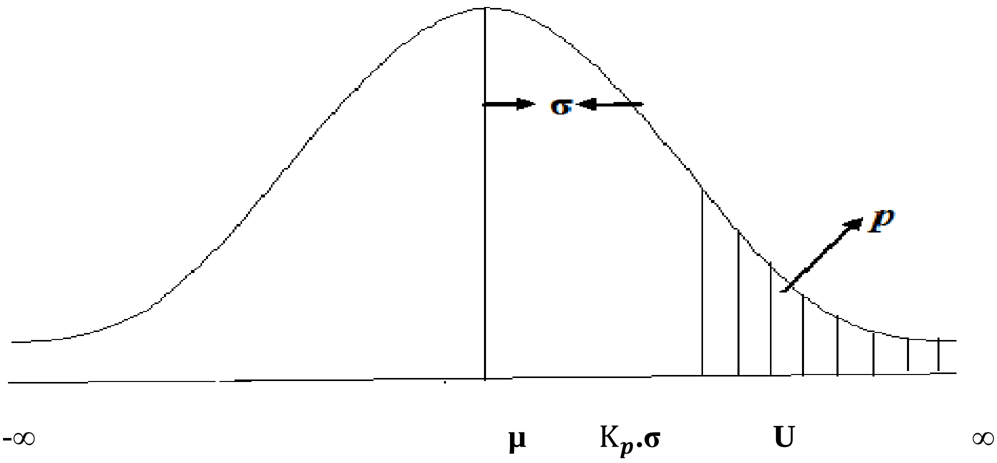


Fig.2.1 Distribution of X

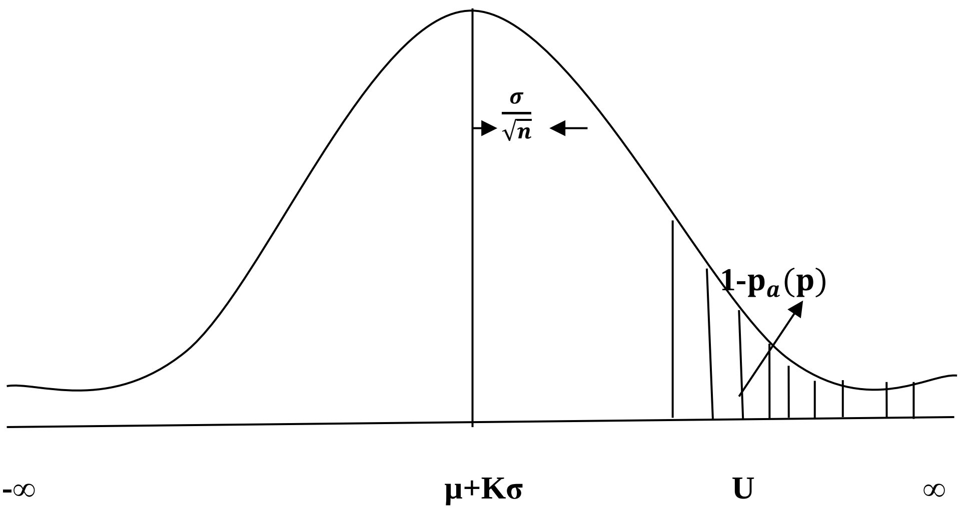


Fig.2.2 Sampling Distribution of $\overline{X}$+kσ

From fig 2.1

$$\frac{U-\mu}{\sigma} = K_p \qquad (2.1)$$

The mean, $\bar{x}$ is distributed normally with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$, where n is the sample size. This implies that $\bar{x} + k\sigma$ follows normal distribution and standard deviation $\frac{\sigma}{\sqrt{n}}$. The distribution of $\bar{x} + k\sigma$ is shown in fig. 2. 2.

From the sampling distribution of $\bar{x} + k\sigma$, one can obtain the probability of acceptance, $P_a(p)$ from fig .2.2

$$P_a(p) = P(\bar{x} + k\sigma \leq U)$$

therefore $1 - P_a(p) = P(\bar{x} + k\sigma > U)$

Also $$K_{-P_a(p)} \frac{\sigma}{\sqrt{n}} = U - (\mu - k\sigma)$$

$$K_{-P_a(p)} = -K_{P_a(p)}$$

Thus $$-K_{P_a(p)} \frac{\sigma}{\sqrt{n}} = U - \mu + k\sigma$$

Simplifying further one obtains,

$$K_p = k - \frac{1}{\sqrt{n}} K_{P_a(p)} \qquad (2.2)$$

Equation (2.2) provides $P_a(p)$ for any given p and vice-versa.

Using this equation one may

i. obtain the desired plan for specified points on Operating Characteristic curve and

ii. compute various points on the OC curve of the desired sampling plan

Determination of n and k for given AQL and LQL

One can obtain the plan parameters n and k of a known variable sampling plan for two points on the Operating characteristic curve $(p_1, 1-\alpha)$ and (p_2, β) by using the relation (2.2) as

$$K_1 = k - \frac{1}{\sqrt{n}} K_{1-\alpha} \qquad (2.3)$$

$$K_2 = k - \frac{1}{\sqrt{n}} K_\beta \qquad (2.4)$$

where K_1 is the value of K_p when $p = p_1$

threrfore $$p_1 = \int_{k_1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du \qquad (2.5)$$

K_2 is the value of K_p when $p = p_2$

Therefore $$p_2 = \int_{k_2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du \qquad (2.6)$$

Equations (2.3) and (2.4) can be written as,

$$K_1 = k - \frac{1}{\sqrt{n}} K_\alpha \qquad (2.7)$$

$$K_2 = k - \frac{1}{\sqrt{n}} K_\beta \qquad (2.8)$$

where K_α and K_β are given by

$$\alpha = \int_{k_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du \qquad (2.9)$$

$$\beta = \int_{K_\beta}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du \qquad (2.10)$$

Multiplying (2.5) by K_β , (2.6) by K_α and adding one get

$$k = \frac{K_1 K_\beta + K_2 K_\alpha}{K_\alpha + K_\beta} \qquad (2.11)$$

subtracting (2.6) from (2.5), one obtains

$$n = \frac{(K_\alpha + K_\beta)^2}{(K_1 + K_2)^2} \qquad (2.12)$$

Significance of the Values of k and n

The choice of the two parameters k and n determine the variable single sampling plan completely. In the known sigma variable sampling plan one may observe from (2.11) and (2.12) that

i. increase in (K_α+K_β) increases n,
ii. given the indifference quality $p = p_0$, the relation (2.2) becomes,

$$K_0 = k - \frac{1}{\sqrt{n}} K_{0.5} \text{ , when } \alpha \text{ is known}$$

But $K_{0.5} = 0$ from normal tables. Thus in this case $k = k_0$ where k_0 is the value of K_p at $p = p_0$. This shows that k is the independent of the sample size n at the indifference quality.

Wallis (1958) approximation is used in obtaining sample size n_s for unknown sigma plans are derived by multiplying corresponding known sigma sample plans n_σ with the factor $(1 + (k^2/2))$. Therefore $n_s = n_\sigma(1 + (k^2/2))$.

In general, n fixes the steepness of the operating characteristic curve and k fixes the location of operating characteristic curve.

Selection of Variable Sampling Plan for given AQL and LQL

Table 2.1 to 2.9 provide the sampling plan parameters namely acceptance criteria, k sample size, n_σ for known standard deviation plan, n_s for unknown standard deviation plan for $\alpha = 0.05, 0.01, 0.10$; $\beta = 0.5, 0.01, 010$ for various values of p_1 and p_2 such that $p_1 < p_2$.

From the numerical values obtained for various p_1 , p_2 with α and β presented in Tables 2.1to 2.9 one could observe that increase in p_2 decreases n_σ and n_s.

These tables can be utilized to select plans for parameters for the desired discrimination. For example, to determine variable sampling plan to the specified values $p_1 = 0.02$, $p_2 = 0.07$, $\alpha = 0.05$, $\beta = 0.10$ from Table 2.2 one gets $n_\sigma = 24.59$, $k = 1.734$ and $n_s = 62$.

From the Table 2.3 one gets $n_\sigma = 42.57$,$k = 1.917$ and $n_s = 121$ for the specified values $p_1 = 0.015$, $p_2 = 0.06$,$\alpha = 0.05$,$\beta = 0.01$,

From the Table 2.6 one gets $n_\sigma = 28.33$,$k = 2.08$ and $n_s = 35$ for specified values $p_1 = 0.01$, $p_2 = 0.05$,$\alpha = 0.10$,$\beta = 0.01$,

From the Table 2.9 one gets $n_\sigma = 56.49$,$k = 1.75$ and $n_s = 143$ for the specified values $p_1 = 0.2$, $p_2 = 0.075$,$\alpha = 0.01$,$\beta = 0.01$,

Method of Construction of Tables

Table 2.1 to 2.9 are constructed to obtain the acceptance constant k, sample size for standard deviation known n_σ for single sampling variable plan. These tables are indexed by the combination of producer's quality level with α values (0.01.0.05 , 0.10) and the consumer's quality level with β values (0.01,0.05 , 0.10). Plans are derived using

$$k = \frac{K_1 K_\beta + K_2 K_\alpha}{K_\alpha + K_\beta}$$

$$n_\sigma = \frac{(K_\alpha + K_\beta)^2}{(K_1 + K_2)^2}$$

Wallis (1958) approximation is used in obtaining sample size , n_s for unknown sigma plans by multiplying corresponding n_σ with the factor $\left(1 + \frac{k^2}{2}\right)$

where K_1 – area of p_1 in upper tail of normal curve

K_2 – area of p_2 in upper tail of normal curve

K_α – area of α in upper tail of normal curve

K_β – area of β in upper tail of normal curve

TABLE 2.1 Variable Single Sampling Plan Parameters indexed by AQL and LQL

p_1	p_2	$\alpha = 0.05$, $\beta = 0.05$				
		K	n_σ	Rounded n_σ	n_s	Rounded n_s
0.01	0.015	2.25	425.39	425	1502.15	1502
0.01	0.02	2.195	149.38	149	509.23	509
0.01	0.03	2.11	56.25	56	181.46	181
0.01	0.04	2.045	33.518	34	103.6	104
0.01	0.05	1	23.55	24	35	35
.015	0.02	2.115	900	900	2912.9	2913
.015	0.03	2.03	138.90	139	425	424
.015	0.04	1.965	64.78	65	189.8	190
.015	0.05	1.91	40.27	40	113.7	114
.015	0.06	1.865	29.26	29	80.14	80
0.02	0.03	1.975	376.81	377	1111.7	1111
0.02	0.04	1.91	121	121	341.7	342
0.02	0.05	1.855	64.78	65	176.23	176
0.02	0.07	1.77	32.372	32	83	83
0.02	0.075	1.75	28.329	28	71.7	72
0.04	0.045	1.73	302.5	303	755.17	755
0.04	0.05	1.705	900	900	2208.16	2208
0.04	0.07	1.62	138.9	139	321.16	321
0.04	0.08	1.585	88.897	89	200.56	201
0.04	0.09	1.555	64.78	65	143	143
0.05	0.06	1.605	1344	1344	3075	3075
0.05	0.07	1.565	376.8	377	838.23	838
0.05	0.08	1.53	189.06	189	410.34	410
0.05	0.09	1.5	121	121	257	257
0.05	0.1	1.47	84.02	84	174.79	175

TABLE 2.2 Variable Single Sampling Plan Parameters indexed by AQL and LQL

p_1	p_2	$\alpha = 0.05$, $\beta = 0.10$				
		K	n_σ	Rounded n_σ	n_s	Rounded n_s
0.01	0.015	2.240	337.6	338	1184.7	1185
0.01	0.02	2.2009	118.5	119	405.7	406
0.01	0.03	2.0325	44.65	45	136.8	137
0.01	0.04	2.01	26.60	27	80.33	80
0.01	0.05	1.735	18.69	19	46.82	47
.015	0.02	2.1082	714.3	714	2301.7	2302
.015	0.03	1.9623	110.25	111	322.51	323
.015	0.04	1.939	51.41	51	148	148
.015	0.05	1.878	31.96	32	88.3	88
.015	0.06	1.827	23.229	23	61.99	62
0.02	0.03	1.914	127.86	128	362.06	362
0.02	0.04	1.891	96.04	96	267.75	268
0.02	0.05	1.829	51.418	51	137.42	137
0.02	0.07	1.734	24.59	25	61.55	62
0.02	0.075	1.712	22.48	22	55.4	55
0.04	0.045	1.7263	2401	2401	5977	5977
0.04	0.05	1.698	714.34	714	1744	1744
0.04	0.07	1.602	110.25	110	251.7	252
0.04	0.08	1.5635	70.56	71	156.8	157
0.04	0.09	1.529	51.419	51	111.5	112
0.05	0.06	1.599	1067	1067	2431	2431
0.05	0.07	1.554	299.08	299	660	660
0.05	0.08	1.515	150.06	150	322	322
0.05	0.09	1.481	96.04	96	201	201
0.05	0.1	1.447	66.69	67	136.5	137

TABLE 2.3 Variable Single Sampling Plan Parameters indexed by AQL and LQL

p_1	p_2	$\alpha = 0.05$, $\beta = 0.01$				
		K	n_σ	Rounded n_σ	n_s	Rounded n_s
0.01	0.015	2.263	618.76	619	2203.1	2203
0.01	0.02	2.175	217.28	217	731.2	731
0.01	0.03	2.147	81.82	82	270.39	270
0.01	0.04	2.09	48.75	49	155	155
0.01	0.05	2.048	34.25	34	106	106
.015	0.02	2.124	1309.1	1309	4261.6	4262
.015	0.03	2.053	202.04	202	627	627
.015	0.04	2.00	94.23	94	282	282
.015	0.05	1.9544	58.58	59	170	170
.015	0.06	1.917	42.57	43	120.7	121
0.02	0.03	1.989	548.1	548	1631.9	1632
0.02	0.04	1.935	176.00	176	505	505
0.02	0.05	1.89	94.23	94	262.5	263
0.02	0.07	1.819	47.08	47	124.9	125
0.02	0.075	1.802	41.20	41	108	108
0.04	0.045	1.735	4400.1	4400	11022	11022
0.04	0.05	1.714	1309.1	1309	3231.7	3232
0.04	0.07	1.643	202.04	202	474.7	475
0.04	0.08	1.614	129.3	129	297.7	298
0.04	0.09	1.59	94.23	94	213.34	213
0.05	0.06	1.612	1955.6	1956	4496	4496
0.05	0.07	1.579	548.11	548	1231	1231
0.05	0.08	1.55	275	275	605.3	605
0.05	0.09	1.525	176	176	380.6	381
0.05	0.1	1.50	122.22	122	259.7	260

TABLE 2.4 Variable Single Sampling Plan Parameters indexed AQL and LQL

p_1	p_2	$\alpha = 0.05$, $\beta = 0.05$				
		K	n_σ	Rounded n_σ	n_s	Rounded n_s
0.01	0.015	2.259	337.64	338	1199	1199
0.01	0.02	2.211	118.56	119	408.35	408
0.01	0.03	2.136	44.64	45	146.47	146
0.01	0.04	2.079	26.6	27	84	84
0.01	0.05	2.031	18.69	19	57.23	57
.015	0.02	2.12	714.34	714	2319.6	2320
.015	0.03	2.047	110.25	110	341.23	341
.015	0.04	1.99	51.419	51	153.23	153
.015	0.05	1.94	31.96	32	92.10	92
.015	0.06	1.902	23.22	23	65.22	65
0.02	0.03	1.985	299.08	299	888.30	888
0.02	0.04	1.928	96.04	96	274.53	275
0.02	0.05	1.88	51.41	51	142.26	142
0.02	0.07	1.80	25.69	26	67.30	67
0.02	0.075	1.787	22.48	22	58.37	58
0.04	0.045	1.733	2401	2401	6006	6006
0.04	0.05	1.711	714.34	714	1759.9	1760
0.04	0.07	1.637	110.25	110	257.97	258
0.04	0.08	1.60	70.56	71	160.8	161
0.04	0.09	1.58	51.41	51	115.57	116
0.05	0.06	1.61	1067	1067	2449.8	2450
0.05	0.07	1.575	299.08	299	670	670
0.05	0.08	1.544	150.06	150	328.92	329
0.05	0.09	1.518	96.04	96	206.69	207
0.05	0.1	1.492	66.69	67	140.91	141

TABLE 2.5 Variable Single Sampling Plan Parameters indexed by AQL and LQL

p_1	p_2	$\alpha = 0.10$, $\beta = 0.10$				
		K	n_σ	Rounded n_σ	n_s	Rounded n_s
0.01	0.015	2.25	260.6	261	920.31	920
0.01	0.02	2.2	91.31	91	312.28	312
0.01	0.03	2.11	34.38	34	110.91	111
0.01	0.04	2.05	20.49	20	63.54	64
0.01	0.05	1.99	14.4	14	42.9	43
.015	0.02	2.115	550.1	550	1780.4	1780
.015	0.03	2.013	84.9	85	256.9	257
.015	0.04	1.965	39.59	40	116.0	116
.015	0.05	1.91	24.61	25	69.51	70
.015	0.06	1.865	17.88	18	48.9	49
0.02	0.03	1.975	230.3	230	679.51	680
0.02	0.04	1.91	73.96	74	208.8	209
0.02	0.05	1.855	39.59	40	107.7	108
0.02	0.07	1.77	19.78	20	50.76	51
0.02	0.075	1.75	17.31	17	43.81	44
0.04	0.045	1.73	1849	1849	4615.3	4615
0.04	0.05	1.70	550.1	550	1345	1345
0.04	0.07	1.62	84.9	85	196.3	196
0.04	0.08	1.58	53.49	53	120.25	120
0.04	0.09	1.555	39.59	40	87.45	87
0.05	0.06	1.605	809.0	809	1851.1	1851
0.05	0.07	1.565	226.7	227	504.45	504
0.05	0.08	1.53	113.7	114	246.93	247
0.05	0.09	1.5	72.82	73	154.7	155
0.05	0.1	1.47	50.56	51	105.18	105

TABLE 2.6 Variable Single Sampling Plan Parameters indexed by AQL and LQL

p_1	p_2	$\alpha = 0.10$, $\beta = 0.01$				
		K	n_σ	Rounded n_σ	n_s	Rounded n_s
0.01	0.015	2.272	511.89	512	1502.15	1502
0.01	0.02	2.233	179.75	180	509.23	509
0.01	0.03	2.17	67.66	68	181.46	181
0.01	0.04	2.126	40.33	40	103.6	104
0.01	0.05	2.08	28.33	28	35	35
.015	0.02	2.11	1083	1083	2912.9	2913
.015	0.03	2.07	167.14	167	425	424
.015	0.04	2.02	77.95	78	189.8	190
.015	0.05	1.984	48.46	48	113.7	114
.015	0.06	1.952	35.21	35	80.14	80
0.02	0.03	1.999	453.43	453	1111.7	1111
0.02	0.04	1.953	145.60	146	341.7	342
0.02	0.05	1.913	77.95	78	176.23	176
0.02	0.07	1.853	38.95	39	83	83
0.02	0.075	1.839	34.09	34	71.7	72
0.04	0.045	1.738	3640.1	3640	755.17	755
0.04	0.05	1.72	1083	1083	2208.16	2208
0.04	0.07	1.66	167.14	167	321.16	321
0.04	0.08	1.635	106.97	107	200.56	201
0.04	0.09	1.613	77.95	78	143	143
0.05	0.06	1.617	1617.8	1618	3075	3075
0.05	0.07	1.589	453.43	453	838.23	838
0.05	0.08	1.564	227.50	228	410.34	410
0.05	0.09	1.543	145.06	145	257	257
0.05	0.1	1.521	101.1	101	174.79	175

TABLE 2.7 Variable Single Sampling Plan Parameters indexed by AQL and LQL

p_1	p_2	$\alpha = 0.10$, $\beta = 0.05$				
		K	n_σ	Rounded n_σ	n_s	Rounded n_s
0.01	0.015	2.236	618.76	619	2165.5	2166
0.01	0.02	2.171	217.28	217	729.32	729
0.01	0.03	2.07	81.82	82	257.11	257
0.01	0.04	1.99	48.75	49	145	145
0.01	0.05	1.93	34.25	34	98	98
.015	0.02	2.105	1309.1	1309	4209	4209
.015	0.03	2.006	202.04	202	608	608
.015	0.04	1.929	94.23	94	269.54	270
.015	0.05	1.865	58.58	59	160	160
.015	0.06	1.812	42.57	43	112	112
0.02	0.03	1.96	54.811	55	160	160
0.02	0.04	1.884	176	176	488	488
0.02	0.05	1.819	94.23	94	250	250
0.02	0.07	1.720	47.08	47	116.7	117
0.02	0.075	1.697	41.208	41	100.5	101
0.04	0.045	1.724	4400.1	4400	10938	10938
0.04	0.05	1.695	1309.1	1309	3189.6	3190
0.04	0.07	1.596	202.04	202	459.35	459
0.04	0.08	1.555	129.3	129	285.6	285
0.04	0.09	1.519	94.23	94	202.9	203
0.05	0.06	1.597	1955.6	1956	4449	4449
0.05	0.07	1.550	548.11	548	1206.5	1207
0.05	0.08	1.509	275	275	588	588
0.05	0.09	1.474	176	176	367	367
0.05	0.1	1.439	122.22	122	248.76	249

TABLE 2.8 Variable Single Sampling Plan Parameters indexed by AQL and LQL

p_1	p_2	$\alpha = 0.01$, $\beta = 0.10$				
		K	n_σ	Rounded n_σ	n_s	Rounded n_s
0.01	0.015	2.227	511.89	512	1781	1781
0.01	0.02	2.156	179.75	180	597.5	598
0.01	0.03	2.04	67.68	68	208.5	209
0.01	0.04	1.963	40.33	40	118.03	118
0.01	0.05	1.892	28.33	28	79	79
.015	0.02	2.099	453.43	453	1452.2	1452
.015	0.03	1.989	145.60	146	433.6	434
.015	0.04	1.906	77.95	78	219.5	220
.015	0.05	1.835	38.95	39	104.5	105
.015	0.06	1.777	35.21	35	90.8	91
0.02	0.03	1.950	453.43	453	1315.5	1316
0.02	0.04	1.866	145.60	146	399.08	399
0.02	0.05	1.796	77.95	78	203.66	204
0.02	0.07	1.686	38.95	39	94	94
0.02	0.075	1.660	34.09	34	83	83
0.04	0.045	1.721	3640.1	3640	9030.5	9031
0.04	0.05	1.689	1083.0	1083	2627.7	2628
0.04	0.07	1.579	167.14	167	375.5	376
0.04	0.08	1.534	106.97	107	232.8	233
0.04	0.09	1.496	77.95	78	165	165
0.05	0.06	1.592	1617.8	1618	3667.9	3668
0.05	0.07	1.540	453.43	453	991	991
0.05	0.08	1.495	227.5	228	481.7	482
0.05	0.09	1.456	145.60	146	299.9	300
0.05	0.1	1.418	101.11	101	202.7	203

TABLE 2.9 Variable Single Sampling Plan Parameters indexed by AQL and LQL

p_1	p_2	$\alpha = 0.01$, $\beta = 0.01$				
		K	n_σ	**Rounded** n_σ	n_s	**Rounded** n_s
0.01	0.015	2.25	848.26	848	2995	2995
0.01	0.02	2.195	297.88	298	1015.47	1015
0.01	0.03	2.11	112.16	112	361.8	362
0.01	0.04	2.045	66.83	67	206.5	207
0.01	0.05	1.99	46.96	47	139.9	140
.015	0.02	2.115	1794.6	1795	5808	5808
.015	0.03	2.03	276.98	277	847.6	848
.015	0.04	1.965	129.18	129	378.57	379
.015	0.05	1.91	80.30	80	226.77	227
.015	0.06	1.865	58.35	58	159.8	160
0.02	0.03	1.975	751.40	751	2216.8	2217
0.02	0.04	1.91	241.28	241	681	681
0.02	0.05	1.855	129.18	129	351	351
0.02	0.07	1.77	64.55	65	165.66	166
0.02	0.075	1.75	56.49	56	142.99	143
0.04	0.045	1.73	6032.1	6032	15058.5	15059
0.04	0.05	1.705	1794.6	1795	4403	4403
0.04	0.07	1.62	276.98	277	640	640
0.04	0.08	1.585	177.27	177	399.9	340
0.04	0.09	1.555	129.18	129	285	285
0.05	0.06	1.605	2680.9	2681	6133.9	6134
0.05	0.07	1.565	751.40	751	1671.5	1672
0.05	0.08	1.53	377.00	377	818	818
0.05	0.09	1.5	241.28	241	512.7	513
0.05	0.1	1.47	167.55	168	348.5	349

CHAPTER III

Analysis of Operating Characteristic Values

This chapter presents the properties of operating characteristic function of acceptance sampling plans with reference to its parameters and quality indices.

The effect of acceptance number, sampling size and fixed ratio of sample size to acceptance number on the operating characteristic values are discussed for single sampling plan with reference to attributes. Also the trend of operating characteristic values of single sampling plans having identical AQL, RQL and IQL values are examined.

For variable single sampling plans the effect of parameters on the OC function are analyzed by considering fixed n with varying k and fixed k with varying n. A comparison is provided to study the efficiency of matched attribute single sampling plan and variable ingle sampling plan

Operating characteristic function plays a major role in the theory of sampling inspection. The two main aspects for evaluation of the effect of sample size and acceptance number on operating characteristic functions are to estimate

i. the dependence of operating characteristic function on the sample size and

ii. the dependence of operating characteristic function on the acceptance number.

Operating characteristic function for attribute single sampling plan may be derived using hyper geometric distribution, binomial distribution and Poisson distribution. The Hypergeomtric distribution is fundamental t acceptance sampling. It is applicable when sampling an attribute characteristic from finite lot without replacement. This model is exact for the case of nonconforming units for isolated lots of smaller size.

Undoubtedly the most used distribution in acceptance sampling is the binomial. It complements the hyper geometric in the characteristic from an infinite lot or from a lot when sampling is done with replacement.

This model is exact for the case of nonconforming units whenever $n / N \leq 0.10$, where n and N are respectively the sample and lot size. Poisson model is exact for the case of non-conformities when $n / N \leq 0.01$, sample size is large and p is small such that np is finite.

The simplest of three functions is based on Poisson distribution which is applied to study the effectiveness of sampling plans. The probability of acceptance depends only on c and λ where λ= np which is Poisson probabilities depends only on λ, It does not matter whether the expected number of defects per sample is occurrence of the product of a small sample size and large proportion of occurrence of defects or a large sample size and small proportion of occurrence of defects.

The OC function under Poisson model is defined by

$$P_a(p) = \sum_{r=o}^{c} e^{-np} \frac{(np)^r}{r!}$$

$$= \sum_{r=o}^{c} e^{-\lambda} \frac{(\lambda)^r}{r!}, \ np = \lambda$$

Where n is sample size and p is the process average.

The following aspects are considered for study of operating characteristics values.

i. Effect of acceptance number with fixed ample size
ii. Effect of sample size with fixed acceptance number to sample size
iii. Effect of fixed ratio of acceptance number to sample size
iv. Effect of fixed AQL
v. Effect of fixed RQL
vi. Effect of fixed IQL

Effect of Acceptance Number with Fixed Sample Size

The acceptance probability depends only on c and λ where λ = np. Table 3.1 presents the probability of acceptance obtained by using expression (3.1) with excel worksheet relating to single sampling plan (50,c) for acceptance number c =0,1,2 as a function of λ. The operating characteristic curves corresponding to these single sampling plans are shown in fig.3.1. The following salient features may be observed

i. Probability of acceptance is an increasing function of c
ii. Probability of acceptance is a decreasing function of λ

Effect of Sample Size with Fixed Acceptance Number

The effect of sample size on operating characteristic function is studied with various n values. Probability for a single sampling plan may be considered s a function of λ for a given value of acceptance number. Table 3.2 presents the probability of acceptance corresponding to single sampling plan (n, 2) for n = 50,100, 200 for various fraction defective. Probability acceptance values are worksheet for the specified parameters with equation (3.1). The operating characteristic curves for single sampling plans (n, 2) for n = 50, 100, 200 are shown in fig.3.2. The following salient features may be seen from the trend of operating characteristics curves.

i. Operating characteristic is a decreasing function of n for fixed c
ii. Operating characteristics is a decreasing function of p for any specified n and c and the point of inflexion may be seen at $p = c / n$.

TABLE 3. 1 OC Values of Single Sampling Plan (50,c)

p	C		
	0	1	2
0.005	0.7788	0.9735	0.9978
0.010	0.6065	0.9097	0.9856
0.020	0.3678	0.7357	0.9195
0.030	0.2231	0.5578	0.8088
0.040	0.1353	0.4060	0.6766
0.050	0.0821	0.2872	0.5438
0.060	0.0497	0.1991	0.4231
0.070	0.0301	0.1358	0.3208
0.080	0.0183	0.0915	0.2380
0.090	0.0111	0.0610	0.1735

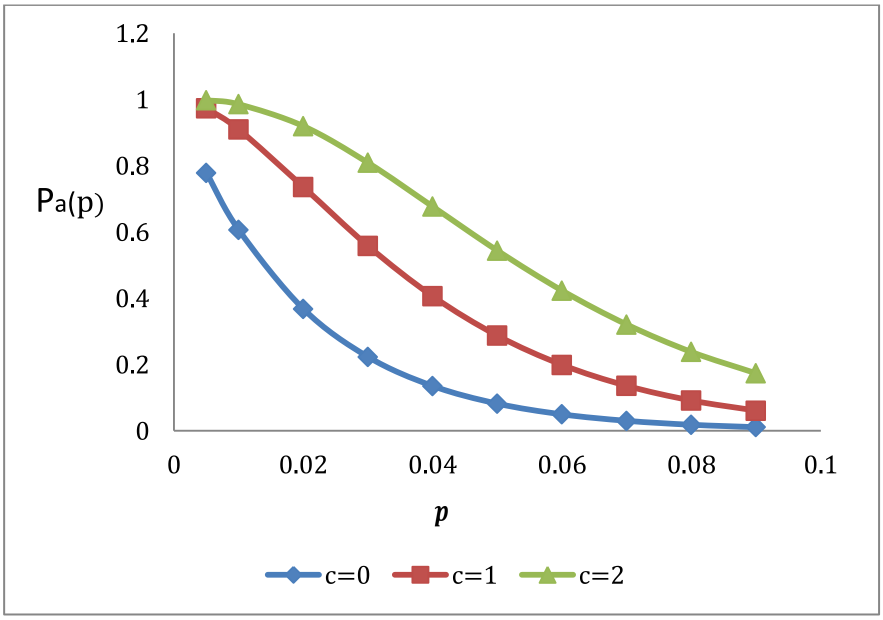


Fig 3.1 OC curves of Single Sampling Plans with fixed n=50

TABLE 3.2 OC VALUES OF SINGLE SAMPLING PLAN (n ,2)

p	N		
	50	100	200
0.005	0.9978	0.9856	0.91967
0.010	0.9856	0.9196	0.67667
0.020	0.9195	0.6766	0.23810
0.030	0.8088	0.4231	0.06199
0.040	0.6767	0.2380	0.0137
0.050	0.5438	0.1245	0.0027
0.060	0.4231	0.0619	0.00052
0.070	0.3208	0.0296	0.00009
0.080	0.2380	0.0137	0.000016
0.090	0.1735	0.090	0.000002

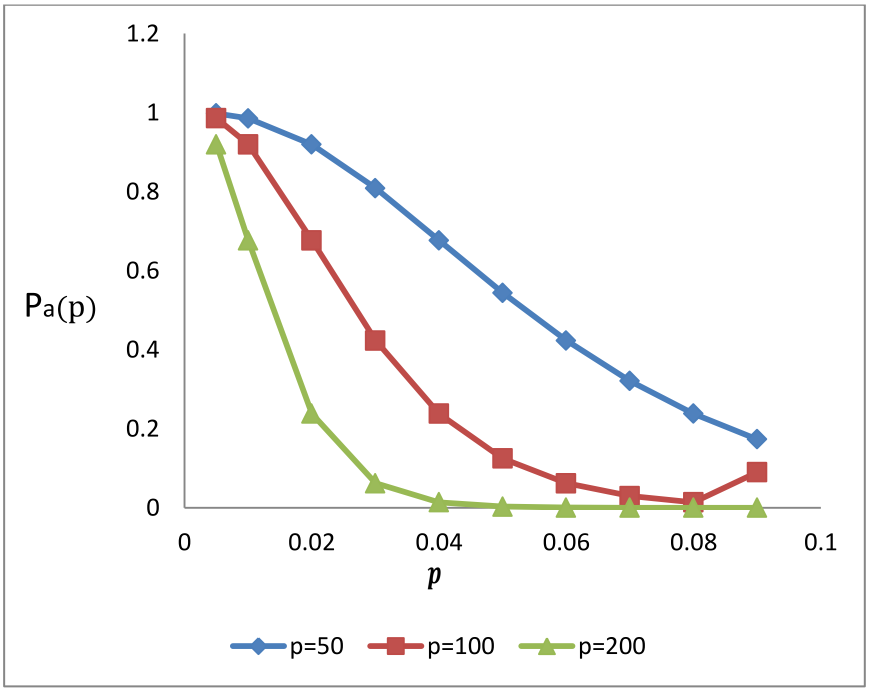


Fig 3.2 OC curves of Single Sampling Plans with fixed c = 2

Effect of Fixed Ratio of Acceptance Number and Sample Size

The effect of fixed ratio of sample size and acceptance number , $h = c / n$ on OC values are studied in this part.

The probability of acceptance for single sampling plans having the fixed ratio of acceptance number to the sample size as $h = 0.04$ are computed and presented in Table 3.3 . The operating characteristic curves of these sampling plans are given in fig.3.3.

For fixed h, fig. 3.3 shows that

i. the operating characteristic values increase for lesser p values and decreases for higher p values and
ii. generally for a fixed c and n the operating characteristic values increase with decrease in p

Effect of same AQL on Operating Characteristic Function

Table 3.4 presents the probabilities of acceptance for certain selected plans having the same AQL which means that the probability of acceptance corresponding to this proportion defective, p is 0.95 for all single sampling plans considered.

Fig.3.4. shows that the operating characteristic curves of single sampling plans with same AQL. These curves indicate that

$$n = \lambda\,(p^{-1} - 1/2\,) + (c/2)$$

TABLE 3.3 OC Values of Singe Sampling Plans with fixed ratio of n and c

p	n=25c=1	n=75,c=3	n=100,c=4
0.005	0.992809	0.999388	0.9998278
0.01	0.973501	0.992707	0.9963401
0.02	0.909796	0.934357	0.9473469
0.03	0.826641	0.809433	0.8152632
0.04	0.735759	0.647231	0.6288369
0.05	0.644636	0.483767	0.4404932
0.06	0.557825	0.342296	0.2850565
0.07	0.477878	0.231669	0.1729916
0.08	0.406006	0.151204	0.0996324
0.09	0.342547	0.095765	0.0549636

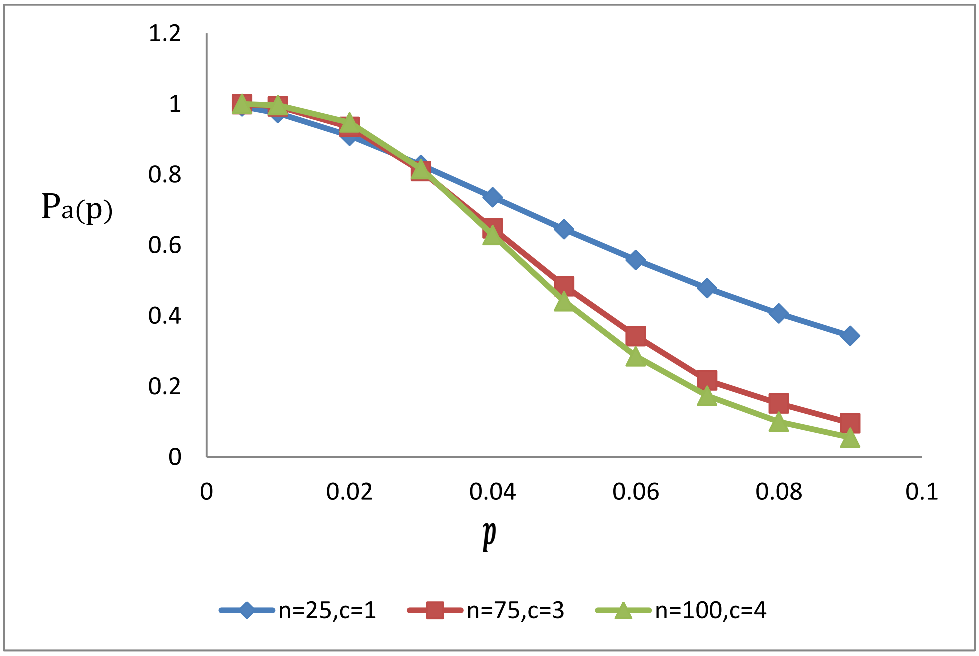


Fig 3.3 OC values of Single Sampling Plans with fixed ratio of n and c

TABLE 3.4 OC Values of single sampling plans with same AQL

p	n=30, c=1	n=78, c=3	n=201, c=7
0.001	0.999559	0.999999	1
0.002	0.998270	0.999978	1
0.003	0.996185	0.999896	0.999999
0.004	0.993351	0.999692	0.999997
0.005	0.989814	0.999293	0.999989
0.01	0.963064	0.991667	0.998868
0.02	0.878097	0.926603	0.947666
0.03	0.772482	0.791166	0.739839
0.04	0.662627	0.620368	0.447391
0.05	0.557825	0.453248	0.215750
0.06	0.462837	0.312845	0.086916
0.07	0.379615	0.206273	0.030423
0.08	0.308441	0.131038	0.009530
0.09	0.248660	0.080792	0.002731

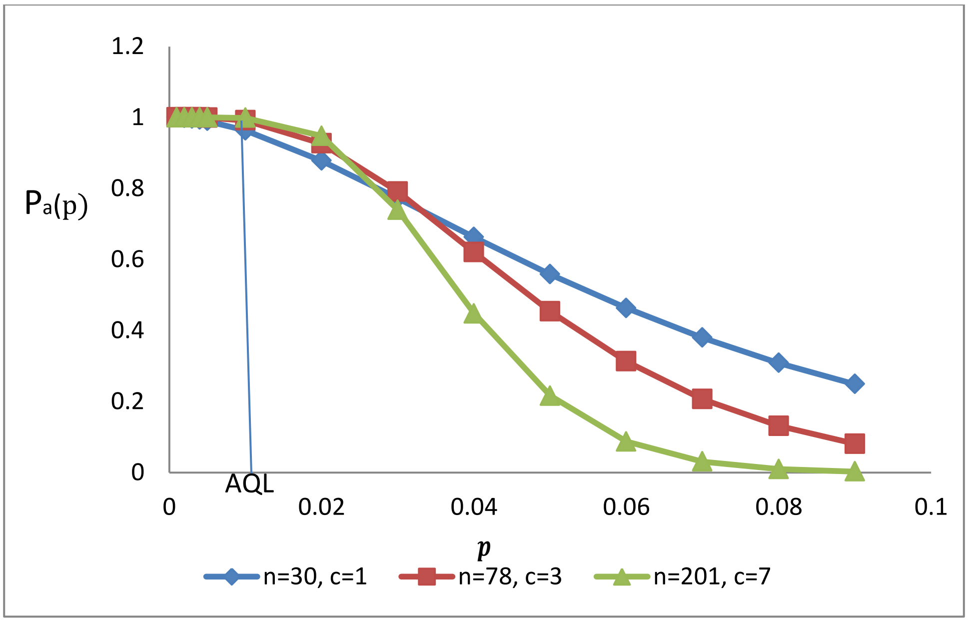


Fig.3.4 OC curves of Single Sampling Plans with same AQL

Effect of same RQL on Operating Characteristic Function

Table 3.5 presents the probability of acceptance for certain selected plan having the same RQL which means that the probability of acceptance corresponding to this proportion defective p, is 0.10 for all single sampling plans considered.

Fig.3.5. shows the operating characteristic curves of single sampling plans with same RQL which implies that probability of acceptance of single sampling plans at RQL is 0.10 for all the plans selected. These curves indicate that

$$n = \lambda \left(p^{-1} - 1/2\right) + (c/2)$$

Effect of same IQL on Operating Characteristic Function

Table 3.6 presents the probabilities of acceptance for certain selected plans which are having the same IQL which means that the probability of acceptance corresponding to this proportion defective p is 0.5 for all single sampling plans considered.

Fig.3.6. shows that the operating characteristic curves of single sampling plans having the same IQL. These curves

i. indicate the relation between c and n as (n +(1/3)) IQL = c+ (2/3)

ii. Show the probability of acceptance decreases for the proportion defective < IQL and increase for the proportion defectives > IQL for any n and c

TABLE 3.5 OC Values of Single Sampling plans with same RQL

p	n=18,c=1	n=69,c=3	n=115,c=15
0.001	0.999332	0.999999	1
0.002	0.997393	0.999989	1
0.003	0.994277	0.999948	1
0.004	0.990071	0.999845	1
0.005	0.984859	0.999641	1
0.01	0.946306	0.995552	0.999973
0.02	0.830178	0.956905	0.997411
0.03	0.695369	0.866031	0.975141
0.04	0.564541	0.736002	0.904949
0.05	0.448126	0.591408	0.777623
0.06	0.349721	0.453247	0.613611
0.07	0.269322	0.33393	0.446004
0.08	0.205203	0.238065	0.301
0.09	0.154984	0.165099	0.190333

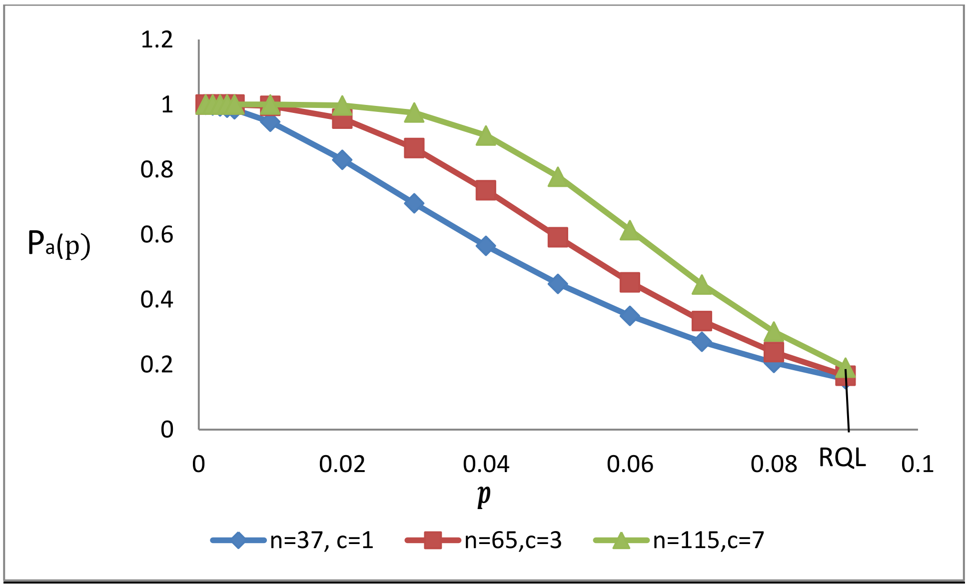


Fig.3.5 OC curves of Single Sampling Plans with same RQL

TABLE 3.6 OC Values of Single sampling Plans with same IQL

p	n=34,c=1	n=75,c=3	n=155,c=5
0.001	0.999467	0.999999	1
0.002	0.997790	0.999981	1
0.003	0.995138	0.999107	1
0.004	0.991549	0.999734	1
0.005	0.987087	0.999388	0.999999
0.01	0.953772	0.992707	0.999806
0.02	0.851116	0.934357	0.986747
0.03	0.728401	0.809433	0.905816
0.04	0.605719	0.647232	0.72730
0.05	0.493245	0.483767	0.502786
0.06	0.395287	0.342296	0.303242
0.07	0.312820	0.231668	0.162925
0.08	0.245054	0.151204	0.079532
0.09	0.190364	0.095765	0.035857

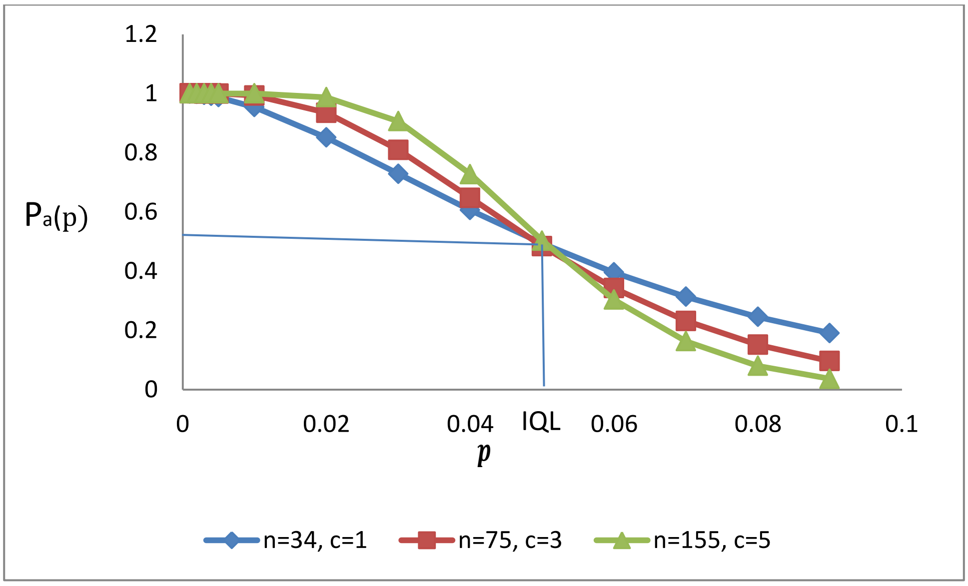


Fig. 3.6 OC curves of Single Sampling Plans with same IQL

Operating Characteristic function for Variable Single sampling plan is derived using Normal Distribution. A random variable is said to have normal distribution, if is probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)} \qquad -\infty < x < \infty$$

where mean= μ

variance= σ^2

It is assumed that the individual measurements of the submitted lot follow normal distribution with mean μ and standard deviation σ . Products which are defectives, have the quality characteristic X > U. The proportional area to the right of the value U of the normal curve is equal to p with given,

$$\frac{U-\mu}{\sigma} = K_p \qquad (3.1)$$

The mean $\overline{X}$ is distributed normally with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$, where n is the sample size.

The implies that, $\overline{X} \sim N(\mu, \sigma^2/n)$

From the sampling plan distribution of $\overline{X} + k\sigma$, one can obtain the probability of acceptance, $P_a(p)$

$$P_a(p) = P(\overline{X} + k\sigma \leq U)$$

therefore $$1 - P_a(p) = P(\overline{X} + k\sigma \leq U)$$

Also $$K_{-P_a(p)} \frac{\sigma}{\sqrt{n}} = U - (\mu - k\sigma)$$

But $K_{-P_a(p)} = -K_{P_a(p)}$ in normal distribution

Therefore $$K_{-P_a(p)} \frac{\sigma}{\sqrt{n}} = U - \mu + k\sigma$$

Simplifying further with (3.1) one obtains,

$$K_P = k - \frac{1}{\sqrt{n}} K_{P_a(p)} \qquad (3.2)$$

Equation (3.2) provides $P_a(p)$ for any given p and vice- versa. Using this equation one can

i. design the plan for desired points on operating characteristic curve.

ii. Compute various points on the OC curve of the desired sampling plan.

Using the normal distribution calculation of OC curves for variable sampling plans may be performed, when the standard deviation is known.

Computation of OC Values with Upper Specification Limit

For any given value of p, the probability of acceptance can be determined as follows for an upper specification limit.

i. determine k_P from p
ii. obtain $K_{P_a(p)} = k_P - k$
iii. convert $K_{P_a(p)}$ to the distribution of sample means as $P_a = \sqrt{n}K_{P_a(p)}$
iv. $P_a(p)$ is the probability of acceptance.

Computation of OC Values with Lower Specification Limit

i. Determine k_P
ii. obtain $K_{P_a(p)} = k_P + k$
iii. convert to the distribution means as $P_a(p) = \sqrt{n}K_{P_a(p)}$
iv. the probability of normal variant equal to or exceeding $P_a(p)$ is the probability of acceptance. Its complement $P_a(p)$,is the probability of rejection.

The relation (3.2) is used to compute the probability of acceptance for various proportion defective. The probability of acceptance values obtained through excel work sheet by using normal probability values corresponding to the given p.

Effect of Acceptance Constant with Fixed Sample Size

Table 3.7 presents the probability of acceptance obtained for Variable Single Sampling Plans with fixed n and acceptance constant k = 1, 1.5, 2. the OC curves corresponding to these Variable Single Sampling Plans are shown in fig 3.7.

From OC values of Variable Single Sampling Plans one observes

i. OC values decrease for increase in k
ii. OC values decrease for increase in p

Effect of Sample Size with Fixed Acceptance Constant

OC values corresponding to various fraction defective are computed for variable single sampling plans (n, 1.55) for n = 5, 10, 12 are given in table 3.8. the OC curves for Variable Single Sampling plans (n, 1.55) for n = 5, 10, 12 are shown in fig 3.8

OC curves of variable singe sampling plans reveal the following facts

i. OC values decrease for increase in p
ii. OC values decrease for increase in n for inferior quality
iii. OC values increase for increase in n for superior quality

TABLE 3.7 OC Values of Variable Single Sampling Plans with known sigma and fixed n = 20

p	OC values		
	k=1	**k=1.5**	**k=2**
0.0050	0.99945	0.99680	0.97548
0.01	0.99543	0.98670	0.87340
0.015	0.89342	0.86439	0.82098
0.02	0.85320	0.82650	0.78651
0.025	0.78492	0.7409	0.71383
0.05	0.65987	0.6289	0.5932
0.075	0.57932	0.54086	0.49703
0.09	0.5145	0.47320	0.44628
0.01	0.47659	0.45812	0.39326
0.12	0.37542	0.34621	0.31960
0.15	0.29540	0.26931	0.24720
0.175	0.1834	0.16480	0.16189
0.2	0.1209	0.0835	0.07450
0.25	0.05490	0.0162	0.0494
0.275	0.0238	0.00439	0.00104
0.3	0.00421	0.00075	0.0005

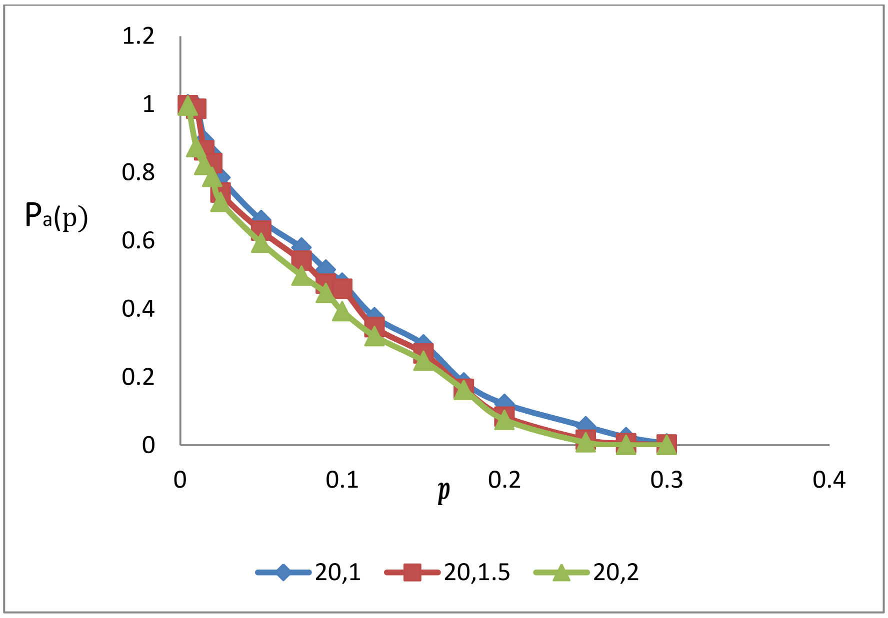


Fig 3.7 OC curves of Variable Single Sampling Plans with fixed n=20

TABLE 3.8 OC Values of Variable Single Sampling Plans with known sigma and fixed k = 2

p	OC values		
	n=5	n=10	n=12
0.0050	0.9014	0.96638	0.97725
0.01	0.86214	0.93943	0.95449
0.015	0.85993	0.93699	0.95352
0.02	0.78524	0.86864	0.89065
0.025	0.70000	0.78283	0.83444
0.04	0.60320	0.71382	0.75721
0.05	0.53317	0.62420	0.67340
0.06	0.4943	0.57340	0.63421
0.075	0.423	0.5233	0.56598
0.09	0.3723	0.4503	0.49352
0.1	0.33143	0.4231	0.4713
0.12	0.2914	0.37142	0.4103
0.15	0.2342	0.32154	0.3654
0.175	0.1949	0.2734	0.3254
0.2	0.16087	0.21634	0.28431
0.25	0.1035	0.1534	0.19350
0.3	0.056	0.1134	0.14340
0.325	0.0193	0.08253	0.12430
0.35	0.0034	0.06781	0.1043
0.375	0.001	0.0345	0.06254
0.4	0.00067	0.00453	0.0524

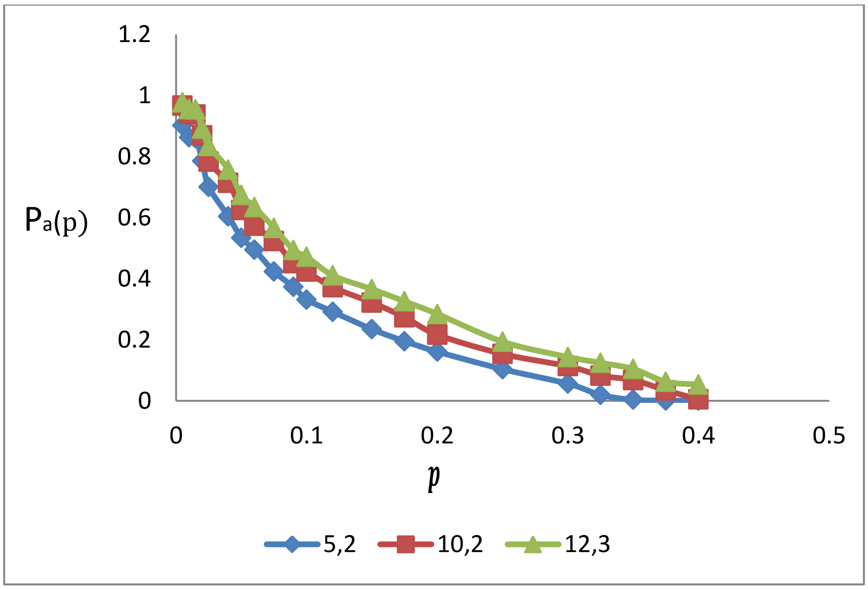


Fig 3.8 OC curves of Variable Single Sampling Plans with fixed k=2.

SUMMARY AND CONCLUSION

In this dissertation attribute single sampling plan and variable single sampling plan are analysed.

Important definitions are presented under Basic Concepts.

In chapter I designing of attribute single sampling plan is explained in detail.

In chapter II designing variable single sampling plan is discussed in detail. Properties of OC function with respect to parameter and quality indices along with comparison are presented in chapter III. At the end a bibliography is added.

Recommendations for future study

i. Cost model using quadratic loss function may be designed.
ii. Attribute single sampling plan and variable single sampling plan may be designed to accommodate the process with non-constant proportion of defective.
iii. Attribute single sampling plan and variable single sampling plan may be designed in the presence of inspection error.

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